THE ASTROPHYSICAL JOURNAL

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An International Review of Spectroscopy and Astronomical Physics

VOLUME XXV

JANUARY—JUNE, 1907

CHICAGO
The University of Chicago Press
1907

THE ASTROPHYSICAL JOURNAL

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXV JANUARY 1907 NUMBER 1

AN INVESTIGATION OF THE INFLUENCE OF ELECTRICAL FIELDS UPON SPECTRAL LINES

By G. HULL

In the autumn of 1905 the writer began an investigation in the Cavendish Laboratory, Cambridge University, upon the influence which electrical fields producing luminous discharges might have upon the spectral lines comprising the luminosity concomitant with those discharges. The particular points toward which attention was to be directed were whether any variation of wave-length (Doppler effect) occurred when the direction of viewing the discharge was changed from that along the discharge to one at right angles to it; whether the width of the spectral line altered when the part of the luminous column used as a source was changed; whether any polarization of the radiation from the luminous column was produced by the discharge; whether Roentgen rays in their passage through the luminous gas produced any change of wave-length, widening of the lines, or polarization of the light.

Looking at these questions from the experimental view-point, evidence could be adduced favoring either positive or negative results. The extremely accurate work of Michelson, and later of Pérot and Fabry, made it clear that no Doppler effects were to be expected in ordinary end-on discharge tubes. On the other hand, the well-known Feddersen experiment, in which the oscillations of a condenser discharging between two spheres in air at ordinary pressure are

made evident by means of a rotating mirror, leads to the usual explanation that the curving of the images of the spark is due to luminous particles traveling out from the spark terminals. Indeed, Schuster and Hemsalech made an extended study of this curvature for the individual lines composing the discharge when various substances were used for the spark-gap. Their conclusions were that in general luminous metallic particles from the electrodes were driven out along the spark-gap, with various velocities reaching a value as high as 2000 meters per second. These experiments made by Schuster and Hemsalech certainly suggest that a Doppler effect will be found in electrical discharges in air between metallic electrodes. But it will be shown here that another interpretation may be given to their results.

The investigation will be discussed here under the following heads:

1. Doppler effects for discharges in air.

2. Doppler effects in end-on discharge tubes between the electrodes.

3. Change of wave-length, widening of the lines, or polarization of the light due to Roentgen rays.

4. Doppler effects in end-on discharge tubes behind a perforated electrode. (Canal rays.)

5. Polarization of the radiation from the luminous column.

6. Influence of the electrical field upon spectral lines.

Instruments.—At the time when this experiment was started the chief instrument available was an echelon spectroscope by Hilger, kindly loaned to the writer by Professor Liveing. The echelon prism consisted of seventeen plates (eighteen steps of 1 mm each), 7.5 mm each in thickness. The dispersive power was equivalent to that of a grating of 9,600 lines to the millimeter, or sixteen times as great as that of the ordinary grating, and the resolving power to that of a grating of 180,000 lines.

The telescope and collimator lenses had an aperture of 5 cm and a focal length of 40 cm. The telescope carried a plate-holder which could be rotated about a vertical axis so that a considerable part of the spectrum could be brought into focus at one time.

The mounting of the instrument was arranged as in the diagram (Fig. 1). Usually the source of light studied was placed directly in front of the slit S of the collimator. The radiations after passing

through the echelon prism *E* were analyzed by a flint glass prism *F*, of 60° angle, so that in the focal plane of *T* there appeared what looked like the ordinary line-spectrum of the source, except that some of the lines might be double and might have a few satellites. These apparently double lines of course were different orders of the same line, their angular distance apart being λ/e, where e was the width of the step. The linear distance between the orders on the photographic plate was approximately 0.2 mm. Except for quite complex sources, there was no difficulty in identifying the lines. This simple method of separating the lines is greatly to be preferred to that of

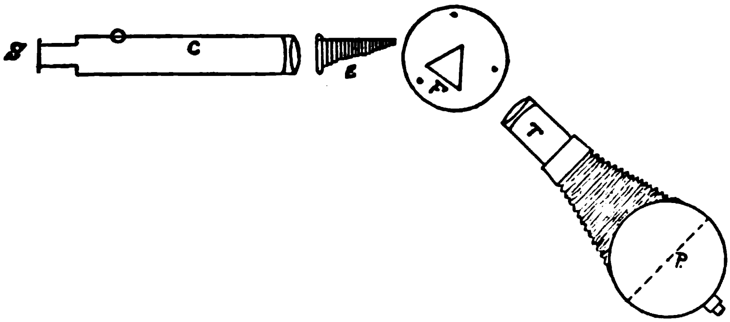


FIG. 1

analyzing the light before it enters the collimator, for in the latter method the lines of the spectrum have to be examined or photographed separately. In this method they all can be photographed at once. The strips of the echelon (1 mm in width), the slit of the collimator and the edge of the prism were made vertical.

In front of the slit of the collimator was a movable diaphragm by means of which the upper or lower half of the slit could be exposed to the source. For example, if the Doppler effect in the canal rays of mercury was to be tested, one-half of the slit was exposed to the canal stream directed at about 10° to the axis of the collimator, then the other half of the slit was exposed to an ordinary mercury-spectrum tube. Fig. 2 shows the appearance of a plate (enlarged two diameters; the fine lines have been lost in the process) when canal rays in helium (*a*) and a helium-spectrum tube (*b*) were sources. If any Doppler effect was present, it would be made evident by a shift of the two halves relatively to one another. There might easily be some doubt

regarding the interpretation to be placed upon this shift. For, confining our attention to one radiation, there would be in general on the plate two maxima corresponding to (say) the 8000 and 8001 orders. If the radiation were shortened in wave-length by about 0.5 tenth-meter, the maximum toward the shorter side of the echelon would move over and take the place of the other maximum, while a new order would follow it and take its place. If the radiation were lengthened, the shift would be in the opposite direction. In both cases the two maxima would look like the original two maxima. Thus, though a variation in wave-length of 0.1 tenth-meter would pro-

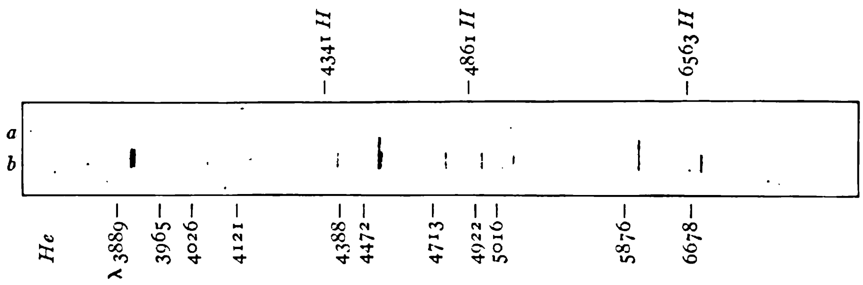


FIG. 2

duce a very marked shift in the lines, one of 0.5 might not cause a noticeable displacement. But shifts as great as 0.2 tenth-meter could be detected and measured by the single prism *F*, the echelon being removed. The method therefore used for determining the order of magnitude and nature of the shift was to remove the echelon and take a photograph of the spectrum through the prism *F*. It is evident that when the echelon is in use the width of the spectral lines used cannot be as great as 0.5 tenth-meter, otherwise the two chief orders would overlap. By width of a line is here meant the wave-length interval between two radiations whose intensities are one-half that of the maximum.

When the echelon and prism were arranged as in Fig. 1, a shift of the lines toward the red end of the spectrum denoted a shortening of those lines.

Adjustment for focus.—The slit and the photographic plate were placed at the foci of their respective objectives by the ordinary method

of parallax. Then through the 60° prism photographs were taken of the upper half of the slit, the right half of the collimator objective being diaphragmed, and of the lower half of the slit with the left half of the lens shadowed. The light source was a mercury, cadmium, or helium discharge tube. The displacements of the two halves of the slit-images were measured for the separate lines, then this process was repeated for a new position of the camera objective, or of the slit or for a new inclination of the photographic plate. At least a dozen plates were taken before a satisfactory adjustment was found. Even then it was not quite possible to bring the upper and lower halves together for all the lines of an extended spectrum. When the echelon was placed in position, it was found that the focus had slightly changed, so that a new position for the camera objective and photographic plate had to be determined.

The echelon prism was adjusted visually. This adjustment was considered satisfactory when the five[1] satellites of the green mercury line were sharp and uniform throughout their length.

An interferometer of the Michelson form (from W. & L. E. Gurley) was also available during these experiments, but of course it could be used only when the light-sources were strong and the radiations simple.

A Rowland concave grating of 10.5 feet radius, 14,438 lines to the inch, 4-inch ruled surface, was also used, but experience proved that for the long exposures required the instrument was affected by temperature-changes and by tremors of its support. The results obtained by this instrument were rejected as unreliable.

1. *Discharges in air.*—As already stated, the Feddersen experiment and the investigation of Schuster and Hemsalech suggest that in electric discharges between metallic terminals in air luminous vapors move along the line of discharge. When the present investigations were begun, the writer did not know of Mohler's paper on "Doppler Effect and Reversal in Spark Spectra."[2] Mohler used Leyden jars of a capacity of 22.5 meters in the secondary circuit of an induction

[1] It is now pretty certain that the large number (twenty) of satellites announced by Lummer and Gehrcke as belonging to the mercury lines were, for the most part, ghosts. The green line λ 5461 has five, or possibly six, satellites.

[2] *Astrophysical Journal*, 15, 125, 1902.

coil operated by a Wehnelt break. The analyzer was a concave grating of 10 feet radius, 14,400 lines to the inch, 4-inch surface. The most noticeable effect observed by Mohler was that, when the spark from magnesium was going away from the slit, the lines $\lambda\lambda$ 2795, 2802, 2856 were reversed. With regard to the Doppler effect the only datum given by Mohler is that the aluminium lines $\lambda\lambda$ 3961, 3944 give an average measured displacement of 0.01 tenth-meter. This shift would be produced by a velocity of 370 meters per second.

The criticism to be made of this result is that the aluminium lines so produced are very broad (probably 2 or 3 tenth-meters)—so broad that it is very doubtful whether a shift of this magnitude could be measured, or if measured whether it might not be due to unsymmetrical broadening.

In considering the most favorable materials for the spark terminals, the writer had chosen an amalgam of mercury and cadmium. These substances give strong, fine lines. Moreover, the velocity of the luminous vapor of mercury, as found by Schuster and Hemsalech, was about 900 meters per second, and of cadmium about 600 meters per second.

Figure 3 shows the first arrangement of the cadmium-mercury and aluminium electrodes. These were held by a hard-rubber plate which could be rotated about an axis passing through the middle of the spark-gap at right angles to the plane of the plate. The line of discharge in general made an angle of 30° with the axis of the collimator. One half of the slit received light directly from the spark, the other half received the light after reflection at the mirrors m_1, m_2 and m_3; the last mirror being a small silvered strip of glass immediately in front of the slit.

If the luminous *Cd-Hg* particles are moving with a velocity v, the difference in wave-length of the two beams through the upper and lower halves of the slit is a $\lambda\frac{2v}{V}\cos 30°$, where V is the velocity of light. If the mercury particles have a velocity of 900 meters per second, the wave-length difference should amount to 0.025 tenth-meters.

Photographing the two halves of the slit simultaneously has the advantage that, if the wave-length of the radiation from the spark

depends upon conditions[1] under which that spark is produced, the simultaneous photographs should eliminate differences in wave-lengths due to those conditions. But this arrangement has a defect. For in using the echelon prism the greatest care must be taken in the alignment of the light beam. Removing the echelon and looking through the objective of the collimator, the electrodes and mirrors may be so adjusted that the object-glass is uniformly illuminated by either source, and thus the alignment is made satisfactory. But a small shift in the light-source, due for example to a movement of the spark along the electrode or to a motion of one of the mirrors, will seriously affect that alignment and may possibly produce a displacement of the two photographic halves of the slit. It would be better then, to allow more freedom in varying conditions. So finally the mirrors were dispensed with entirely. A movable diaphragm allowed either half of the slit to be exposed. The direction of the discharge was changed by rotating the electrodes about the axis at right angles to the spark-gap. Before every set of readings the echelon was removed, and the position of the spark-gap so adjusted that the light beam fell centrally on the objective when either half of the slit was open and for either direction of the discharge. A lens of 5 cm focal length between the electrodes and the slit, and about 5.5 cm from the spark gap, assisted in the alignment and insured light falling upon all parts of the objective of the collimator. The discharge was produced by an induction coil (with a hammer break) capable of giving a 20-cm spark. The spark-gap circuit had in series a small self-induction (0.01 henry). To make sure that the discharge passed in only one direction, a rectifier or electrical valve was included in the spark-gap circuit. The plates were measured by a Zeiss microscope whose micrometer screw was of ½ mm pitch. The distance between two successive orders of a line was taken for various lines, and all other readings were referred to this interval. It happened that tenths of divisions of the micrometer head corresponded nearly to 0.001 tenth-meters. The average error of setting on a good line was 0.002 tenth-meters.

[1] Haschek (*Sitzungsberichte der Kais. der Wiss. in Wien*, 110, 1901) states that the wave-length from a spark depends on the pressure in the spark and also upon the density of the vapor. But later investigations failed to verify his results. See note in *Astrophysical Journal*, 14, 201, 1901.

Although a great number of plates were taken, only those were measured on which the upper and lower images were nearly equal in intensity and were neither over- nor under-exposed. A photograph in which the upper half of the slit had been exposed to the approaching spark and the lower half to the opposite was combined with one for which the conditions were reversed.

FIG. 3

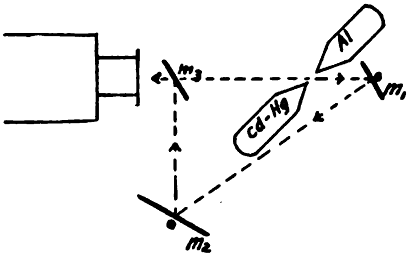
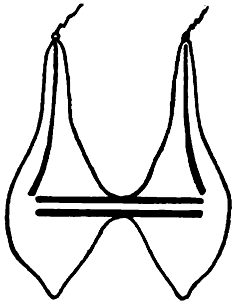


FIG. 4



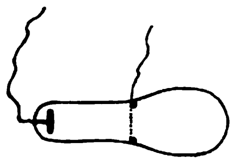


FIG. 5

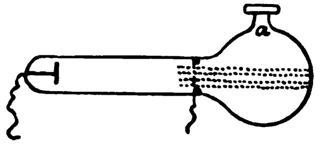


FIG. 6

The mercury and cadmium lines on the plates, making allowance for the width of the slit, which was about 0.02 mm, had a width of between 0.02 and 0.08 tenth-meters. Several of the satellites of the mercury lines appeared on the plates. This gives an indication of their fineness, as does also the fact that interference fringes for a difference of path of 80,000 waves were obtained for the green mercury line.[1]

The following table shows the variations of the readings. A plus sign means that the shift of the upper and lower photographic halves

[1] Nutting, in the *Astrophysical Journal*, **23**, 66, 1906, states that the widths of the lines for sparks in air are never less than 0.5 tenth-meter. This limit is considerably greater than that obtained by the writer.

of the slit was of the same sign as a Doppler effect, and a negative sign that it was opposite to that effect. The readings are in tenth-meters, and are reduced to motion along and at right angles to the discharge.

TABLE I

Line	Plates					Mean
	Nos. 111 and 114	Nos. 112 and 113	Nos. 115 and 116	Nos. 125 and 126	Nos. 127 and 128	
	t.-m.	t.-m.	t.-m.	t.-m.	t.-m.	t.-m.
Cd λ 4800	+0.004	−0.009	+0.005	−0.009	−0.005	−0.002
Cd 4678	+0.003	−0.005	....	−0.010	−0.003	−0.004
Hg 4359	+0.004	−0.001	....	−0.007	+0.004	0.000
Hg 4047	....	....	....	−0.002	+0.007	+0.002
Mean	....	....	....	....	....	−0.001

These data show that for the spark-gap here used, viz., *Cd-Hg* electrodes, 3 mm length of spark-gap, without capacity but with self-induction, there is no motion of the luminous particles as great as 100 meters per second.

When a small capacity was inserted, the lines broadened and were often reversed, so that it was difficult to measure them accurately. The discrepancies were about three times as large as those in the table, and were both positive and negative. Schuster and Hemsalech did not find that the velocity which they measured decreased with the capacity. There was rather more evidence of an increase of velocity with decrease of capacity.

Hence, if we had analyzed our spark by means of a rotating mirror, we should have found that the discharge was curved, just as it is curved when large capacities are joined to the spark-gap. (The feeble illumination of our spark does not readily permit a direct experimental test.) We are therefore led to the conclusion that this curving of the image of the spark is not due to a motion of the luminous particles (for there is no Doppler effect great enough to account for the curving), but to the propagation among those particles of a condition of luminosity.

2. *Discharges in a partial vacuum.*—Discharges in air having failed to give a Doppler effect, motion of the luminous particles was next looked for in vacuum tubes. Here again mercury vapor was chosen on account of the fineness and strength of its lines. The

interferometer and the echelon spectroscope were used to detect the expected change of wave-length.

An end-on tube of the form shown in Fig. 4 was used. The tube connecting the two bulbs was of about 3-mm internal diameter and about 10 cm long. The aluminium electrodes were brought down near the ends of the tube which terminated near the glass walls.

Before sealing off from the pump, exhaustion and sparking were continued until all gases but mercury vapor (as far as could be seen by the spectrum) had been removed. Consequently, at ordinary temperatures the discharge passed with difficulty.

During an experiment the tube rested in an asbestos-covered brass box which could be maintained at a fairly constant temperature up to 250° C. for a considerable time.

Interference fringes for a difference of path of 400,000 waves (this limit was fixed by the shortness of the instrument) were easily found for the green line. These fringes were observed by means of a telescope in the eyepiece of which was a system of cross-hairs. The position of the central fringes could be read to approximately one-tenth of the distance between them. No change could be seen in the fringes when the discharge was reversed. This means that no change of wave-length occurred greater than one part in four million. The velocity of the luminous particles, assuming that it was reversed with the discharge, therefore could not have been greater than 50 meters per second. The results obtained using the echelon prism as analyser were in accord with the interferometer result.

No systematic record of the temperature of the tube was made. Conditions were merely varied from one in which there was a great deal of vapor in the tube to one in which there was very little.

3. *The influence of the passage of Roentgen rays through the luminous gas.*—While the previous part of the investigation was in progress it was thought well to test whether Roentgen rays in their passage through the luminous gas would in any way affect the radiation. The theoretical bearing of this problem is very interesting.

Professor J. J. Thomson represents a cathode ray as a negative corpuscle moving with great velocity and carrying with it lines of force. When this particle is suddenly stopped, a pulse runs out along these lines of electric force. This pulse is the X-ray. It is polarized, its electrical component being in the plane containing

the line of force and the direction of the original motion of the cathode particle. When this pulse is arrested by coming in contact with a new medium, there is generated another radiation the direction of propagation of which is at right angles to the plane just described. These theoretical conclusions have been verified by Barkla[1] in a series of most interesting experiments. We know that this electric pulse or X-ray passing through a gas ionizes it. That is, the electric pulse tears from each of a number of molecules one or more electrons. If the molecule were luminous, this disturbance, amounting to a collision, would affect the phase of the radiation, and would therefore produce a widening of the line. Moreover, if the wave-length depends upon the number of electrons in a molecule, the length of the wave would be changed. Again, this sudden tearing-off of an electron might give rise to an increased radiation in a direction at right angles to that of the pulse. With interference fringes corresponding to a difference of path of 400,000 waves no change of wave-length nor widening of the line was noticeable.

The polarization was looked for with a Nicol prism and sensitive Savart plate, but none was found.

This experiment is of interest in connection with the optical phenomenon of absorption. Ether pulses or X-rays passing through a gas are absorbed to a very small extent, but produce marked ionization. Ether waves (of the order of light-waves) may experience considerable absorption, as is the case for ultra-violet light in iodine vapor, but produce no ionization;[2] the present experiment shows that an increase in the ionization of a gas (for the gas here studied) does not affect its absorption or emission. Apparently absorption of energy is always necessary to produce ionization, but ionization need not accompany absorption. It requires the absorption of a special kind or form of energy, for example that of electric pulses, of short-period light-waves,[3] or of waves specially suited to the gas,[4] to produce ionization.

[1] *Phil. Trans.*, A, **204**, 467–479, 1905.

[2] Henry (*Proc. Camb. Phil. Soc.* **9**, 1897) found that there was no increase in conductivity in iodine vapor when illuminated by ultra-violet light.

[3] P. Lenard (*Ann. der Phys.* (4) **1**, 486; **3**, 289, 1900) found that easily absorbed radiation from an electric spark produced ionization.

[4] Sodium vapor is rendered specially conducting by yellow light, potassium by blue. Elster and Geitel, *Wied. Ann.*, **52**, 433, 1894.

4. *Doppler effects in end-on discharge tubes behind a perforated electrode.*—As discharges through mercury vapor in end-on tubes had produced no Doppler effect, attention was turned to other forms of tubes and other gases, when Stark announced the phenomenon here looked for—the Doppler effect in the canal rays of hydrogen. This gas had not been used at the outset of the present experiments, because the principal lines of hydrogen are so broad as not to lend themselves readily to analysis by the echelon prism. Moreover, it was not expected that an effect would be discovered large enough to be detected by other means at the writer's disposal. But the first test showed that the effect was easily seen when the prism F was used as an analyzer. One half of the slit was illuminated by light from an ordinary hydrogen spectrum tube, the other by the canal rays in hydrogen. An observer at the telescope could clearly see the lines from the canal rays shift from one side to the other of the ordinary hydrogen lines as the canal stream was directed toward or away from the slit. The photographic plates showed that the shift was about 5 tenth-meters, corresponding to a velocity of the luminous particles of about 3×10^7 cm per second.

An attempt was made to obtain the effect for mercury vapor. The tubes used had the form shown in Fig. 5. The distance between the electrodes was about 10 cm, the diameter of the tube about 3 cm. The discharge was produced by an induction coil, the potential ranging from 1,000 to 30,000 volts, though for any one exposure the potential was kept as nearly constant as possible by regulating the temperature of the tube.

The echelon was put in its place, and photographs of the two halves of the slit were taken, one half being illuminated by the ordinary mercury discharge tube, the other half by the canal stream. Great care was taken to have the objective of the collimator full of light. The tube was inclined at an angle of about 20° to the axis of the collimator, and so placed that no light but that from the canal stream passed through the spectroscope.

The first tube used to test the Doppler effect in the mercury canal stream was that which showed that effect for hydrogen, for there were present in the canal stream not only the hydrogen but also the mercury lines. Using the interferometer as analyzer, with fringes

corresponding to a difference of path of 120,000 waves of the green line λ 5461, no variation in wave-length was seen when the canal tube was rotated through 120°. The visibility of the fringes rapidly decreased for a greater path-difference. With interference for this difference of path a change of wave-length of one part in six hundred thousand could be detected. That is to say, the Doppler effect for the mercury particles in that tube could not have been as great as 0.01 tenth-meters, or, considering that the direction had been altered from +60° to −60°, the velocity of the particle could not have been greater than 300 meters per second. The echelon results placed the limit rather lower than this.

The discharges that were used with this tube had been so heavy that the anode melted and became distorted, rendering the tube useless. Another tube, similar in form and dimensions, was prepared. When this tube was sealed off from the pump (pressure about 0.06 mm of mercury), both hydrogen and mercury lines were present, but the former disappeared after a few plates were taken. The discharges used were heavy, the potential being about 4000 volts. The time required for an exposure (slit-width being 0.02 mm, Lumière "Sigma" plates being used) varied from 8 to 15 minutes. With this tube eight plates were taken, two of which showed a shift of about one-fifth of the interval between the two maxima. But the interpretation placed upon this at the time was that the shift was not one-fifth but six-fifths of that interval. The reason for this interpretation was that there were irregularities in the measurements which could not be accounted for if the smaller fraction were correct, but which seemed reasonable if the larger one were chosen. Moreover, the larger value gave a velocity of the mercury particles of 2.5×10^4 meters per second, which value was consistent with that found for the hydrogen particles. It was intended to find the order of the shift by the single prism, but unfortunately before this could be done the tube had hardened considerably, and the conditions had so changed that the plates now showed no certain shift. A test with the interferometer gave fringes (for the green line λ 5461) for a difference of path of 120,000 waves. These fringes were not changed by turning the tube through 120°. Consequently the velocity of the luminous

particles (in the later history of this tube) could not have been greater than 300 meters per second.

Three other tubes were made, one of which contained hydrogen and mercury vapor, the other two containing only mercury vapor as far as any spectroscopic test could determine. The last two tubes required heating before the discharge took place; even then the potential was apt to run high, up to 60,000 volts or more. A number of plates were taken with the canal streams in these tubes as sources, but no shift of the mercury line was found.

At the time these experiments were performed it was thought that the shift obtained in two of the plates was real and that the nterpretation placed upon it was correct. In a preliminary note published in the *Proceedings of the Royal Society*, June 1906, it was stated that the Doppler effect for mercury had been found, but a re-inspection of all the data, and new data obtained by using the canal stream in helium as a source, leads the writer to suspect that the shift obtained on the two plates was accidental. If it was due to a Doppler effect, then that effect is dependent upon conditions which are not evident. Neither increasing the potential or the current nor freeing the tube from gases other than mercury vapor produces the effect.

Since the preliminary note was written the writer has made other attempts to obtain the Doppler effect in mercury vapor, but without success.

Helium next suggested itself as a substance with which to work. In contrast to mercury, it has a low atomic weight. Moreover, it has an increasing spectrum consisting of six series, according to Runge and Paschen, two principal series each with two secondary series. If the Doppler effect differed for lines of different series, certainly that difference should be made evident in the canal rays of helium.

The gas obtained from Tyrer & Co., London, proved to be quite pure. The photographs taken through the prism F with a spectrum tube filled with this gas as a source gave all the lines from λ 7281.8 to λ 3819.8 without a trace of impurity[1]. The canal ray tube was the one which had been used for obtaining the rays in hydrogen.

Table II, column 3, gives the relative intensities of the lines as

[1] After long usage, however hydrogen lines made their appearance.

TABLE II

	λ	Relative Intensity in Plücker Tube	Relative Intensity Canal Discharge	Shift per 1000 Volts
2d Sub *B*	7281.80	1	0.00	
2d Sub *A*	7060.00	1	0.00	
	7065.50	4	0.00	
1st Sub *B*	6678.40	10	0.04	
1st Sub *A*	5876.21	2	0.27	
	5875.87	20	0.27	+0.004 t.-m.
2d Sub *B*	5047.80	2	0.00	
2d Prin. *B*	5015.70	10	0.27	+0.004
1st Sub *B*	4922.00	6	0.14	
2d Sub *A*	4713.48	1	0.11	
	4713.25	8	0.11	+0.004
1st Sub *A*	4471.86	2	0.33	
	4471.65	20	0.33	+0.004
2d Sub *B*	4437.70	2	0.00	
1st Sub *B*	4388.10	8	0.26	+0.005
2d Sub *B*	4169.10	1	0.00	
1st Sub *B*	4143.90	2	0.13	
2d Sub *A*	4121.00	6	0.03	
1st Sub *A*	4026.51	1	0.30	
	4026.34	10	0.30	+0.006
2d Prin. *B*	3964.87	6	0.10	
1st Prin. *A*	3889.00	40	0.17	+0.004
2d Sub *A*	3867.60	1	0.00	
1st Sub *A*	3819.80	2	0.25	

they appeared on the plate (Wratten & Wainwright "pan-chromatic" plates, Plücker tubes as source) with the classification as given by Runge and Paschen. Here the first and second principal series are denoted by *A* and *B* respectively. The fourth column gives the intensity, estimated from seven different plates, of a line in the canal spectrum compared with that of the same line in the ordinary Plücker tube. It is evident from an inspection of this column that the lines of the first sub-series *A* are relatively stronger in the canal ray spectrum than they are in the Plücker tube spectrum. The two principal series are next in order, while the lines of the two second subordinate series are seldom visible in the canal stream. The relative intensities for the various lines, comparing canal rays with Plücker tube, are: first sub-series *A*, 0.30; principal series *A* and *B*, 0.18; first sub-series *B*, 0.13; second sub-series *A*, 0.05; second sub-series *B*, 0.00.[1]

[1] When the direction of the current was changed so that cathode rays in place of the canal stream appeared behind the perforated plate the spectrum changed from yellow to green-blue. The intensities of the lines were completely changed. The red and yellow lines disappeared while λ 5016 came out strong.

The discharge was obtained from an induction coil. A spark-gap, whose terminals were brass spheres one inch in diameter in multiple with the canal ray tube, allowed one to estimate the potential difference between the electrodes.

Two plates were taken with the flint glass prism as analyzer. The dispersion of this prism gave a variation of from 25 to 35 tenth-meters per millimeter. These plates showed that there was no Doppler effect in the helium canal stream as great as 0.1 tenth-meter.

A number of plates were taken with the echelon in place, and of these six were measured. The tube was refilled with helium after plate No. 218 was taken. Table III gives the estimated potential, the mean shift in the direction of the Doppler effect for all the lines on that plate, and the times of exposure for the Plücker and canal ray tubes.

TABLE III

Plate	Potential in Volts	Mean Shift	Time in Minutes
217	9,000	+0.008 t.-m.	6 and 100
218	16,000	+0.006	5 and 150
219	3,000	+0.008	5 and 100
220	7,000	+0.025	6 and 165
222	22,000	+0.003	7 and 150
223	36,000	+0.022	8 and 165

It will be noticed that, assuming these shifts are due to real Doppler effects, there is no apparent connection between the shifts and the potentials. But if it be assumed that the shift is proportional to the square root of the potential, then column 5 of Table II follows. Other lines than those given in the table were measured, but on account of their faintness or fuzziness through poor focus they are not included.

These data indicate that the Doppler effect for the lines in the canal stream of hydrogen is of the order of 4 tenth-meters for a potential of 1,000 volts. Assuming the velocity to vary inversely as the square root of the molecular weight, we deduce from this displacement for the lines in the canal rays of helium a Doppler effect about six hundred times as great as that which is given by this experiment.

It therefore appears from these investigations that the luminous hydrogen particles in the canal stream in that gas have a velocity of

the same order as that deduced by Wien from the electric and magnetic deflections. But the luminous particles of helium and generally those of mercury have velocities very small compared with that which we should expect from the electric experiments.

It follows that the canal particles in helium and mercury either do not have nearly as great a velocity as they are generally supposed to have, or that the *luminous particles* do not take part in the large velocity. In the latter case non-luminous particles must be streaming through the gas with great velocity, producing luminosity in other particles, probably by bombardment, and giving to those particles a fraction of their own velocity. The mass of the non-luminous particle would have to bear to that of the luminous the ratio of the mass of the electron to that of the hydrogen atom to explain the results in the case of helium. But we have no experimental evidence that the canal rays in helium and mercury have a velocity greater than that here determined optically. Experiments are now under way to measure the Doppler effect in the canal stream of other gases, and also to test the velocity by means of the magnetic and electrostatic deflections.

5. *A test of the polarization of the radiation from the canal rays.*—A glass tube (*a*) was sealed on to a canal ray bulb at right angles to the direction of the rays as in Fig. 6. On this tube was sealed a piece of optical glass as free as possible from strain. Outside of this plate of glass was arranged a series of diaphragms which prevented light from the sides of the tube *a* from passing through the analyzer, which was a very sensitive combination of Savart plate and Nicol prism. The canal stream was that in hydrogen and showed the Doppler effect for the lines of that gas. No polarization of the radiation from the canal rays could be detected.

Voigt,[1] from theoretical considerations, predicts a polarization of the light emitted from luminous particles in an electrical field. This polarization should persist during the subsequent motion of the particle in the canal rays.

Stark,[2] in a paper published during the present investigation, claims to have found this polarization in the light emitted by the

[1] *Ann. der Phys.*, (4) **4**, 197, 1901.

[2] *Deutsch. Phys. Gesell. Verh.*, **8**, 6, 1906.

canal rays. He used a Nicol prism as detector, but gives no details regarding precautions taken to eliminate the effects of reflections from the glass surfaces or of transmission through the strained glass forming the walls of the tube.

Although no accurate quantitative measurements were taken of the sensitiveness of the Savart plate and Nicol prism here used, observations on the polarization of the light from the sky near the Sun and on the light reflected at small angles of incidence from black glass surfaces, taken with a knowledge of the sensitiveness of other Savart plates, showed that the detector here used would show a partial polarization of 0.5 per cent. of the total light. Incandescent lamps and electric discharges in all the tubes used and in all parts of the tubes showed strong polarization when viewed through this analyzer. In other words, this analyzer must have been forty or fifty times as sensitive as the Nicol prism used by Stark. But the canal stream in our case showed no polarization. The writer therefore feels that the effect has not been observed, and that the various theories which have been constructed upon it as an observed phenomenon must be re-examined.

6. *The influence of the electrical field upon spectral lines.*—The theoretical deductions regarding the electrical analogue of the Zeeman effect do not lead us to expect that such an effect may be observed. Voigt (*loc. cit.*), for example, concludes that when a luminous vapor in an electrical field of 300 volts per cm is viewed at right angles to the lines of force, the spectral lines will be widened by one two-thousandth part of the distance between the D lines, or by 0.0003 tenth-meter—a quantity which is outside the limit of observation.

From another point of view, however, we may expect an effect. According to Larmor,[1] the motion of a source through the ether may affect the intrinsic free period of the radiant vibrations, the amount depending on the square of the ratio of the translatory velocity to the velocity of radiation. In all ordinary cases this effect would be negligible, but when the velocity of translation approaches that of the canal rays in hydrogen (5×10^5 meters per second), the square of the ratio would give a wave-length change of the order of 0.015 tenth-meter—a quantity which may readily be observed.

The light produced by the electrical discharge in uniform tubes

[1] *Aether and Matter*, p. 46.

3 or 4 cm in diameter was examined perpendicularly to the direction of discharge at various points between the electrodes and also behind the perforated cathode. Thus the electrical field varied from zero, behind the electrodes, to its highest value (about 500 volts per cm) near the cathode. These tubes gave the Doppler effect for the hydrogen lines, but not for those of mercury. About thirty different plates were taken, the echelon prism being the analyzer, the times of exposure varying from 30 minutes to 3 hours. A comparison hydrogen and mercury tube was used.

The data may be gathered together as follows. The hydrogen lines $H\beta$, $H\gamma$, $H\delta$, have widths in the cathode layer, the canal stream, and the positive column of 0.5, 0.4, 0.12 tenth-meter. They are not shifted in any of these cases more than 0.1 tenth-meter. This is an upper limit determined by the use of the single prism *F*. The amount of shift, if any (on account of the width of the hydrogen lines), could not be measured by means of the echelon prism. The lines of the second hydrogen spectrum were not broadened, but appeared to be shifted both in the cathode layer and in the canal rays by 0.01 tenth-meter toward the blue. The lines of the mercury spectrum λλ 5461, 4359, 4047 were not shifted; they might have been slightly broadened. It should be noted that the lines of the second hydrogen spectrum were very faint in the canal stream, so that only the strongest lines appeared on the plate, viz.: λλ 4642 (?), 4639 (?);[1] 4586, 4581, 4574, 4503, 4223, 4213, 4205, 4176.

Attempts were made to photograph the spectrum of helium using the luminous layer covering the cathode as source, but all such attempts failed owing to the very heavy discharge of the metal (aluminium) of the cathode on the tube. The canal rays showed no broadening of the lines. But the velocity of the luminous particle in the canal stream of helium as found by the writer is very small, so that no broadening was to be expected on account of the motion of the luminous mass centers.

Since the broadening of the principal hydrogen lines is found

[1] The lines λλ 4642, 4639 appeared in the spectra of the hydrogen comparison tubes loaned by Professor Liveing and prepared by him with the greatest care; also in the tubes prepared by the writer. If they are oxygen lines, they are the only lines of that element in the spectra used.

behind the cathode where the electric field is very small, that result cannot be attributed to the electrical analogue of the Zeeman effect, unless the change of period persists after the particles escape from the influence of the electrical field in front of the cathode.

Nor does it fall in completely with the prediction made by Larmor, whose theory requires a shift and only a small broadening.

Nor does the theory recently advanced by Professor Thomson[1] account for the effect. That theory accounts for the widening by considering the influence which vibrating electrified corpuscles have upon one another. The widening so obtained is proportional to or increases with the density of the synchronously vibrating corpuscles. But the density of the luminous particles in the canal rays cannot be nearly as great as it is in the ordinary discharge tubes. Consequently, on that theory the lines in the latter case should be the wider. This is contrary to fact.

We may, however, account for the widening here obtained in the following way. The gas in the neighborhood of the cathode is in part the source of the cathode rays. Let us consider, then, an atom with a negative corpuscle vibrating about it, and suppose that this atom ejected a negative corpuscle with great velocity; there would be a new differential motion of the atom and its satellite. In some cases this new relative motion would be tangential to the satellite's orbit. In consequence, the major axis of the orbit would in some cases be increased and in other cases decreased. If a represents the semi-major axis of the orbit, v the tangential velocity, and μ a constant, then $\delta a = \frac{2a^2}{\mu} v\delta v$, where δa and δv are the variations in major axis and tangential velocity. From $T = 2\pi\sqrt{\frac{a^3}{\mu}}$ the relative change in period

$$\frac{dT}{T} = \frac{3a}{\mu} v \cdot \delta v \,.$$

$$= \frac{3}{4} \frac{T^2}{\pi a^2} v \cdot \delta v \,.$$

[1] J. J. Thomson, "A Theory of the Widening of the Lines in Spectra," *Proc. Camb. Phil. Soc.*, 13, 318, 1906.

Substituting the approximate values of T, a, and v, 10^{-15}, 10^{-8}, and 10^8, we find that

$$\frac{\delta T}{T} = \pm 10^{-6} \delta v \, .$$

Since the value of $\frac{\delta T}{T}$ found in the case of the hydrogen lines is of the order of 10^{-4}, we find that $\delta v = 100$ cm. That is to say, if the explosion which takes place when the negative electron is ejected as a cathode corpuscle is sufficient to cause a relative tangential velocity of the nucleus and electron vibrating about it as great as 100 cm per second, the radiation emitted by a group of particles would be rendered less homogeneous to the extent here found. The atoms which have thrown off a negative corpuscle are also those which will be set in motion by the electric field, so that, if luminous, they will exhibit the Doppler effect. On the contrary, if they do not show this effect, they should not on this theory show broadening. The lines in the canal rays of mercury and helium fall in with this test.

In review of the topics treated in this paper, it may be noted that for discharges in air, for the luminous column between the electrodes of end-on discharge tubes, or for the canal rays, the only certain effect obtained has been a Doppler effect and widening of the lines in the canal rays in hydrogen, while for the influence of the electrical field upon spectral lines the only effect has been found to be a widening of the hydrogen lines. It looks as though these phenomena were connected in the manner indicated in this paper. The other results are all negative.

Roentgen rays in their passage through a luminous gas do not affect the radiations in any (as yet) measurable way, nor does a strong electric field impose upon luminous particles any measurable polarization.

These experiments indicate that hydrogen molecules easily acquire a positive charge (lose a negative electron), while the molecules of helium acquire this charge with difficulty. Does not this lead us to favor the view that the alpha particles given out by radioactive substances are molecules of hydrogen rather than of helium, as held by Rutherford? The radioactive phenomena, however, all favor helium.

The writer's thanks are due to Professor Liveing for the loan of apparatus, and to Professor Thomson for the facilities accorded to him. In general, he wishes here to record his tribute to the admirable spirit of comradeship in research which is found in Cambridge, and especially in the Cavendish Laboratory.

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ON THE RADIATION OF CANAL RAYS IN HYDROGEN[1]

By J. STARK

PART I. CARRIERS OF LINE SPECTRA

§ 1. *Introductory.*—As the Zeeman effect indicates both by its magnitude and by its sign, the *centers* of emission for line-spectra are the negative electrons. These form one part of a distinct system of material or electrical centers; and it is this system which we propose to call the "carrier" (*Träger*) of the line spectrum.

In a gas rendered luminous by electrical means, there are, besides neutral atoms, positive atoms—i. e., atoms which, during the process of ionization, have lost one or more negative electrons. It is in these positive atoms (*Atomionen*) that one may expect to find the carriers[2] of the line spectra.

A glow-discharge (*Glimmstrom*) was established in the open air between two electrodes placed in the same vertical line, the anode above, the cathode below. Into the middle of the discharge was introduced a bead of a lithium salt, when it was observed that the red color of the lithium traveled downward toward the cathode, in spite of the convection currents in the gas tending to drive it upward. This result[3] may be explained in various ways upon the assumption that the positive lithium ions are transported by the current toward the cathode. First, one may suppose that the lithium ion is not merely the carrier of the positive charge, but is at the same time the carrier of the line spectrum. Or, secondly, it might be that the light is emitted by the neutral lithium atoms which are formed by the reunion of positive ions and negative electrons at points between the bead of salt and the cathode. In view of the experiments described in this paper, the first explanation appears to be the correct one.

Imagine an electric arc established between mercury electrodes in a vacuum tube to which has been fused a condensing chamber. A

[1] Translated from advanced proofs of an article to appear in the *Annalen der Physik*, communicated by the author.

[2] J. Stark, *Die Elektrizität in Gasen*, p. 447, 1902.

[3] E. Riecke and J. Stark, *Physikalische Zeitschrift*, 5, 357, 1904,

jet of luminous mercury vapor will then flow into the condensing chamber from the path of the electric current. If now an electric field be passed through this vapor, ending upon a secondary electrode, then, before this field of force is made, the jet of vapor above the cathode yields the line spectrum of mercury; but as soon as the field is established the line spectrum of mercury disappears from the region above the cathode. This result[1] can also be explained in different ways. First, it is possible that the positive mercury ions are the carriers of the line spectrum; so that, when these ions are dragged out of any region to the cathode, the line spectrum goes with them. Or, secondly, it may be that the line spectrum is emitted by the neutral atoms which are excited by collision with negative electrons; the electric field drives these out of the stream of vapor to the cathode; and therefore in the region above the cathode the previously observed emission disappears. According to the results presented in this paper, it is the first of these explanations which is the correct one.

In order to test these hypotheses concerning the carriers of line spectra, one might proceed as follows.[2] According to the researches of W. Wien, canal rays are always partly composed of positive ions moving with high velocities. If, now, they are at the same time emitting spectral lines, the positions of these lines must change, according to Doppler's principle, when the direction along which they are observed changes in reference to the direction of translation in the canal rays. This phenomenon is the subject of the present paper. Preliminary reports[3] on my earlier observations have already been published. In the meantime I have extended and improved upon these; but unfortunately my work has been interrupted by change of residence from Göttingen to Hannover, and it has been impossible for me to substitute, for the tentative results in Parts II and III, exact quantitative measurements. Nevertheless, I now publish this account, in the hope that someone more favorably situated than I will take up these experiments and carry them through.

§ 2. *Method of the experiment.*—The laboratory had at its dis-

[1] J. Stark, *Annalen der Physik*, **14**, 520, 1904.

[2] J. Stark, *Die Elektrizität in Gasen*, p. 457, 1902.

[3] J. Stark, *Physikalische Zeitschrift*, **6**, 892, 1905.

posal four high-tension batteries and one high-tension dynamo; these could be joined up in series so as to yield anywhere from 1,300 to 9,000 volts. When still higher voltages were needed, say from 10,000 to 60,000 volts, a large induction coil with a mercury turbine interrupter was used. With the arrangement of electrodes and with the low pressures which here had to be employed, the negative glow always extended up to the anode. Therefore the cathode-drop (defined as the potential difference between the cathode and the negative glow) is always equal to the voltage between the electrodes; these quantities were measured by means of Braun electrometers, reading from 0 to 3,000 and from 0 to 10,000 volts. This was the mode of working in all cases in which a constant and direct voltage was used. During the entire time of exposure the cathode-drop was maintained to within 15 per cent. of a constant value. If at any time the cathode-drop decreased by more than 15 per cent., the circuit was opened and the tube pumped out until the desired higher value was obtained. If, on the other hand, the vacuum became too high in consequence of the electric discharge, some additional gas was admitted until the cathode-drop again assumed its normal value. The strength of the current was measured by a Deprez ammeter.

The vacuum tubes employed were cylindrical in shape and from 3 to 6 cm in diameter. The cathode was arranged as follows (see Fig. 2). A strip of sheet aluminium, 1 or 2 cm wide, was bent into a cylinder which would just fit into the vacuum tube. Across one end of this cylinder was riveted a circular aluminium disk which was perforated as densely as possible with holes of ½ to 1 mm in diameter. At the other end of the cylinder a platinum wire was attached, as shown in Fig. 2, and then passed through the side of the tube, where it served as one electrode. For anode was used an aluminium disk completely filling the cross-section of the tube. It was not permissible to make the anode small or to place it in a side tube; for if the cathode rays, instead of falling upon the metallic anode, struck the end of the tube, the glass became so hot that the atmospheric pressure was sufficient to crush it. With heavier currents, especially in case the induction coil was used, the aluminium anode, like the anti-cathode in Röntgen bulbs,

was so highly heated that it disintegrated (*zerstäubte*) faster than the cathode.

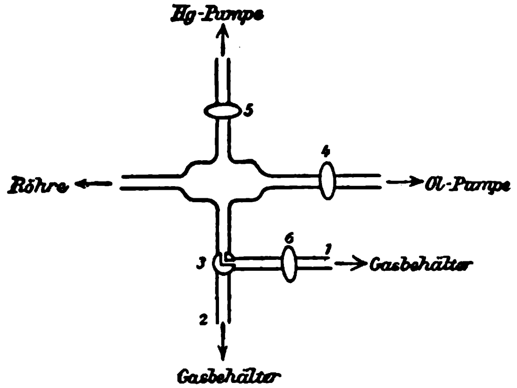


Fig. 1

The distance between anode and cathode was always chosen so large that the Crookes dark space did not reach the anode; for if this were not the case, then the current would cease altogether or the discharge become irregular. Accordingly this distance was always greater than 10 cm and generally amounted to 25 cm. In the observations on the Doppler effect with mercury lines it reached 40 cm.

That portion of the vacuum tube which lay behind the cathode and into which the canal rays passed varied in length, according to circumstances, between 4 and 20 cm.

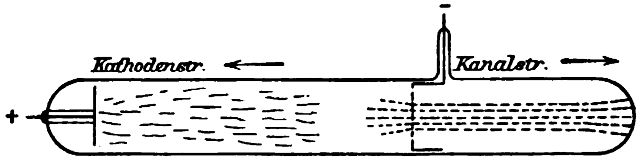


Fig. 2

As is well known, the cathode-drop increases as the gas-pressure diminishes, and simultaneously the length of the Crookes dark space increases; this length may therefore serve as a rough measure of the cathode-drop. This method was employed in the case of the induction coil, since it here became impracticable to use the electrometer; that is, the current was interrupted as soon as the length of the dark space fell below a certain definite limit; then gas was pumped out until on again closing the circuit the dark space assumed a sufficient length.

The exhaustion of the tubes was accomplished by means of two air pumps of the oil type, one yielding 200 cc, the other 400 cc per stroke. The pumps were electrically driven, were carefully kept dry, and so were capable of producing in a short time a vacuum of 0.01 mm. In some experiments as, for instance, that with mercury, even lower pressures were desirable in order to obtain a high value for the cathode-drop. For this purpose a mercury pump was used in conjunction with the larger oil pump. Fig. 1 shows how the two pumps, the gas reservoir, and the tube were connected. At first, cock No. 4 was open, cock No. 5 closed, and the tube exhausted to the limit of the oil-pump; then No. 4 was closed, No. 5 opened, and the mercury-pump set into operation.

The method of introducing the gas will also be seen from Fig. 1. No. 3 was a three-way stopcock, one opening of which was closed with wax, so that the remaining opening formed an elbow. If a large amount of gas was to be admitted, the reservoir was attached to opening No. 1, and cock 3 was set so as to be closed against 1 and 2; then 6 was opened so that the portion of the tube lying between 3 and 6 would fill with gas; next cock 6 was closed and cock 3 set so as to admit to the tube the portion of gas just entrapped. If, however, it was desired to admit only a small quantity of gas, then the reservoir was attached to the opening marked 2; cock 3 was so set as to be closed to the tube and to cock 6, while remaining open to 2. The elbow in the cock would now fill with gas; and, on turning cock 3 through 90°, this elbow would empty its charge of gas into the tube, while remaining closed toward 2 and 6.

The hydrogen for these experiments was sometimes taken from a cylinder of the commercial gas, but generally it was prepared from sulphuric acid and chemically pure zinc. Before admission to the vacuum tube it was passed through two tubes of phosphorus pentoxide, the first 30 cm and the second 25 cm long.

In determining the Doppler effect in canal rays the purity of the gas is a matter of great importance; for upon this depends not only the intensity of the radiation of the canal rays themselves, but also the straightness of their path (§ 9). It does not suffice merely to fill the tube with pure gas; first of all, one must remove the gas which the electric discharge drives out of the electrodes and out of

the glass walls of the tube. The color of the canal rays in hydrogen is a sensitive means for judging of the purity of the gas. When pure, the color is a beautiful red, and as impurities increase the color deviates proportionately from red. At the beginning a new tube will develop so much foreign gas that, after the hydrogen is admitted and the electric current started, the canal rays immediately change color. In these experiments each new tube was purified as follows before any spectrograms were taken. First of all, a strong current was sent through the tube for half an hour, the oil pump being in continuous operation during the entire time. Then hydrogen was admitted until a pressure of 10 mm was attained, after which the current was broken and the hydrogen, together with the foreign gases, pumped out. If now, on closing the circuit for a moment, the dark space proved long enough, the current was applied for 1 minute; the canal rays generally showed at first a red color, but changed quickly into a bluish white. When this happened, then as before fresh hydrogen had to be admitted and the impurity pumped out. These operations were repeated until the canal rays in hydrogen would maintain their red color for about 15 minutes. In general, this state of purity was obtained after something like 5 hours of treatment such as that described.

Tubes were employed for making spectrograms only after they had shown the proper degree of purity; but it often happened that, during exposure, tubes gave off new impurities, so that the canal rays would change color. Or, on the contrary, the gas-pressure would diminish and the cathode-drop rise in consequence of self-exhaustion due to the electric current. For this reason the gas-pressure had to be renewed constantly—every 5 to 25 minutes—during the exposure; fresh gas was introduced either to sweep out impurities or to raise the pressure.

It has been shown by E. Goldstein that the path of the canal rays, in the region behind the cathode, depends upon the curvature of the front surface of the cathode; and I have elsewhere[1] fully described the manner in which the direction of the canal rays is connected with the curvature of the electric lines of force in front of the cathode. If the front surface of the cathode is convex outward, then the

[1] "Die Electrizität in Gasen," *Winkelmann's Handbuch*, 4, 602, 1905.

canal rays converge behind the cathode (Fig. 3*a*); but if the front surface is concave, they diverge (Fig. 3*b*). Behind a plane cathode the canal rays generally converge to a slight extent; accordingly, one must make the front surface of the cathode slightly concave in order to obtain parallel rays (Fig. 2). This was done in most of the tubes used for my photographs.

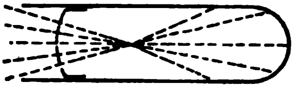


Fig. 3*a*

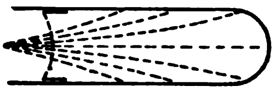


Fig. 3*b*

The spectrograms of canal rays were taken either with a prism spectrograph or with a Rowland concave grating. The former has already been described elsewhere;[1] the latter was kindly loaned me by Professor Runge near the end of my observations. The grating had a radius of 1 m, with 590 rulings to the millimeter, and an aperture of 8.6 cm. It was mounted in the manner employed by Professor Runge, with slit, spectrum, and center of grating on a circle; the camera was large enough to photograph at once the entire first order, and the second as far as λ 5000. The scale of the first order was 16.4 A. U. to the millimeter. Sometimes I projected the image of the canal rays upon the slit with the aid of a lens; generally, however, the slit was placed close up against the tube to obtain greater intensity.

The canal rays take their rise in different cross-sections of the region in front of the cathode, so that they complete a part of their course before reaching the canals of the cathode. The first cathode glow, so called, emits the spectrum of canal rays around this region. I have also taken several spectrograms of the first cathode glow. But these I shall not discuss here, confining my attention to spectrograms of canal rays obtained behind the cathode. On the front of the cathode there occurs a partial reflection of canal rays (§ 9), which is liable to introduce complications.

§ 3. *Character of the Doppler effect.*—To begin with, the direction of observation was chosen perpendicular to the beam of canal rays. An examination of the spectrograms, in the case of hydrogen, shows

[1] *Annalen der Physik*, 16, 493, 1905.

that the region traversed by the canal rays in the rear of the cathode emits both the line[1] spectrum and the band[2] spectrum. By band spectrum we mean the so-called second or many-lined spectrum of hydrogen, which will be more fully discussed in § 10; at this point we consider only the line spectrum, including the series $H\alpha$, $H\beta$, ... But $H\alpha$ was not observed, owing to the small sensitiveness of photographic plates in this region.

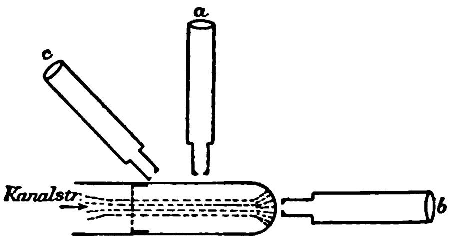


Fig. 4

The normal direction of observation is indicated by a in Fig. 4; in the position marked b the canal rays are directed "end on" toward the observer; while in position c they make an angle of 45° with the direction of observation.

In Figs. 5a, 5b, and 5c the wave-lengths are plotted as abscissae, the intensities as ordinates. These figures represent the appearance of the hydrogen lines observed ($H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, $H\zeta$, $H\eta$) for the three different positions of observation. In the case represented by

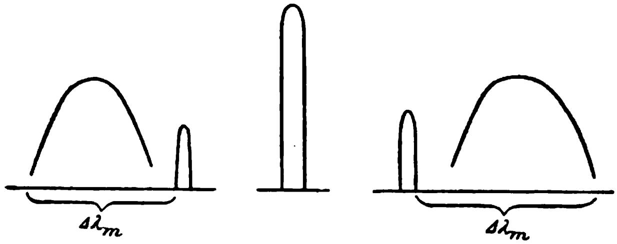


Fig. 5c Fig. 5a Fig. 5b

Fig. 5a the hydrogen line appears strong at the position in the spectrum where it is ordinarily seen. In the cases represented by Figs. 5b and 5c the hydrogen line is also seen in its usual position; but now its intensity is much less than in Fig. 5a; besides, it is accompanied (on the blue side in 5b, and on the red side in 5c) by a broad,

[1] A. Wüllner, *Physikalische Zeitschrift*, 1, 132, 1899; E. Goldstein, *Ann. d. Physik*, **64**, 44, 1898; *Physikalische Zeitschrift*, 1, 133, 1899.

[2] J. Stark, and W. Hermann, *ibid.*, **7**, 92, 1906.

hazy band. The presence of this band is explained as a Doppler effect produced by the translation of the carrier of the hydrogen series with reference to the observer. The intensity of the band is ascribed to the hydrogen particles in motion and may be called the "displaced intensity;" the intensity of the line which remains at its usual wave-length may be assigned to those hydrogen particles which have a small velocity relative to the observer, and may be called the "stationary intensity."

As is shown by Figs. 5*b* and 5*c*, the region behind the cathode emits simultaneously both displaced and stationary intensities; but the intensity observed in 5*a* results from the addition of the stationary and displaced intensities.

If in the canal rays behind the cathode there were only a single velocity, then we might expect, according to Doppler's principle, a displaced line, which would be sharp and of the same breadth as the stationary line. From the fact that the displaced line is a broad band, we may therefore infer that in the rear of the cathode the canal rays travel with various speeds. These different speeds are to be measured by the difference of wave-length $\Delta\lambda$ between the displaced intensity and the nearest edge of the stationary line. Then by Doppler's principle we have $v = c\left(\frac{\Delta\lambda}{\lambda}\right)$, where v is the velocity of the luminous source with reference to the observer, c the speed of light, and λ the stationary wave-length.

There are two causes which may bring about these different velocities in the rear of the cathode. First, the canal rays may originate in different cross-sections of the region in front of the cathode, and may therefore drop through different ranges of potential before reaching the canals; or, secondly, those rays which leave the canals with the same velocity may lose different proportions of their translational energy by collision with gas particles in the rear of the cathode.

One striking feature of the Doppler effect is that between the stationary line and the displaced intensity there is an intensity minimum. This may arise either from the fact that low velocities are not very frequent or that they are in some way connected with small intensities. In § 14, Part II, we shall again consider the origin of this minimum and of the stationary line.

What concerns us here especially is the maximum value of the displacement $\Delta\lambda_m$, that is, the distance of the outer edge of the band from the inner edge of the stationary line. This maximum

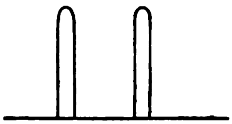


Fig. 6*a*

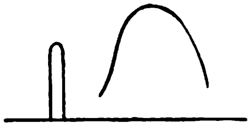


Fig. 6*b*

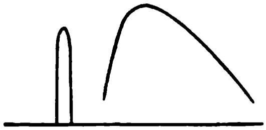


Fig. 6*c*

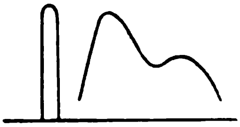


Fig. 6*d*

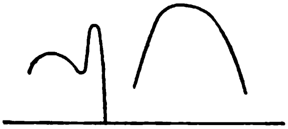


Fig. 6*e*

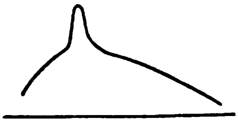


Fig. 6*f*

a, a single speed;
c, { many low speeds; few high speeds;
e, reflection on glass wall;
b, a single maximum;
d, two maxima;
f, dispersion in a heavy gas.

displacement measures the maximum speed of the line-sources in the rear of the cathode, namely, $v_m = c\left(\frac{\Delta\lambda_m}{\lambda}\right)$.

The maximum velocity of the canal is computed as follows:

$$V_m = \sqrt{2\left(\frac{\epsilon}{\mu}\right)\Delta V},$$

where ϵ is the charge and μ the mass of a canal ray particle, and ΔV the cathode-drop in potential.

The Doppler effect, as described above, may be called ***normal;*** it is what one obtains when the gas is pure, and the cathode-drop is kept constant during the exposure for the spectrogram. For the sake of completeness, however, some other types are described in Figs. 6*a*–*f*. Fig. 6*a* illustrates the hypothetical case in which the canal rays have only one velocity; Fig. 6*b* represents the normal type; Fig. 6*c* shows the effect on a plate exposed for a short while

with a large cathode-drop and a long while with a smaller cathode-drop; Fig. 6*d* is obtained when the two different values of the cathode-drop are widely different; Fig. 6*e* illustrates a case to be described later, namely, that of the reflection of canal rays on a glass wall; and Fig. 6*f*, the case of dispersion of canal rays in a heavy gas—a phenomenon which will also be described below.

§ 4. *Constancy of the Doppler effect within any one series.*—There are two constant errors which may affect the measurement of the quantity which we have called the maximum displacement, i. e., the distance between the outer edge of the band and the nearest edge of the stationary line. First, if the stationary line is strong, its image spreads upon the photographic plate and gives an abnormal width, thus making the maximum displacement too small. Secondly, the outer edge of the band is generally diffuse, so that in setting upon its boundary one is apt to make the maximum displacement again too small.

Table I, obtained from a grating spectrogram, gives the maximum displacement for the hydrogen lines $H\beta$, $H\gamma$, $H\delta$, and $H\epsilon$, and also the largest velocity of the corresponding carrier (*Träger*). Taking into consideration the constant error, one might say that the maximum velocity of the carrier is the same for all hydrogen lines.

TABLE I

Wave-Length λ	Max. Displacement $\Delta\lambda_m$	Mean Error	Constant Error	Velocity $c\frac{\Delta\lambda_m}{\lambda}$
$H\beta$ 4861.5	7.58 Å.	0.18	$\Delta\lambda_m$ too small	$4.67 \cdot 10^7$ cm.sec^{-1}
$H\gamma$ 4340.7	7.17	0.12	......	$4.95 \cdot 10^7$
$H\delta$ 4101.8	6.25	0.12	$\Delta\lambda_m$ too small	$4.57 \cdot 10^7$
$H\epsilon$ 3970.2	5.12	0.12	$\Delta\lambda_m$ too small	$3.87 \cdot 10^7$

All hydrogen lines have, therefore, the same carrier.

Arguing from the homology which exists between the series lines of the chemical elements, one may perhaps make the following generalization. All lines of one and the same series belonging to a chemical element have the same carrier. This theorem has been verified also in the case of mercury, for which see communications in the *Annalen der Physik*, No. 13 of 1906. If all the lines of the series are emitted by one and the same carrier, then it is to be expected

that their wave-lengths will be connected by formulae such as have in fact been discovered by Balmer, Rydberg, Kayser and Runge.

§ 5. *Electric charge of the carrier of the hydrogen series.*—Is the carrier of the hydrogen series a neutral hydrogen atom or a positively charged hydrogen ion? The canal rays themselves, in hydrogen, consist, according to the measurements of W. Wien,[3] of positive hydrogen ions moving with high velocity. If the ions of the canal rays are the sources of the series lines, then it is clear that the series lines must show the Doppler effect. If, however, the series lines have their origin in the neutral atom, then it remains to be explained how the neutral atom acquires the high velocity which the Doppler effect would seem to indicate.

It may be that the ions of the canal rays have a high velocity and yet emit no spectral lines; when now they collide with neutral atoms, they transfer velocity to these, and thus produce emission of the series lines. There are several facts, however, with which this explanation is not in harmony. First, the displaced line does not diminish in intensity when the gas-pressure is lowered, thus making the collisions less frequent (see Part II.) Secondly, in the case of mercury, one finds three groups of lines for which the squares of the reduced maximum displacement $\left(\frac{\Delta\lambda_m}{\lambda}\right)^2$ are in the ratio 1 : 2 : 3. Thirdly, neutral hydrogen which has not been ionized does not absorb the wave-lengths $H\alpha$, $H\beta$, ... even when thick layers of the gas are employed.

The assumption, on the other hand, that the series lines have their origin in the charged ions of the canal rays implies these very facts as necessary consequences. It is therefore highly probable that the carriers of the series lines in hydrogen are the positive hydrogen ions.

From the cathode-drop (ΔV) and the maximum displacement ($\Delta\lambda_m$) the specific charge $\left(\frac{\epsilon}{\mu}\right)$ for the hydrogen rays may be computed. Table II contains the data for this purpose derived from one series of spectrograms. The first column gives the line on which the maximum displacement was measured; the second, the cathode-

[3] *Annalen der Physik*, 5, 421, 1901; 8, 257, 1902; 9, 660, 1902; 13, 669, 1904.

TABLE II

Lines Measured	Observed Cathode Drop (Volts)	Computed Speed (Volts)	$\frac{\epsilon}{\mu}$ from Observed Cathode Drop and Maximum Displacement
$H\delta$	5000	1800	$3.42 \cdot 10^3$
$H\delta$	4200	1600	$3.61 \cdot 10^3$
$*H\delta$	4200	1600	$3.61 \cdot 10^3$
$H\delta$	2500	1400	$5.32 \cdot 10^3$
$H\gamma$	2500	1300	$4.94 \cdot 10^3$
$H\delta$	2000	1100	$5.22 \cdot 10^3$
$*H\delta$	3000	2100	$6.65 \cdot 10^3$

drop as observed on an electrometer during exposure; the third, the kinetic energy ($\frac{1}{2}\mu v^2$) in volts as computed from the maximum displacement, on the assumption that for the "carriers" $\frac{\epsilon}{\mu}=9.5\times10^3$ electromagnetic units; the fourth column gives the value of the specific charge $\frac{\epsilon}{\mu}$, computed from the formula

$$\frac{\epsilon}{\mu}=\frac{c^2}{2\Delta V}\left(\frac{\Delta\lambda_m}{\lambda}\right)^2,$$

where ΔV is the observed cathode-drop.

We shall now consider whether the method of computing the fourth column is justified. $\Delta\lambda_m$ is affected by a constant error which makes its observed value too small, whence the computed value of $\frac{\epsilon}{\mu}$ is always less than it should be. The value employed for ΔV is the largest value of the potential-difference through which the ions of the canal rays pass. Now, first of all, it is possible that the ions in front of the cathode do not experience the whole, but only a part, of the potential drop ΔV; and, secondly, the results of Part II show it to be a fact that, in the region behind the cathode, the canal rays lose kinetic energy through radiation; and hence the farther the canal rays travel, the less their speed. If, therefore, in the computation of $\frac{\epsilon}{\mu}$ the observed values of the cathode-drop ΔV are employed, too small a value of $\frac{\epsilon}{\mu}$ will always be obtained.

Accordingly, the values of $\frac{\epsilon}{\mu}$ given in column 4 of Table II represent merely the lower limit of the actual value. In other words, the specific charge of the "carrier" of the hydrogen series is greater than 6.6×10^3.

W. Wien has determined the largest value of $\frac{\epsilon}{\mu}$ for hydrogen to be 9.5×10^3; this is for a univalent positive hydrogen ion. A bivalent hydrogen ion would have a specific charge $\frac{\epsilon}{\mu} = 19 \times 10^3$. It is not at all likely that the value obtained by me is the lower limit of 19×10^3; it is more probable that the actual value of $\frac{\epsilon}{\mu}$ for the carrier of the series is 9.5×10^3, and that this carrier is therefore the univalent positive hydrogen ion.

One seems to be justified, therefore, in assuming the value 9.5×10^3 in computing the third column. A comparison of this with the second column shows that the actual kinetic energy of the canal rays behind the cathode is greater in proportion as the cathode-drop is greater, but that the former is always from 30 to 60 per cent. less than the latter.

A word may be said here concerning the diminution of speed in canal rays, mentioned above and described more fully below. In Table II there are two horizontal rows marked by an asterisk. In the case of the upper line, the collimator tube was placed in the position *b* of Fig. 4, while in the case of the lower line it was in the position *c;* the portion of the tube in the rear of the cathode was 15 cm long. In the *b* position the ratio of the computed to the observed cathode-drop was, according to the table, $\frac{16000}{42000} = \frac{8}{21}$; while for the *c* position this amounted to $\frac{21000}{30000} = \frac{7}{10}$.

§ 6. *Electric charge of the carriers of series of doublets and triplets, complexity of the line spectrum of an element.*—Using the notation of Kayser and Runge, the hydrogen series $H\alpha$, $H\beta$, ... is a first subordinate series of doublets, the frequency-difference of its components being 0.33. From what precedes we may infer that the carrier of this first subordinate series of doublets is a univalent positive hydrogen ion.

In another article it will be shown that the carrier of the principal

series of doublets of potassium is probably a univalent potassium ion.

As will be seen from a later paper, on the Doppler effect with mercury, there appears in the mercury spectrum a line (λ 2536) whose carrier is a univalent mercury ion. From the homology existing between this and certain lines in the spectra of zinc and cadmium we may infer that this line also belongs to a series of doublets.

These three cases, in which the series of doublets have a univalent positive ion for carrier, lead one to conclude, on the basis of a widespread homology, that in general the carriers of series of doublets are univalent positive ions.

Among series of doublets are to be reckoned the principal series, the so-called first and second subordinate series of doublets, and the line-series, called by Rydberg "secondary series," which accompanies the first components of the first subordinate series on the blue side. We may suppose all of these series to have the same univalent positive ion as carrier; indeed it is quite possible that this ion emits, besides the above mentioned and known series, still other lines whose place in the series of doublets has not yet been recognized.

If the different series are due to one and the same carrier, as is the case with the different members of a single series, then we might expect some systematic relation between the different series such as has already been found between the various members of a single series; in consequence of their having a common source, there would be certain conditions connecting them one with another. And this is, in fact, the case. For, according to Rydberg,[1] the pairs of the first and second subordinate series have the same frequency difference; in addition to this, the two series have the same convergence frequency. The second subordinate series of doublets is related to the principal series by the fact that their first terms ($m=1$) are numerically identical, as shown by the equation

$$\frac{n}{N_0}=\frac{1}{(1+\mu)^2}-\frac{1}{(m+\sigma)^2}\ .$$

The components of each member of this subordinate series exhibit the same Zeeman effect as those of the principal series but in reversed order.

[1] *Svenska Vet. Ak. Handl.*, 23, No. 11.

As will be shown in the following paper on mercury, the two subordinate series of mercury triplets have a bivalent mercury ion as carrier; this probably emits, besides the series, other lines which, having a simultaneous origin in similar sources, are probably related with the series of triplets. The two subordinate series of triplets of course satisfy the condition that their convergence frequencies should be the same, and that the frequency difference between the components of the members should be equal. In the case of mercury there appear, in addition to lines which have a univalent or bivalent carrier, also lines whose carrier is trivalent, as, for instance, $\lambda\lambda$ 4078 and 4347. According to § 15, these apparently belong to a term in which no less than seven components are associated. It is probable that trivalent ions emit series, and that the number of components in each term of the series is greater than three.

Univalent ions emit series lines of two components; bivalent ions emit series of triplets; ions of higher valency should emit series of which the members have more components in proportion as the valency increases.

Series having members of more than three components have not yet been discovered. However, Kayser and Runge[1] have found that in the spectrum of bismuth there are quadruplets, and in antimony sextuplets, with constant frequency differences which repeat themselves. C. P. Snyder[2] has found in the spectrum of rhodium a group of 19 lines repeated as many as 54 times. The chemist finds bismuth and antimony to be trivalent and pentavalent; in like manner rhodium is bi-, tri-, and quadri-valent.

From what precedes it would appear that the higher the valency of an ion becomes, the more numerous are the components in its series lines, and therefore the more complex is its spectrum. If we are to believe that chemical valence also cuts an important figure in spectroscopy, we may also expect that any element will give a spectrum richer in lines in proportion to its chemical valence and in proportion to the number of ions of different valencies which it is able to form. And this appears to be the fact of the case. In Table III are given the elements of a single horizontal row of the periodic

[1] *Abhandl. d. Berlin Akad.*, 1894.

[2] *Astrophysical Journal*, 14, 170, 1901.

system, together with their chemical valencies and their arc and spark lines.

If at the temperature of the arc or of the spark any element produces several different kinds of ions, then the resulting line spectrum consists in the superposition of the spectra of these different sorts of ions.

TABLE III

Element	Valency	Lines in the Arc	Lines in the Spark	Region of Spectrum	Observer
K	1	41		2942.8—7699.3	Kayser and Runge
			133	2341.7—7699.3	Eder and Valenta
Ca	1, 2	106		2200.84—6499.85	Kayser and Runge
			165	2081.53—6499.85	Eder and Valenta
Sc	1, 3	110		3907.62—5717.54	Lockyer and Baxandall
			142	2232.98—4744.02	Exner and Haschek
Ti	2, 3, 4	865		3477.33—5899.65	Hasselberg
			1337	2154.80—4698.93	Exner and Haschek
Vd	1, 2, 3, 4, 5	627		3094.79—5786.41	Rowland and Harrison
			2431	2131.8 —4670.65	Exner and Haschek
Cr	2, 3, 6	770		3433.42—5797.02	Hasselberg
			1572	2173.44—4601.2	Exner and Haschek
Mn	2, 3, 4, 7	1233		2346.58—5748.75	Fritsch
			1152	2215.31—4709.90	Exner and Haschek
Fe	2, 3, 4, 6	4620		2230.01—6750.36	Kayser and Runge
			2290	2068.25—4736.96	Exner and Haschek

The ratio of intensities in the different spectra depends upon the degree of dissociation for each valency, and upon the temperature when the amount of dissociation remains constant. In the spectrum of copper there are triplets[1] as well as doublets; so also in the spectra[2] of *Mg, Ca, Sr, Ba, Ra, Zn, Cd, Hg;* in the spectra of the last three elements there appear, beside doublets and triplets, still other lines. The line spectra of *Li, Na, K, Rb,* and *Cs* are dominated by series of doublets; and they are found again, together with other lines in the spectra of *Au, Ag, Al, In,* and *Tl.*

§ 7. *Conclusions concerning ionization and emission of light.*—According to the preceding pages, the carriers of line spectra of the elements are the positive ions. A vapor which is emitting a

[1] J. R. Rydberg, *Astrophysical Journal*, 6, 237, 1897.

[2] Runge and Paschen, *Abhandlungen der Berlin Akad.*, 1902.

line spectrum is therefore necessarily an ionized or electrically conducting substance. A special case of this general theorem is the following. When a bead of some alkali salt is introduced into a Bunsen flame, there appears simultaneously the line spectrum of the alkali and a marked increase of electric conductivity in the flame. The particles which emit the line spectrum are the positive ions and not the neutral atoms of the alkali; the number of luminous particles involved in the line spectrum is therefore less than the number of atoms introduced into the flame; it is proportional to the degree of electrical dissociation, i. e., to the ratio of the number of positive ions to the total number of atoms introduced. H. A. Lorentz[1] shows, on theoretical grounds, that the number of luminous particles in the Bunsen flame producing the line spectrum is much smaller than the total number of atoms of the alkali introduced. From his computation and from the results of E. Wiedemann[2] the ratio of these numbers is, in the case of a sodium salt, 2.4×10^{-2}.

The converse of the above theorem is that when a gas becomes electrically conducting or is ionized it emits a line spectrum. This proposition, however, is not true; for the emission of a line spectrum depends not simply upon the presence of positive ions, but requires also that the temperature shall be sufficiently high. Yet absorption is perceptible at all temperatures. We may therefore conclude that a gas absorbs its line spectrum from a transmitted beam when there is present in the gas a large number of positive ions; but if this condition is not satisfied, the gas does not absorb its line spectrum. It is known that the oxygen and nitrogen of the Earth's atmosphere do not absorb their line spectra from sunlight even though passing through such an enormously thick layer; the number of positive ions in the atmosphere is exceedingly small. In like manner, thick layers of non-ionized mercury vapor or of hydrogen will not, at moderate temperatures, absorb the slightest trace of their line spectra.

§ 8. *Widening of spectral lines by increase of density.*—The width of spectral lines can be increased without rise of temperature by merely increasing the density of the luminous vapor. This kind of widening is not caused by the Doppler effect, but rather by a force

[1] *Proceed. Akad. Amsterdam,* 1905, p. 591.

[2] *Annalen der Physik,* **37**, 212, 1889.

which deforms the radiant ion and thus changes the period of the lines emitted.

W. Voigt[1] has shown on theoretical grounds that the electrical force acting upon a radiant particle would by deformation alter the wave-length of its spectral lines. According to H. A. Lorentz,[2] a disturbance of the radiation, and hence a widening of the line, would occur long before the radiant particle had traversed the free path assigned to it by the theory of gases; in other words, a radiant particle is subject to distorting forces even when its distance from other particles is much greater than the diameter of the sphere of action demanded by the kinetic theory.

The radiation of the positive ion occurs in the electric field of the positive charge; and this field, even at a considerable distance from the ion, has a large value. The sphere of action of the positive ion is therefore greater than that of the neutral atom. The distorting force between a positive ion and another particle would be effective at greater distances than the force acting between two neutral particles.

§ 9. *Reflection and dispersion of canal rays.*—The reflection of hydrogen rays by glass has already been described by W. Hermann and S. Kinoshita,[3] and we here merely add some numerical data. With a grating spectrogram I have measured the maximum displacement $\Delta\lambda_r$ (see Fig. 6*e*) due to the Doppler effect for the reflected $H\beta$ and $H\gamma$ rays; the speed of the incident rays as computed from their maximum displacement $\Delta\lambda_i$ was from 4.67 to 4.97 $\times 10^7 \frac{\text{cm}}{\text{sec}}$ (1300 volts). The quantity $\left(\frac{\Delta\lambda_r}{\Delta\lambda_i}\right)^2$ gives the fraction of the incident energy which is reflected by the glass wall. In the case just cited this amounts to 0.35 for $H\beta$, and 0.18 for $H\gamma$. The impact of a hydrogen ion against a solid glass wall is therefore not perfectly elastic; nor is it perfectly inelastic; with a velocity due to a drop of 1,300 volts the ion loses, on impact, from 65 to 82 per cent. of its initial energy and retains, after reflection, from 35 to 18 per cent. Hermann and Kinoshita have already observed the dispersion

[1] *Annalen der Physik*, 4, 197, 1901.

[2] *Proceed. Akad. Amsterdam*, 1905, p. 591.

[3] *Physikalische Zeitschrift*, 7, 564, 1906.

of hydrogen rays in a heavy gas. The following observations deal with the dispersion of hydrogen canal rays in hydrogen itself.

As already indicated in § 3, the sum of the stationary and displaced intensities is obtained when the tube is viewed in a direction at right angles to that of the canal rays. I have taken a larger number of spectrograms in this position under both high and low gas-pressures. On comparing these I find the following facts: With high pressures the strong line when viewed normally appears upon a weakened darkened background, as indicated in Fig. 7*a*.

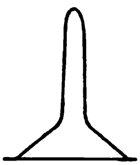


Fig. 7*a*

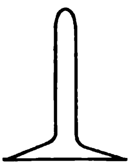


Fig. 7*b*

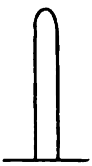


Fig. 7*c*

When the gas-pressure is lowered, the intensity of the background diminishes and the line itself becomes sharper (Fig. 7*b*); at still lower pressures the background ceases to be visible even when the intensity of the line is very great (7*c*). This peculiar widening of the hydrogen lines at higher pressures may be explained in the following manner. A small portion of the canal rays originally moving along a straight path at right angles to the line of sight collide with hydrogen molecules, and are reflected by these to one side so as to acquire a velocity component in the line of sight, either toward or away from the observer. Therefore both to the right and to the left of the undisplaced line we have the Doppler effect. The intensity, however, will diminish as the number of collisions of canal rays with neutral gas particles diminishes, i. e., as the gas-pressure diminishes. Even with the naked eye it is seen that canal rays in hydrogen are less diffuse as the pressure diminishes.

§ 10. *The line at* λ 4688, *and the banded spectrum of canal rays.*—Starting from the second subordinate series, J. R. Rydberg[1] has computed the wave-length of the first line in the principal series of hydrogen to be 4687.88. In some bright stars there is, besides the known hydrogen lines, a strong line at λ 4688.

Hasselberg[2] has measured the wave-lengths of a large number of lines in the banded spectrum of hydrogen; and among these he

1 *Astrophysical Journal*, 6, 233, 1897.

2 *Mem. de l'Acad. de St. Petersburg*, VII, ser. XXXI, No. 14.

counts the lines at λ 4689.39 and λ 4685.97. Both of these lines I have found on all of my spectrograms. In the spectrograms of the negative glow and in the spectrograms of the canal rays, observed when the line of sight is perpendicular to the rays, the space between the two lines is bright; but in the spectrograms taken in the positions *b* and *c* of Fig. 4 the interval between the two lines is filled with a dark, weak, hazy band.

In my preliminary paper I thought I saw in this band the Doppler effect of a line at λ 4688, and considered it the first line of the principal series. Yet, in spite of many attempts with later and better spectrograms, I have not succeeded in accurately determining the wave-lengths of the edges of this band; the difficulty lies in the small intensity of the band and in the proximity of the two lines in the banded spectrum. I cannot therefore conclude that the line at λ 4688 and its Doppler effect have been shown on my plates.

As already mentioned, the region which is traversed by canal rays emits both the linear and the banded spectra of hydrogen. The lines of the banded spectrum are sharp and narrow, but they never show any Doppler effect either in the negative glow, or in the glow which covers the cathode, or in the space traversed by the canal rays; and this is true whatever be the direction of observation. From these facts we may infer the following: The banded spectrum and the line spectrum of hydrogen do not have the same carrier; the banded spectrum is not emitted by the positive hydrogen ion. The carrier of the banded spectrum has neither a positive nor a negative charge; for an electric field does not impress upon it a velocity in any direction. Nor is the banded spectrum emitted by neutral hydrogen atoms which are brought into luminosity by collision with canal rays; for in this event the colliding hydrogen atom would receive from the incident canal ray, not merely internal energy, but also energy of translation. Besides this, it is to be remembered that hydrogen which is not ionized will not absorb, even in great thicknesses, the slightest trace of its banded spectrum.

A good while ago I suggested[1] that the banded spectrum of an element is produced at the instant when positive ions unite with negative electrons and is emitted by the neutral system formed by

[1] *Annalen der Physik*, **14**, 525, 1904.

the parts thus uniting. In a later paper[1] I have shown how easily and naturally the properties of banded spectra flow from this hypothesis. But to avoid misunderstanding, the following remarks are added.

By banded spectra are meant those which we observe as a secondary phenomenon in ionization and in electric currents in gases. The banded spectrum which appears in certain gases, such as oxygen and iodine vapor, independently of ionization or of electric conduction, is not the one here discussed. This belongs rather in the group of spectra of compounds, as illustrated by carbon dioxide and water vapor; also by a series of oxides and halogen compounds when used in the Bunsen flame. This latter spectrum apparently has its origin in the reaction between the parts of the molecule, just as the former has its origin in the reaction of the negative electrons upon the positive ions.

The lines of a banded spectrum differ from those of a line spectrum in the following respect: at high temperatures the former disappear in a continuous background, while the latter increase in intensity. The Doppler effect is not to be expected in the lines of bands, but only in those lines which become intensified by rise of temperature; these are precisely the lines which belong to the line spectrum; they are strong in the arc and still stronger in the condensed spark.

This hypothesis concerning the carriers and the constitution of banded spectra resembles, in some points, the view expressed long ago by E. Wiedemann,[2] that the line spectrum is a function of the separate atoms in the molecule, and that the quantity of heat which is necessary to transform the banded into the line spectrum is measured by the maximum value of the heat of dissociation of the hydrogen molecule.

[1] *Physikalische Zeitschrift*, **7**, 355, 1905.

[2] *Wied. Ann.*, **5**, 506, 1878; **10**, 252, 1880.

[*To be continued*]

THE ABSORPTION OF SOME SOLIDS FOR LIGHT OF EXTREMELY SHORT WAVE-LENGTHS

By THEODORE LYMAN

The observations recorded in this paper deal with that part of the spectrum discovered by Schumann which lies between λ 2000 and λ 1250 Ångström units. The region possesses considerable interest, for to the ordinary problems of spectroscopy—the radiation and absorption spectra of solids and gases, and the reflecting power of metals—must be added questions connected with the photo-electric effect and with the physiological action of light. The author's[1] determination of the wave-lengths of the hydrogen spectrum afford, it is to be hoped, trustworthy landmarks by which future investigations in the region may be guided. It next becomes necessary to improve, as much as possible, the facilities for such explorations.

The grating spectroscope[1] which the author used in previous work, though well adapted for wave-length measurement, is not convenient for general investigation. The large size of the vacuum receiver, 110 cm long by 11.3 cm in diameter, renders the process of exhaustion tedious; the length of the light-path within the apparatus, almost 2 m, exaggerates the importance of absorbing impurities in the gas; and the relatively small amount of light obtained in any one grating spectrum renders the study of feeble radiation difficult. The use of a prism spectroscope offers improvement in all these respects. The difficulty, however, of obtaining suitable prisms and lenses is considerable; white fluorite is the only substance which has been used in the past for this purpose, and even this very expensive material absorbs completely in the neighborhood of λ 1230—a point nearly two hundred units above the present limit of the spectrum.

The purpose of the present investigation was therefore purely a practical one—namely, to discover, if possible, some substance which should be as transparent as fluorite, and which should exist in clear

[1] *Astrophysical Journal*, 23, 181–210, 1906.

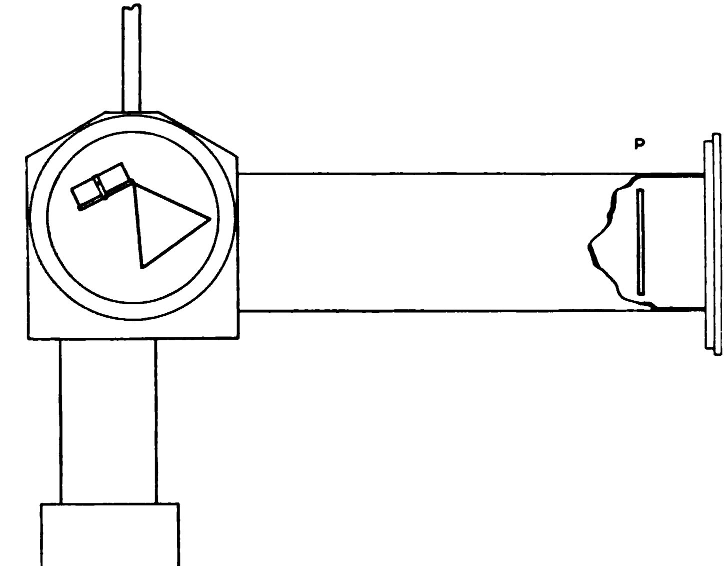
masses of sufficient size to permit the manufacture of prisms and lenses. Of course, the discovery of some material more transparent than fluorite would be most welcome, even if it existed only in small crystals; for such a substance would serve the much-needed purpose of a window for the large vacuum spectroscope.

In testing the transparencies of a large number of substances for light of very short wave-length the method described in a former paper,[1] though accurate, is tedious. The author has, therefore, constructed a new instrument for the purpose. In it the photographic plate is replaced by a fluorescent screen coated with willemite, and the place of grating is taken by a concave mirror and fluorite prism. The construction may be understood by referring to Fig. 1. A brass casting, with a cavity of circular cross-section 8 cm in diameter, contains the mirror and the prism; the top of the casting is closed air-tight by a plug which screws into place. Into the sides of the casting, and nearly at right angles to each other, fit two brass tubes. One of these tubes 21 cm long carries a flange at its far end closed air-tight by a glass plate. The fluorescent screen *P* is placed a few centimeters from this plate and may be observed through it. The other tube carries the source of light. In order to secure the necessary brilliancy of illumination, no slit is used, the end of the capillary of a discharge tube serves itself as the source. The details of the arrangement are shown in Fig. 2. One electrode of the tube is formed by a platinum wire sealed in the usual manner, while the other electrode is the brass casting itself. Thus, when the apparatus is exhausted the discharge passes from the platinum along the capillary to the sides of the casing, the end of the tube, some 0.9 mm in diameter, furnishing a bright point of light. The bits of the substances whose transparencies are to be examined are fastened over the holes of the disk shown in the illustration. This disk can be rotated through a simple cog mechanism, by a key fitting air-tight in the screw cover which closes the end of the casing tube. By this rotation the specimens are brought in turn before the end of the capillary and at a distance of about 6 mm from it.

The mirror employed is of speculum, radius of curvature 25 cm, and the mounting is of a very simple form permitting movements

[1] *Loc. cit.*, p. 106.

about horizontal and vertical axes. The prism is of excellent white fluorite, by Zeiss, angle 60°; it is mounted on a table fitted with leveling screws.



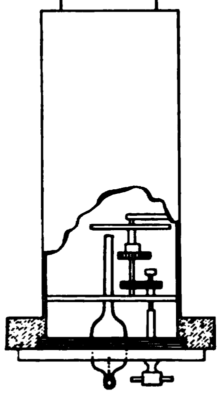


FIG. 1

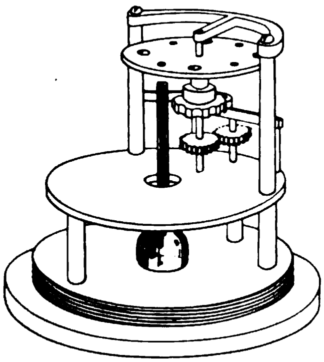


FIG. 2

The exact dimensions are not essential to the success of the apparatus; Fig. 1 is one-third, and Fig. 2 is two-thirds, of natural size. It is worth noting that the screw-joints which close the prism-chamber and one of the casing tubes have proved more satisfactory than

flanges with ground plates. In order to secure an air-tight joint, it is necessary only that the screw-threads be carefully cut and well covered with stopcock grease. Brass apparatus made from castings are almost sure to contain pin-holes, but the resulting leak may be reduced to a minimum by the use of thin shellac.

It is obvious that the optical arrangement of the apparatus is such that no great perfection of definition is to be expected. For the rough tests for which the instrument was constructed this, however, is not a serious drawback.

The operation of the instrument is as follows. The mirror and prism are so adjusted that the lines at the extreme ultra-violet end of the aluminum spectrum at λ 1854 fall on the end of the fluorescent screen. This arrangement is secured by the use of an aluminum spark in air and a slit temporarily placed in the casing tube at the point usually occupied by the end of the discharge capillary. A piece of fluorite of known transparency is then fastened over one of the holes in the movable disk, a second hole is left open, while the remainder are closed with pieces of the substances whose transparencies are to be tested. The temporary slit is now removed, and the face-plate with its discharge capillary and revolving disk is screwed into place. The casing is now connected with a vacuum pump, and the pressure is reduced to a value of about 0.3 mm of mercury. The apparatus is then washed three or four times with dry hydrogen, the pressure being finally reduced to 3 or 4 mm. The observer, seated in a darkened room, starts the current through the discharge capillary; with no absorbing medium in the light-path, the spectrum will then be seen to extend quite across the fluorescent plate—a distance of about 5 cm. Next, by turning the key on the outside of the face-plate, the fluorite and other absorbing substances come in succession in front of the capillary, and their effect on the length and brilliancy of the spectrum is determined. In practice it was found convenient to use a piece of quartz for comparison in many cases, since most of the substances tested were far inferior in transparency to colorless fluorite. The results obtained by these eye observations were checked in all important cases by the photographic method, the large grating spectroscope being used for this purpose in a manner described in the former paper. Comparison showed that the rough prism instru-

ment could be trusted to detect the difference in transparency of two substances the length of whose spectra, when obtained photographically, differed by not more than fifteen Ångström units. If it be remembered that the object of the investigation was a purely practical one, it will be seen that this prism instrument fulfils the purpose for which it was intended. The arrangement of mirror, prism, and fluorescent screen may be employed for other purposes than that of measuring the absorption of solids; for, if the open capillary is replaced by an "end-on" discharge tube closed by a fluorite window, sealed into the face-plate and separately exhausted, the apparatus serves to demonstrate the absorption of the air. If the discharge tube is excited and the casing is attached to a vacuum pump, the spectrum is seen to extend itself across the fluorescent screen as the pressure is diminished.

A further modification of the arrangement will permit of an approximate determination of the limit of reflection of metals in this region.

The transparency of the following substances has been tested.

Quartz.—Quartz in thicknesses of from 1 to 2 mm shows considerably greater transparency than was to be expected. Spectrum No. 5 of Plate I obtained with the grating spectroscope, shows lines to wave-length λ 1500. The slit-width was 0.09 mm, as in all the following cases; the source was a vacuum tube filled with air at about 1 mm pressure; the time of exposure was eight minutes. The light passed through the usual fluorite window before reaching the quartz plate.

Spectrum No. 4 shows the transparency of a column of quartz 2 cm long. The rapid increase of absorption with thickness here indicated explains why quartz prisms and lenses were found by Schumann to be useless for work beyond λ 1600.

Spectrum No. 3 shows the absorption of a quartz plate 0.2 mm in thickness. All these spectra as well as those which follow, were obtained with the same time of exposure, namely eight minutes.

As far as can be determined from an examination of fourteen specimens of colorless quartz between 1 and 2 mm thick, some selected from right-handed and some from left-handed pieces, and some from specimens cut parallel and perpendicular to the optic axis, the direction of rotation and of the axis makes no practical difference in the transparency of the material.

Fused quartz seems somewhat less transparent than the crystalline substance.

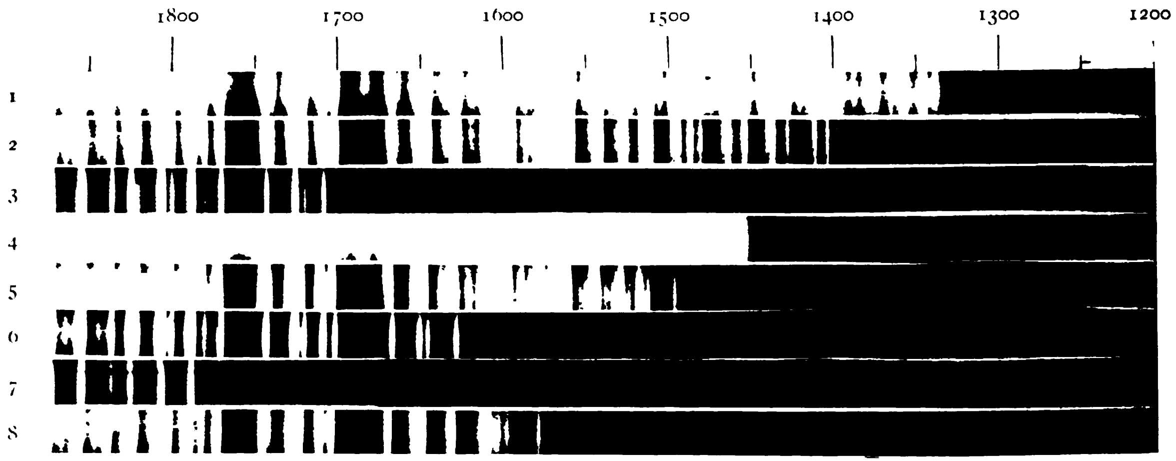
It appears, therefore, that in the region from λ 2000 to λ 1500 quartz in thickness of about 1 mm may be used in place of fluorite. Thick prisms or lenses, however, are useless in this region.

Fluorite.—Though clear, colorless fluorite is rare and expensive, the colored varieties are common enough, and sometimes may be obtained in clear masses of considerable size. Specimens cut from purple, green, pink, and yellow fluorites of various shades have accordingly been tested, which were usually in the form of plates from 1 to 2 mm thick. They show considerable range of transparency. A typical spectrum taken through a purple fluorite is shown in No. 3. Of fifty-seven specimens, forty-two were less transparent than this purple sample, ten were nearly equal to quartz 1 mm thick, and five were almost as good as colorless fluorite from Zeiss. From this test it appeared that the depth of color was a good indication of the absorption; the most deeply colored were the most opaque, while the five most transparent pieces were nearly, if not quite, colorless. There was one notable exception to this rule, however: four specimens from a light-green crystal from Westmoreland, N. H., were nearly as transparent as the plates from Zeiss, No. 1. No. 2 illustrates this fact. This crystal was free from flaws. There is some hope, therefore, that colored fluorite may yet be found which will serve for prisms and lenses, and which will possess the requisite transparency for use in the region between λ 2000 and λ 1250.

It is well known that the color may be removed from fluorite by heating, and, if the process is carefully carried on, no cracks are developed. This loss of color, however, is accompanied by only the very slightest gain in transparency, if any. In ten cases out of twelve no difference could be noted at all either by the visual or by the photographic method.

A microscopic examination with a power of two hundred diameters showed little difference in constitution between the transparent Westmoreland fluorite and the more opaque colored varieties. In general, the specimens which showed the greater absorption seemed to contain the larger number of those fluid-filled cavities which have been

PLATE I



TRANSMISSION OF EXTREMELY SHORT WAVES BY VARIOUS SUBSTANCES

1. Colorless Fluorite.
2. Green Fluorite.
3. Purple Fluorite.
4. Quartz, 0.2 mm thick
5. Quartz, 2 mm thick.
6. Quartz, 20 mm thick.
7. Rock-Salt.
8. Ceylon Topaz.

so often observed. The colorless variety of fluorite from Zeiss was quite free from these microscopic inclosures.

The fluorescence excited by cathode rays was somewhat less brilliant in the colorless than in the colored fluorite, but the differences were not of such a magnitude as to give an indication of the relative transparencies of the specimens.

Topaz.—Next to the colored fluorites, topaz from Ceylon shows the greatest transparency of all the substances examined. The result obtained by the photographic method is shown in spectrum No. 8. The specimen was 1.5 mm thick; it is thus inferior to quartz. Topaz from Japan, Utah, and Siberia is much less transparent than that from Ceylon. It is possible, however, that this difference may be considered rather as a peculiarity of the individual specimen than as a distinctive property connected with a region.

Gypsum.—This substance, when examined in bits 1 mm thick bounded by cleavage surfaces, shows a spectrum which extends to the region between λ 1700 and λ 1650.

Celestite.—This substance, when examined in polished pieces 1 to 2 mm thick, shows a transparency about equal to that of gypsum.

Rock-salt.—The transparency is shown by No. 7, Plate I. The specimen was 2 mm thick, with well-polished faces. It is worthy of note that up to the present time some authorities have considered that rock-salt was as transparent as quartz to the extreme ultra-violet.[1] It is evident that in the region beyond λ 2000 for thicknesses of 1 or 2 mm this is far from true.

Barite.—In polished pieces 1 to 2 mm thick the transparency is about equal to that of rock-salt.

Alum.—Pieces cut from crystals and 1 mm thick show a spectrum ending near λ 1750, but of rather stronger intensity than that obtained with rock-salt.

Colemanite.—Transparent to the neighborhood of λ 1750.

Sugar.—Plates 1 mm thick cut from crystals of rock candy are less transparent than colemanite.

The test pieces of all the substances just mentioned were clear and free from flaws. The surfaces were carefully polished.

In addition to the above, the following substances were tested and

[1] Kayser, *Handbuch der Spectroscopie*, Vol. 3, p. 386.

found perfectly opaque to light of shorter wave-lengths than 2000; borax, adularia, calcite, chrysoberyl, sanidin, arragonite, apophyllite, silver chloride (horn silver), Kunzite, and diamond. Several of these are known to be opaque to light of longer wave-lengths than 2000. The tests were made in the hope of discovering some material which showed selective absorption to a marked degree.

In the case of diamond we have an example of a crystalline substance which is transparent to the visible part of the spectrum and to X-rays, yet which is apparently opaque to light of very short wave-length.

SUMMARY

Fluorite still remains the only known substance transparent to λ 1250 Ångström units. While colorless fluorite is in general far more transparent than the tinted varieties, yet colored fluorite does exist of sufficient transparency to serve in apparatus intended for the exploration of the region between λ 2000 and λ 1250.

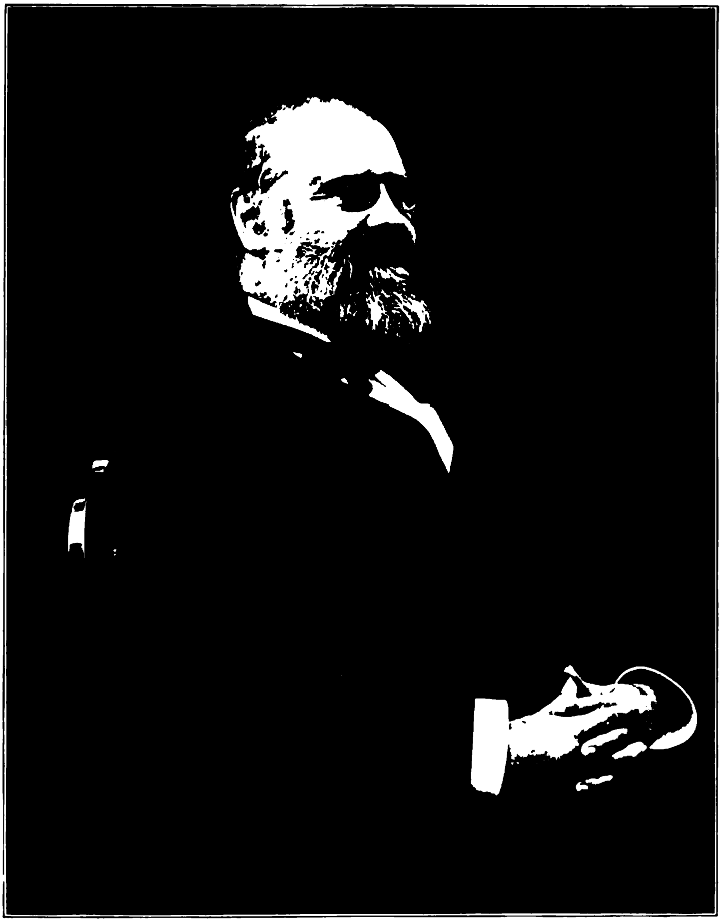
Quartz in thicknesses of from ·1 to 2 mm is transparent to λ 1500. This is a useful fact, as it indicates that quartz windows may be employed to replace fluorite in those cases where the electric or physiological action of the strong portion of the hydrogen spectrum lying near λ 1600 is to be studied.

Of all the substances examined none show absorption bands in the region between λ 2000 and λ 1250. The apparatus employed in this examination, however, does not permit the investigation of this point by methods of great delicacy.

It is interesting to note that the fluorescent substance willemite responds to the action of light of extremely short wave-length.

Jefferson Physical Laboratory
Harvard University
December 26, 1906

PLATE II



GEORGES RAYET

Minor Contributions and Notes

GEORGES RAYET

Science has suffered a severe loss in the sudden decease, on June 14, 1906, of M. Georges Rayet, founder, and for more than twenty-five years director, of the Observatory of Bordeaux.[1]

Born at Bordeaux on December 12, 1839, he became attached to the Observatory of Paris in 1862. Here he was more particularly occupied with the weather service, but he was also attracted by spectroscopy, then a new branch of research, and he undertook the spectroscopic study of numerous celestial objects. His published works during the following years deal with the spectrum of the Sun, prominences, and sun-spots; with the spectra of several comets, and with terrestrial magnetism and auroras. Some of his investigations were made in conjunction with M. C. Wolf, and together, in 1867, they discovered the three stars in *Cygnus* having bright lines in their spectra. The names of MM. Wolf and Rayet have been always associated with the spectra of this interesting type, the number of stars belonging to which has since been greatly enlarged. Present-day students of stellar spectra, dealing with photographs made with powerful instruments, are perhaps unable to fully appreciate the difficulties of the early visual spectroscopic observations of faint stars; such discoveries certainly deserve far more credit than those which may be made upon modern spectrograms.

At the solar eclipse of August 18, 1868, M. Rayet gave his attention to the spectra of the prominences, which were then objects of great interest and supposedly observable only at the time of an eclipse. He established the fact that the prominences contained other substances than hydrogen, and found the line D_3, subsequently ascribed to helium.

From 1874 to 1876 M. Rayet occupied the chair of physical astronomy of the faculty of sciences of Marseilles, and in 1876 became professor at Bordeaux in the same chair, which he was the first to

[1] This note is based upon facts kindly communicated by M. Ernest Esclangon.

occupy; and the duties of which he discharged for thirty years until his death.

The construction of an observatory at Bordeaux was decided upon in 1877, and M. Rayet was designated as director. The observatory began its activity in 1881, and for twenty-five years M. Rayet here worked steadily, making numerous observations on nebulae, double stars, and comets. He was a successful executive as well as scientist, and was devoted to the interests of the observatory to which he had given so many years of his life.

A LIST OF EIGHT STARS WHOSE RADIAL VELOCITIES ARE VARIABLE

The following stars have been found to be spectroscopic binaries. All of them, with the exception of *SU Cygni*, were discovered with the Mills spectrograph in the course of the regular observing program. Velocities expressed without decimals indicate either that the measures are preliminary and approximate, or that the type of spectrum does not permit accurate measurements to be made.

1 Geminorum ($\alpha = 5^h\ 58^m.0$, $\delta = +23°\ 16'$)

Plate	Date	Velocity	Measured by
2643 D	1903, January 4	+32. km	Curtis
		+33.7	Burns
4027 D	1905, September 27	+19.	Moore
4471 E	1906, October 1	+13.	Moore
		+15.7	Newkirk
4538 C	1906, November 9	+20.	Moore
		+20.5	Newkirk

This star is of type H. Its binary character was discovered by Mr. Moore.

B. A. C. 5890 = *D. C.* 7579 ($\alpha = 17^h\ 21^m.3$; $\delta = -5°\ 0'$)

Plate	Date	Velocity	Notes
1343 B	1899, July 18	+ 7.± km	lines single
1738 C	1900, May 20	−27.±	
3851 B	1905, June 21	+69.	strong component
		−82.	faint component
4314 B	1906, July 16	+ 4.	lines single
4324 B	1906, July 22	+29.	strong component
		−31.	faint component
4348 A	1906, August 1	+30.	strong component
		−25.	faint component
4353 A	1906, August 6	− 4.	lines single
4467 A	1906, September 30	+ 3.	lines single

The star is an F type, with broad lines. Both spectra are visible. The ratio of their intensities is about two to one, although all lines do not behave alike in this respect. The variable velocity and doubling of spectrum were discovered by Mr. Burns, by whom all of the above measures were made. From his observations of the variation from coincidence of the two spectra it seems probable that the masses of the stars are not very different.

δ Sagittae ($\alpha = 19^h\ 42^m.9$; $\delta = +18°\ 17'$)

Plate	Date	Velocity	Measured by
1371 B	1899, August 7	+10. km	Campbell
		+ 8.	Stebbins
1392 B	1899, August 14	+10.	Campbell
1750 D	1900, May 24	+ 9.	Wright
2174 D	1901, June 18	+ 5.	Campbell
		+ 2.	Wright
		+ 5.	Stebbins
2293 A	1901, October 22	+ 3.3	Burns
2407 E	1902, June 8	+ 6.	Stebbins
2932 A	1903, September 9	+ 4.	Moore
4282 E	1906, July 3	− 5.	Burns
		− 3.	Moore

The spectrum is of type M, and is classified by Miss Maury as XVII c. Its variable velocity was suspected by Mr. Campbell from the fourth plate and confirmed by the recent measures.

o² Cygni ($\alpha = 20^h\ 12^m.3$; $\delta = +47°\ 24'$)

Plate	Date	Velocity	Measured by
3819 D	1905, June 7	−25. km	Moore
3947 B	1905, August 13	−24.	Moore
4326 D	1906, July 22	+ 2.	Burns
4350 D	1906, August 1	+ 5	Burns

The type of spectrum is H. Its binary character was discovered by Mr. Burns from the third plate.

ε Cygni ($\alpha = 20^h\ 42^m.1$; $\delta = +33°\ 36'$)

Plate	Date	Velocity	Measured by
165 B	1896, September 23	−13.6 km	Campbell
480 A	1897, August 31	−14.	Burns
941 A	1898, September 15	−11.8	Campbell
2190 E	1901, July 3	−12.	Wright
2821 B	1903, June 20	− 9.	Curtis
4263 C	1906, June 27	− 7	Burns
		− 6.3	Newkirk
4375 B	1906, August 24	− 7.	Burns

This is a K-type star. The variation in velocity was suspected by Mr. Curtis and confirmed by Mr. Burns.

ζ Cygni ($\alpha = 21^h\ 8^m.7$: $\delta = +29°\ 49'$)

Plate	Date	Velocity	Measured by
183 B	1896, October 5	+19.6 km	Campbell
764 A	1898, June 8	+21.	Burns
772 A	1898, June 15	+21.	Campbell
865 B	1898, August 8	+20.	Campbell
2398 D	1902, June 2	+15.	Stebbins
4277 F	1906, July 2	+14.	Burns
		+13.9	Newkirk
4364 D	1906, August 13	+14.	Burns

The type of spectrum is K. The variable radial velocity was shown by the measures of Mr. Stebbins and Mr. Burns.

ι Capricorni ($\alpha = 21^h\ 16^m.7$; $\delta = -17°\ 15'$)

Plate	Date	Velocity	Measured by
1788 D	1900, June 26	+12. km	Wright
1890 B	1900, September 19	+13.	Wright
2923 D	1903, September 7	+ 8.	Moore
4267 D	1906, June 29	+18.	Burns
		+16.	Moore
4383 B	1906, August 26	+14.	Burns
4430 B	1906, September 16	+ 9.	Moore

This star is an H-type. The variable velocity was suspected by Mr. Moore and confirmed by the recent measures.

SU Cygni ($\alpha = 19^h\ 40^m.8$; $\delta = +29°\ 01'$)

This variable, of the *δ Cephei* type, was found to be a spectroscopic binary by Mr. J. D. Maddrill[1] from observations of its radial velocity made during the past summer with the one-prism spectrograph. Its spectrum is of type F–G.

W. W. Campbell
J. H. Moore

Mount Hamilton
November 12, 1906

A LIST OF SOUTHERN STARS HAVING VARIABLE RADIAL VELOCITIES

From an examination of plates secured by the D. O. Mills Expedition to the Southern Hemisphere, the following stars have been found to have variable radial velocities:

[1] Announced by Mr. Maddrill in *Publ. A. S. P.*, **18**, 252, 1906.

β Reticuli ($\alpha = 3^h\ 42^m.9$; $\delta = -65°\ 7'$)

Date	Velocity	Negative by[2]	Measured by[2]
1903, December 7	+54.5 km	W.	R. H. C.
1904, January 4	+53.8	W.	P.
December 18	+48.2	P.	W.
1905, November 5	+47.4	W.	W.
November 5	+47.1[1]		A.
December 28	+48.1	W.	W.

The spectrum of this star is of type K.

m Velorum ($\alpha = 9^h\ 47^m.9$; $\delta = -46°\ 5'$)

Date	Velocity	Negative by[2]	Measured by[2]
1905, January 10	− 3.6 km	W.	W.
January 28	− 0.9	W.	P.
March 13	+ 8.2	P.	P.
April 3	+12.8	W.	W.
April 3	+10.9[1]		A.
December 6	− 2.6	W.	W.

[1] Independent measure of above plate.

The type is G 5 K.

ν Centauri ($\alpha = 13^h\ 43^m.5$; $\delta = -41°\ 12'$)

Date	Velocity	Negative by[2]	Measured by[2]
1904, April 20	− 1. km	P.	P.
1905, January 25	+ 4.	P.	P.
March 18	+28.	P.	P.
April 3	+ 9.	W.	P.
April 13	+33.	P.	P.
April 27	− 1.	P.	P.

[2] The observers indicated in these columns are Messrs. Palmer, R. H. Curtiss, Albrecht, and Wright.

The type is B 2 A.

Dr. Palmer assigns a period of about thirty-one days to this star.

υ² Centauri ($\alpha = 13^h\ 55^m.4$; $\delta = -45°\ 7'$)

Date	Velocity	Negative by[2]	Measured by[2]
1904, April 6	+4.3[2] km	P.	W.
April 6	+3.0[1]		P
May 16	+5.4	W. & P.	W.
1905, March 15	−7.8	W.	W.
April 4	−7.8	W.	W.
1906, January 22	+3.1	W.	W.

[1] Footnote page 1.

[2] Poor plate, underexposed.

The type is F 5 G.

The variable velocities of β *Reticuli* and ν *Centauri* were detected by Dr. Palmer from preliminary measurements of the third plates.

MOUNT HAMILTON,
November 14, 1906

W. H. WRIGHT

THE VARIABLE RADIAL VELOCITY OF *ANTARES*

Approximate measures of a number of the earlier Mount Hamilton spectrograms of this star placed its velocity at −6 km per second. The star was on the working list of the D. O. Mills Expedition, at Santiago, and rough measures of a spectrogram secured on March 22, 1905, indicated a velocity of −1 km. A number of other observations were secured and an earlier plate was measured. The following are the observations obtained by the Expedition:

Date	Velocity	Measured by	Notes
1904, June 24	−2.3 km	Palmer	Underexposed
1905, March 22	−1.	Wright	
March 28	−0.9	Wright	
March 29	+1.2	Wright	

These determinations differed by about 6 km from the Mount Hamilton observations referred to above, and afforded a strong indication of variable velocity. At the direction of Dr. Campbell, the star was re-observed at Mount Hamilton, and the old plates were definitively measured. Mr. Burns's results, as given below, fully confirm the variability:

Date	Velocity	Measured by
1897, April 8	−3.0 km	Burns
May 19	−6.2	Burns
July 8	−5.4	Burns
1899, July 18	−1.4	Burns
1902, May 27	−4.6	Burns
1905, May 14	+2.0	Burns
May 23	+0.4	Burns
1906, July 18	−2.4	Wright

Miss Maury finds indications of a secondary or faint superimposed spectrum in the case of *Antares*, but this is referred to as being probably due to the telescopic companion.[1]

MOUNT HAMILTON
September 10, 1906

W. H. WRIGHT

[1] *Annals H. C. O.*, **28** 100, Remark 155.

THE RADIAL VELOCITY OF *POLARIS*

The following radial velocities of *Polaris* were secured from Mills spectrograms:

Gr. M. T. 1906	Velocity	Measured by
July 8^d 22^h 43^m	-14.2 km	Campbell
9 00 23	-13.9	Campbell
9 16 33	-14.4	Campbell
10 00 01	-17.8	Campbell
10 17 25	-18.9	Campbell
11 17 05	-17.3	Campbell
12 00 11	-16.2	Campbell
19 00 19	-20.8	Campbell

According to these results the velocity-curve for the four-day binary system attained a minimum value of $-19.8\pm$ km, determined graphically, at the epoch 1906.5. The minimum at 1904.5 was $-18.5\pm$ km.[1] At 1896.9 it was -20.7 km.

W. W. CAMPBELL

November 23, 1906

NINE STARS HAVING VARIABLE RADIAL VELOCITIES

I. VARIABLE STARS

RZ Cassiopeiae = 77 1906 = *B D.* 69°179
($\alpha = 2^h\ 40^m$; $\delta = +69°\ 13'$; Mag. = 6.5 to 8.0)

The spectrographic observation of this *Algol* variable was begun by Mr. Parkhurst on August 26, 1906, in connection with its photometric investigation by photography. The plate showed that the spectrum was well measurable, so that the star was placed on the programme for future observation. Mr. Parkhurst's second plate, on September 2, was taken at nearly the same phase of the star's variation (about three hours before minimum) and shows very little change in radial velocity. The radial velocity on these plates is roughly -9 km. The third plate, however, secured by Frost and Barrett on September 24 closely at the predicted time of greatest velocity of approach, 7 hours after minimum, yields a radial velocity of -114 km, according to my provisional measures. The fourth spectrogram, taken by Mr. Fox on November 4, eleven hours before minimum, gives a value of approximately -5 km. Unfavorable

[1] Lick Observatory Bulletin, 3, 86, 1905; *Astrophysical Journal*, 21, 191, 1905.

weather has so far defeated our attempts to secure a plate at the phase of maximum positive velocity.

After this star had been included in the first draft of this note, *Astronomische Nachrichten* No. 4135 arrived, in which Professor Hartmann reports his recent observations of the star. Our value for maximum approaching velocity is in good agreement with the data of his preliminary note.

The star has the *Orion* type of spectrum, as is commonly found in the case of *Algol* stars, with the helium lines faint. The hydrogen lines are much sharper than usual, and numerous enhanced lines of the metals may be seen, particularly of *Ti* and *Fe*.

X Cygni ($\alpha=20^h\ 39^m$; $\delta=+35°\ 14'$; Mag. = 6.4 to 7.5)

In 1905 one one-prism plate of this short-period variable star (type of δ *Cephei*) was secured by Mr. Barrett, and in 1906 three by Mr. Parkhurst and one by the writer. I have made approximate measurements of the last four plates and find a range of over 50 km in the radial velocity. The spectrum is of an advanced F type, suitable for fairly accurate measurement, but the spectrocomparator would probably prove particularly advantageous for it. The period of the star's variation of light is given as $16^d\ 9^h$—from minimum to maximum $6^d\ 5^h$, from maximum to minimum $10^d\ 4^h$. A considerable number of plates will therefore be necessary to cover the different phases. The largest positive velocity on these four plates was shown two days after minimum, with a small negative velocity two days after maximum. The velocity is also small half-way between maximum and minimum. Such results are to be expected from analogy with the spectrographic observations of δ *Cephei* and other stars of this class.

II. VISUAL BINARIES

13 *Ceti* ($\alpha=0^h\ 30^m$; $\delta=-4°\ 9'$; Mag. = 5.3)

Spectrograms of this star have been secured by Mr. Barrett or myself in the course of our regular programme of observations of visual binaries, for the ultimate determination of their parallaxes. The spectrum is an excellent example of the F type and can be accurately measured. Thus far I have had time to make only approxi-

mate measurements, depending chiefly upon the iron lines λ 4384, 4405, and 4415. The first plate, taken September 29, 1905, is rather too faint for measurement, as was the second, on August 20, 1906. The third, of date October 1, showed about the same velocity, but the fourth spectrogram, taken November 9, indicated to the eye a large change of velocity, which was measured to be over 30 km. Four additional plates have since been secured, Mr. Fox assisting in the exposure of the last three. The dates are November 23, 24 (two-prism plates), and December 1 and 3. A total range of over 50 km is indicated, and the last four plates make it appear probable that the period is about two days. The exposure time is kept as short as possible, to avoid the effect of the second star of the visual pair, which is about one magnitude fainter. The short period of revolution of the visual pair, probably less than twenty-five years, makes this an interesting system, and I hope that we can secure enough data during the present opposition for the definitive determination of the spectrographic orbit.

ω Leonis ($\alpha = 9^h\ 23^m$; $\delta = +9°\ 30'$; Mag. $= 5.6$)

This star is also on our regular programme of visual binaries, and two plates were taken in 1905, on February 3 and March 3. Approximate measures showed that the velocity changed by over 20 km. The third plate, on March 30, 1906, increased the range by about 5 km. The iron lines above mentioned are very sharp, and they were used for the provisional measurement. Nothing can be said at present as to the period of variation.

An apparent variation in the distinctness of other lines of the spectrum (early solar type) may be due to the effect of the light of the second star of the visual pair, which is called by double-star observers about one magnitude fainter than the principal star. The period of the visual pair seems to be somewhat over a hundred years, from the best recent observations.

85 Pegasi ($\alpha = 23^h\ 57^m$; $\delta = +26°\ 34'$; Mag. $= 5.8$)

I have been suspicious for the past year as to the constancy of the radial velocity of this star. The spectrum is of the solar type, and the accuracy of the measurements would be proportional to the

dispersion used; of the eleven plates we have so far secured, nine were taken with one prism and two with two prisms. An exposure of from two to three hours is required with our apparatus for a strong one-prism spectrogram. Four hours of exposure are needed with two prisms. The use of three prisms is therefore precluded for our instrument.

The construction of the Bruce spectrograph is so rigid that I am not much concerned as to internal flexures. I have been unable to detect any effect of displacement on trial exposures of the comparison spectrum made first with the telescope on the meridian, and second with the telescope pointing at an hour angle of six hours. But it is certain that with very long exposures the illumination of the collimator lens by the star's light must be progressively changing, due to flexure of the tube of the refractor; since prisms and lenses are not perfectly homogeneous, this must have an unfavorable effect. I should therefore hesitate to determine radial velocities from spectrograms requiring much over four hours of exposure. Where the star's declination permitted, the telescope would in such cases be run under the pier for the latter half of the exposure, so that the hour angle would be divided as nearly equally as possible on the east and west sides of the meridian. But I should never regard it as safe to reverse the telescope and conclude the exposure on the other side of the pier.

On account of the small dispersion and rather long exposures, I therefore give the results of the observations of *85 Pegasi* with some reserve; and most of the measures are to be regarded as provisional. When our spectrocomparator arrives, I expect to go over all the plates, as that instrument doubtless will be of great assistance in measuring such spectra.

The comparatively small range of the variation is no argument against its reality, however, and it is difficult for me to believe that errors of observation can account for the range of 10 km shown, for instance, by the excellent plates of October 31 and December 1, when the conditions of the spectrograph were very similar. The results so far obtained are as follows:

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Remarks
IB 580	1905 Aug. 28	21^h 2^m	B.	..	.....	Comp. lines too faint
604	Oct. 2	16 41	F.	5	—42 km	fair plate
612	Nov. 10	14 8	F.	13	—31	good
IIB 21	Nov. 20	16 41	F. & B.	14	—35	fair
IB 823	1906 Aug. 13	19 45	B.	4	−34	good
830	Aug. 20	18 20	F. & B.	4	(−28)	weak
861	Sept. 21	20 35	F.	5	−39	good
891	Oct. 31	14 37	F.	6	−33	good
913	Nov. 9	15 57	B.	4	(−30)	too weak; exposure 56^m
IIB 88	Nov. 23	17 7	F. & B.	14	−38	fair
IB 920	Dec. 1	15 38	Fox	5	−43	good

Like *13 Ceti*, this visual binary system is of particular interest on account of the shortness of its period, which cannot much exceed twenty-five years. In this connection it should be noted that Campbell found *κ Pegasi* and *ε Hydrae*, two visual binaries having the shortest periods after *δ Equulei*, to be spectroscopic binaries.

III. STARS OF *ORION* TYPE

19 τ⁵ Eridani ($\alpha = 3^h\ 29^m$; $\delta = -21°\ 58'$; Mag. $= 4.2$)

This spectrum is characterized by exceedingly broad and vague lines, which by their recurrent duplicity prove the star to be a spectroscopic binary with components of about equal brightness. It will not be possible to measure the radial velocity accurately. The *Mg* line at λ 4481 is the best in this part of the spectrum. The plates so far obtained are as follows:

Plate	Date	G. M. T.	Taken by	Remarks
IB 638	1905 Dec. 15	14^h 45^m	F.	λ 4481 single, also *Hγ* and *Hβ*
884	1906 Oct. 19	18 12	F. & B.	λ 4481, *Hγ* & *Hβ* double
902	Nov. 1	17 55	P.	λ 4481 double; *Hγ* faintly so; *Hβ* not visibly separated
922	Dec. 14	16 30	Fox	λ 4481 double, broader comp. displaced toward red
930	Dec. 17	16 19	F.	λ 4481 double, broader component displaced toward violet

Rough measurements of plate 884 give an almost equal displacement of the two components of λ 4481, corresponding to velocities of about −95 km and +110 km (after applying a correction of

+7 km for the Earth's velocity). The broader component was the one displaced toward violet.

Similar measures of No. 922, a weak plate, give values of about −60 and +140 km. On Plate 930 the positions of the components are reversed, whence we may infer that the period is short.

33 τ^8 Eridani ($\alpha = 3^h\ 49^m$; $\delta = -24°\ 55'$; Mag. = 4.7)

Five spectrograms have been obtained of this star, of our regular *Orion*-type programme. The spectrum is an example of the better measurable variety of the type, with the helium lines fairly sharp and λ 4481 especially so. Rough measures show a range of over 25 km in the radial velocity. On two of the plates the helium line at λ 4472 gives evidence of duplicity. Nothing can yet be inferred as to the period from our plates, of which the dates are 1905, December 15, 25; 1906, January 26, October 19, November 1.

20 τ Orionis ($\alpha = 5^h\ 13^m$; $\delta = -6°\ 57'$; Mag. = 3.6)

This spectrum has quite good lines; $H\gamma$ is narrower than usual for the type; λ 4481 is also good, and the helium lines are fairly sharp. The first two plates were measured by both Mr. Adams and myself, and the values given are the means of our determinations. The rest of the plates have been measured by Mr. Naozo Ichinohe, Fellow in Astronomy. The data are not yet adequate for finding the period.

Plate	Date	G. M. T.	Velocity
IB 206	1903 Dec. 1	$17^h\ 51^m$	+ 6km
262	1904 Jan. 23	15 56	+12
414	Nov. 1	22 7	+27
628	1905 Dec. 11	16 58	+34
880	1906 Oct. 12	21 24	+22
886	Oct. 19	20 16	+27
896	Oct. 31	21 32	+19
905	Nov. 1	20 40	+27
910	Nov 4	23 00	+16
914	Nov. 9	20 44	+18

4 ξ^1 Canis Majoris ($\alpha = 6^h\ 28^m$; $\delta = -23°\ 21'$; Mag. = 4.2)

The lines in the spectrum of this star are excellent, and it was accordingly selected for early measurement from the large number of unmeasured spectrograms in our cabinet. The first plate was

first measured by Miss F. A. Graves, formerly computer at this observatory. Her result was +36 km. The measures below are by Mr. Ichinohe.

Plate	Date	G. M. T.	Velocity
IB 508	1905 Feb. 6	16^h 20^m	+37 km
523	Mar. 3	14 43	+32
632	Dec. 11	20 10	+22
923	1906 Dec. 14	17 31	+23

With the exceptions noted, the spectrograms referred to in this communication were made with the one-prism arrangement of the Bruce spectrograph. The lower dispersion is better suited for most stars of the *Orion* type, and was necessary for securing the spectra of the faint stars of the solar type without unduly prolonged exposure.

As usual, Mr. Sullivan has rendered valuable assistance in guiding.

EDWIN B. FROST

YERKES OBSERVATORY
December 20, 1906

NOTE ON THE INTERPRETATION OF THE SPECTRA OF THE COMPONENTS OF DOUBLE STARS SHOWING CONTRASTED COLORS

In 1897 we sent a short note to the *Astrophysical Journal*[1] on the spectra of the strongly contrasted colored components of some double stars, which the newer form of my reflecting slit had made it possible for us to photograph separately.

Assuming these double stars to have originated by tidal evolution from one original mass, the pairs of stars at their birth would have been in the same evolutionary stage, and composed of the same substances, though not necessarily in the same relative proportions. The spectra of such pairs of stars should then indicate the evolutional stages which the components had severally reached; the successive stages coming in, it may be presumed, at an earlier time in the case of the component which has the smaller mass. We were met at once, as we then pointed out, by the apparent anomaly that it is the brighter, and as was then assumed the more massive star, which

[1] 6, 322, 1897.

shows a more advanced phase of development, while the relatively faint companion remains still in the earlier white-star stage.

We ventured to say that there is no certain ground for assuming that the brighter star is actually the more massive one. I had already in 1891[1] pointed out that "the brightness of a star depends upon several conditions and must be largely affected by the nature and the condition of the substances by which the light is chiefly emitted, as well as by the absorbent atmosphere through which it has to pass."

We suggested, therefore, that the smaller star, usually bluish in color, though so much less brilliant, might have the greater mass, and for this reason be still at an earlier evolutionary stage.

This view of the relative masses of the components of double stars was further supported by the spectra of other double stars reproduced in our *Atlas of Representative Stellar Spectra* (Plates XI and XII), 1901.

Quite recently our contentions have received remarkable confirmation from the work, on wholly independent lines, of Mr. T. Lewis on the relative masses of the components of eighteen binary stars.[2]

Mr. Lewis' results are summed up as follows: "They [the relative masses] establish the curious persistency of an opposite disparity, among the members of unequal pairs, between light and mass. The apparent satellite is in fact the primary of the system."[3]

UPPER TULSE HILL
December 15, 1906

SIR WILLIAM AND LADY HUGGINS

PRELIMINARY NOTE ON THE SPECTRUM OF *o CETI (MIRA)*

The spectrum of this well-known long-period variable has been photographed on four evenings during its present bright maximum.

The dates of the plates are as follows:

	1906	Spectrograph	Region Included
No. 1	Dec. 13	Three-prism	λ 4300 to λ 5000
No. 2	Dec. 18	Single-prism	$H\delta$ to $H\alpha$
No. 3	Dec. 21	Single-prism	$H\delta$ to $H\alpha$
No. 4	Dec. 24	Single-prism	$H\delta$ to $H\alpha$

[1] Address Brit. Assoc. 1891, pp. 16, 17.

[2] "Measures of Double Stars," *Mem. R. Astron. Soc.*, **56**, xxi, 1906.

[3] *Ibid.*, p. xx.

I am indebted to Mr. E. C. Slipher, Fellow at this Observatory, for exposing the last plate.

Vanadium, iron, and sodium were used for the comparison spectrum.

No. 1. Only two hydrogen lines, $H\beta$ and $H\gamma$, are comprised in the region covered by this plate. These lines are both strong and bright.

No. 2. The spectrum on this plate extends from below C to and including $H\delta$. The continuous spectrum is, however, somewhat weak from λ 6250 to 5000 and from λ 4300 toward the violet. The plate shows the following bright lines: $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$. Although $H\alpha$ (C) is far from weak it is not nearly so strong as the other hydrogen lines, which appear to increase in strength in the order given.

No. 3. This plate contains a splendid impression of the continuous (absorption) spectrum from λ 4200 to λ 6200. It shows a great many absorption bands which are sharp and intense on the more refrangible edge and gradually fade out toward the red. Of the hydrogen lines, $H\beta$, $H\gamma$, and $H\delta$ are strong and bright, and, although the plate is weak beyond λ 6200, there is a faint impression of bright $H\alpha$.

No. 4. The continuous spectrum, although somewhat lacking in density, extends from λ 4300 to below $H\alpha$. The hydrogen lines $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$ are all bright. $H\alpha$ is very sharp, and has bordering it on the violet a strong and rather broad absorption line. This absorption line is also well shown on plate 2.

A word of caution may not be out of place here regarding the remarks on the continuous spectrum of the different plates; for, as a different kind of photographic plate was used in each case, no conclusions are to be drawn as to changes in the continuous spectrum during the period of these observations.

This is the first time, to my knowledge at least, that bright $H\alpha$ has been observed either visually or photographically in *Mira* or any of the long-period variables of its class. However, I believe that this is not to be interpreted as an unusual behavior of $H\alpha$. It is probable that its faintness relative to the other hydrogen lines has made it difficult to observe visually.

Vanadium absorption lines are very strong in *Mira* as compared with other metallic lines.

LOWELL OBSERVATORY, FLAGSTAFF, ARIZ.
December 29, 1906

V. M. SLIPHER

A VERTICAL COELOSTAT TELESCOPE[1]

One of the most difficult problems of the Solar Observatory is that of designing a telescope to meet the rigorous requirements of solar research. The distortion of mirrors, and the local disturbances of the atmosphere, caused by the Sun's heat, are obstacles more serious than any encountered in the case of night observations. The peculiar advantage of the equatorial refractor—viz., that the object glass is not seriously distorted, nor the focus greatly changed, by exposure to sunlight—is offset by the necessity of attaching all spectroscopes and other accessory apparatus to a movable telescope tube. This defect of the equatorial has greatly retarded the progress of solar research, by delaying the application of long-focus grating spectrographs to the study of the solar image.

Many important investigations require the provision of a telescope giving a sharply defined solar image, of large diameter, at a fixed position within a laboratory. The focal length of such a telescope must not change rapidly when the instrument is exposed to the Sun. The image must not rotate, and the laboratory conditions must permit the successful use of the largest and most powerful spectrographs and spectroheliographs.

The Snow telescope meets most of these requirements in a very satisfactory manner. Under suitable atmospheric conditions the large solar image given by it is so sharply defined as to resemble a steel engraving. The five-foot spectroheliograph has permitted excellent photographs of this image to be obtained with the lines of calcium, hydrogen, iron, and other substances. A Littrow spectrograph of eighteen feet focal length has provided the means of photographing the spectra of sun-spots, and the results have proved very useful in the study of the strengthened and weakened lines. The one difficulty with the Snow telescope is the distortion of the image and the change of focus when the mirrors are exposed for some time to the Sun. This is not serious in our present work, since short exposures suffice in photographic observations with the five-foot spectroheliograph and the Littrow spectrograph. But if long exposures were needed, as in the case of a thirty-foot spectroheliograph, the

[1] *Contributions from the Solar Observatory*, No. 14.

change of focus during the exposure would be a serious obstacle. Various ways of overcoming this difficulty have suggested themselves, and one of these will shortly be tried. But since the Snow telescope is fully occupied with its present work, for which it has proved to be well adapted, it has become necessary to devote special attention to the design of a second coelostat telescope. The daily program of observations keeps the Snow telescope so constantly in use that all new work, such as the application of the thirty-foot spectroheliograph, and the study of sun-spot spectra with the remarkable eight-inch grating, recently made and offered for our use by Professor Michelson, must be deferred until the new telescope is completed.

A careful study of the requirements of solar research on Mount Wilson has led me to the following conclusions:

1. A coelostat should be employed, in preference to any form of heliostat or siderostat, since it causes no rotation of the solar image.

2. The coelostat should be mounted at a considerable height above the ground, to reduce the effect of the hot air rising from the Earth.[1]

3. The ground about the instrument should be shaded, for the same reason.

4. The telescope should be best adapted for early morning and late afternoon observations, the former being the more important, because of the better definition.

5. All the mirrors employed should be completely filled with sunlight, to obviate irregular distortion.

6. The mirrors should be extremely thick, to reduce the degree of bending caused by heating.

7. The backs of the mirrors should be silvered, and exposed to sunlight, to compensate bending.

8. The beam of light, after reflection from the second mirror, should be vertical, to reduce the danger of disturbances across the wave-front.[2]

9. The image should be formed by an object-glass instead of a

[1] See report of observations made from a tree, etc., in "A Study of the Conditions for Solar Research at Mount Wilson, California," *Contributions from the Solar Observatory*, No. 1, pp. 23, 25, 26.

[2] The advantages of a vertical beam have been pointed out by Plummer in *Monthly Notices*, **65**, 500, 1905. They have also been mentioned by Frost in a personal letter to me.

concave mirror, in order to bring the focal plane near the ground, to reduce variations of focal length, to shorten the path between coelostat and image, and to facilitate the use of a large spectroheliograph.

10. The walls below the coelostat should be constructed so as to heat as little as possible, when exposed to the Sun. Other devices may also be employed to decrease the evil effects of rising currents of heated air.

11. The spectroscopic laboratory should be below ground, to insure constancy of temperature and great stability of the instruments.

The vertical coelostat telescope shown in outline in the figure, is the outcome of experience with the Snow telescope, and is intended to satisfy the conditions named above.

Two entirely independent galvanized steel towers (about 60 feet high) are required: one (in lieu of a pier) for the support of the coelostat, the other to shield the inner tower from the wind.[1] The outer tower is to be covered with canvas louvres (not shown in the sketch) similar to those used on the Snow telescope house.[2] To prevent vibrations of the outer tower from being transmitted through the ground to the inner tower, the concrete base of one tower will stand on the surface of the ground, while the foundation of the other will extend below the surface. Guy ropes will also be used where necessary. Indeed, the calm which prevails on Mount Wilson during the best observing hours of the dry season has made it seem advisable to try the experiment of using the inner tower alone at first, depending for stability on a large number of steel guy ropes. The inner tower will be used as a skeleton, without covering, so as to expose but little area. If it does not prove to be steady enough, the other tower, with louvres, will be erected later.

The second mirror stands near the middle of the tower. It is to be elliptical in form, with major axis of 22¼ inches (56.5 cm) and minor axis of 12¾ inches (32.4 cm). Both this mirror, and the coelostat mirror (17 inches = 43.2 cm in diameter), will be 12 inches (30.5 cm

[1] President Woodward assures me, after his experience in observing from the summit of lofty wooden towers, similarly shielded, in the work of the Coast Survey, that no trouble from vibration need be feared.

[2] *Contributions from the Solar Observatory*, No. 4; *Astrophysical Journal*, **23**, 6, 1906.

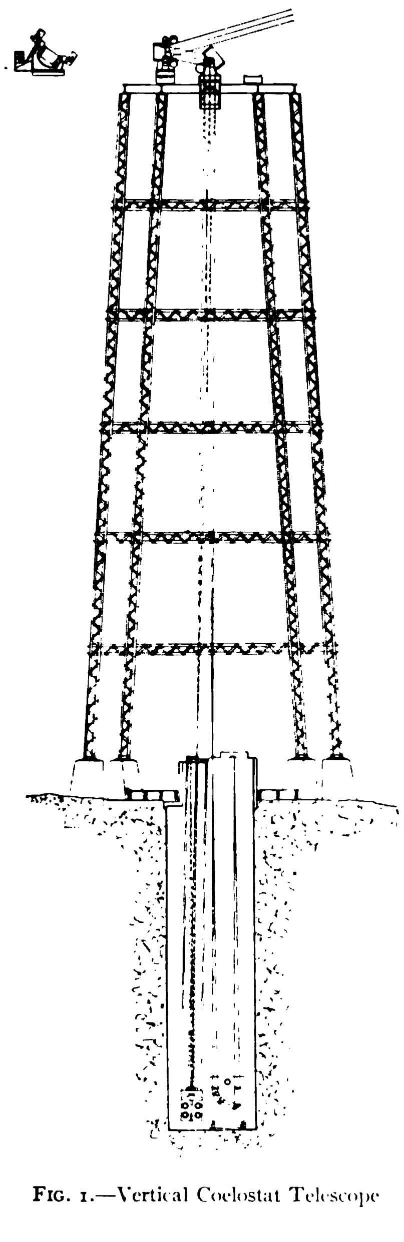


FIG. 1.—Vertical Coelostat Telescope

thick). Experiments made by Professor Ritchey have shown that when a silvered mirror is heated by radiation, the front (warmer) surface becomes convex and the rear surface concave. By placing an electrically heated disk near the rear surface the form can be quickly restored, though the figure, when thus corrected, is not perfect. This experiment, and others involving a test of the curvature of the surfaces, show that the effect of exposing the mirror to sunlight is to produce an actual bending of the mirror. The simple expedient of greatly thickening the disk, so as to increase the resistance to bending, naturally suggested itself to me.

The second mirror is given an elliptical form, so that it may be completely filled with sunlight without undue increase in the size of the coelostat mirror. Its position in this design is such that the polished and silvered back can easily be exposed to the Sun. The angle of incidence is not ordinarily the same as that at the front surface, but the difference is partially offset by the fact that the heating effect of the beam on the face is reduced through the loss incurred by reflection from the coelostat mirror. It is hoped that the great thickness of the mirrors will render more accurate compensation unnecessary.

The coelostat stands on rails, which permit it to be moved north and south. For northern declinations of the Sun it stands north of the second mirror, for southern declinations south, and when the Sun is at the equinox the coelostat is due east or west. The arrangement shown in the figure is that employed with the low morning Sun, for which the telescope will most frequently be used. For the low afternoon Sun the coelostat is transferred, by means of its carriage (which moves easily across east-and-west rails connecting the north-and-south rails at one end) to the rails east of the second mirror. The second mirror is then rotated 180°, and the beam sent vertically downward as before. The back of the coelostat mirror will be heated, if necessary, by sunlight reflected from a large mirror, of ordinary thin plate glass, mounted just behind it. The rails are accurately planed and defined in position, so that the polar axis of the coelostat will always remain in adjustment.

The coelostat will resemble the one used on the Snow telescope,

except for the much greater thickness of the mirror and some improvements in the driving mechanism.

The beam will be reflected vertically downward from the second mirror to a 12-inch (30.5 cm) object-glass, by Brashear, mounted immediately below it. The use of an object-glass, instead of a concave mirror (as in the Snow telescope), is somewhat objectionable, because of its imperfect achromatism. But in practice this is not a serious matter, and the other advantages greatly outweigh this single disadvantage. In the first place, the path of the beam between the second mirror and image, for the same focal length of 60 feet, (18.29 m) is less than half as great as in the Snow telescope, thus reducing the disturbance of the image. Again, the image is formed at a point near the ground, rather than at the top of the tower, where large spectroscopes could not be used to advantage. Finally, the object-glass can be mounted in a carriage, moving on steel balls along rails, thus providing a simple means of producing the motion of the solar image across the collimator slit of the 30-foot (9.14 m) spectroheliograph.

An electric motor, of variable speed, mounted on a pier at the ground level, will drive the photographic plate across the camera slits, and at the same time give synchronous motion to the 12-inch object-glass by means of a vertical shaft passing up through the tower. There will be three collimator slits, so that three photographs of the same part of the Sun may be taken simultaneously with the aid of different lines. The high dispersion of the spectroheliograph should permit some of the narrower dark lines to be used. The collimator and camera lenses (by Brashear) have an aperture of 8 inches (20.3 cm) and a focal length of 30 feet (9.14 m).

The prisms were ordered early in 1905,[1] but have not yet been completed, owing to the difficulty of making suitable glass of the necessary size. For this reason fluid prisms, provided with special means of stirring the liquid contents and of maintaining it at a constant temperature, may be adopted. Slight variations in the position of a spectral line, due to residual fluctuations of temperature, may be compensated by rotating the mirror in the optical train of the spectro-

[1] It was originally intended to use this spectroheliograph with the Snow telescope.

heliograph. For guiding purposes, a line near the one in use would be held in a fixed position, just as in the analogous case of stellar photography.

A small change in the inclination of the mirror at the summit of the tower will cause the solar image to fall on the slit of the Littrow spectrograph, shown at the left of the spectroheliograph. Through the great kindness of Professor Michelson, which I wish especially to acknowledge, this spectrograph will contain an 8-inch (20.3 cm) plane grating, having 500 lines to the millimeter. The aperture and focal length of the objective which serves for both collimator and camera will be 6 inches (15.2 cm) and 30 feet (9.14 m) respectively. The photographic plate will be placed near the slit, as in the case of the Littrow spectrograph used with the Snow telescope. This spectrograph is intended especially for the study of the solar rotation and the photography of the spectra of sun-spots.

George E. Hale

October 1906

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXV MARCH 1907 NUMBER 2

SECOND PAPER ON THE CAUSE OF THE CHARACTERISTIC PHENOMENA OF SUN-SPOT SPECTRA[1]

By GEORGE E. HALE and WALTER S. ADAMS

In a previous study[2] of more than two hundred lines photographed in the spectra of sun-spots it was shown that the lines which are strengthened in spots are, in general, strengthened in the laboratory when the vapor producing them is reduced in temperature. The same condition decreases the relative intensity of lines that are weakened in spots. In a note added to the paper it was stated that at least one of the flutings of titanium appears in our photographs of spot spectra. In the present paper we bring forward additional evidence favoring the view that most of the characteristic phenomena of sun-spot spectra are due to a reduction in temperature of the spot vapors below that of the reversing layer.

In an important paper[3] Fowler, though apparently inclined to retain his former view of a general intensification of the lines of certain elements in spots, gives some results with which ours are in good agreement. He points out that certain lines of iron, titanium, vanadium, and scandium, which are strong in arc or flame, are

[1] *Contributions from the Solar Observatory*, No. 15.

[2] George E. Hale, Walter S. Adams, and Henry G. Gale, "Preliminary Paper on the Cause of the Characteristic Phenomena of Sun-Spot Spectra," *Contributions from the Solar Observatory*, No. 11, *Astrophysical Journal*, 24, 185–213, 1906.

[3] *Trans. International Union for Co-operation in Solar Research*, 1, p. 201.

strengthened in spots. He also shows that many enhanced lines are weakened in spots. Fowler's general conclusion is as follows:

> The general result of the preliminary discussion is to suggest that, while the enhanced lines of some elements are usually reduced in intensity in the spot spectrum, the arc lines are intensified in accordance with their intensities in the arc-flame, thus suggesting that the additional absorption is produced by relatively cool vapors. This result was, in fact, long ago arrived at by Sir Norman Lockyer, who stated that "many of the lines seen in spots are lines seen at low temperatures (some of them in the oxyhydrogen flame), and none of them are those brightened or intensified when we pass from the temperature of the electric arc to that of the electric spark." Such a reduction of temperature also accords well with the presence of spot bands, as already remarked by Cortie and others.

Fowler's views are thus in perfect harmony with our own, except as regards the cause of the weakening of the "enhanced" lines. On this point he writes as follows:

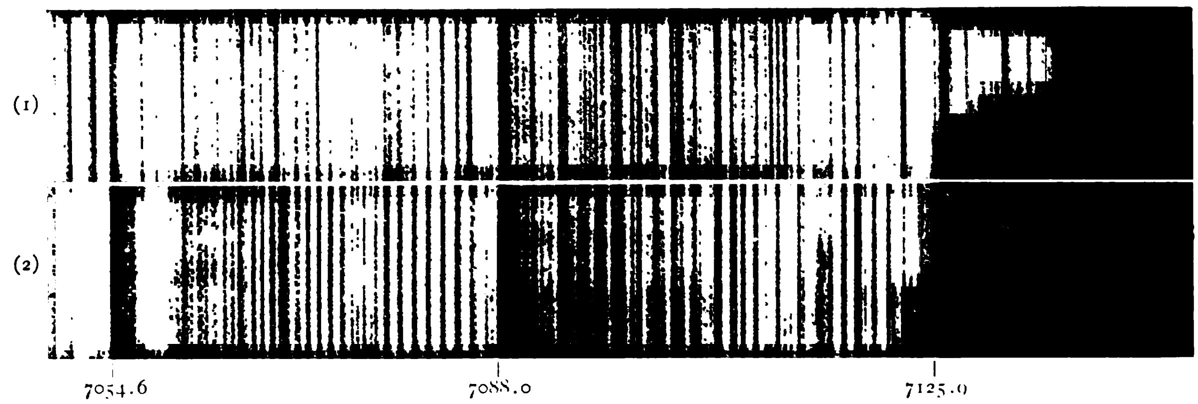
> It is probable that the vapors producing the enhanced lines are chiefly restricted to the higher chromospheric levels, but it is not yet clear whether the reduced intensities in spots is due to the withdrawal of these vapors from over the spot, or to the absence of a sufficiently luminous background to strongly exhibit their absorption (p. 220).

It has seemed to us that, if the strengthening of the flame lines may be accounted for by the reduced temperature of the umbral vapors, the weakening of the spark lines may equally well be attributed to the same cause. This subject, however, is still under investigation.

It is evident, from Fowler's failure to identify the titanium flutings in spot spectra, that photography may be considered to offer special advantages for the study of the fainter lines and bands.

In view of Lockyer's early observations, it is odd that he did not adopt the hypothesis that spots are regions of reduced temperature. He has long held, on the contrary, that at times of sun-spot maximum (when our spectra were photographed) the spot temperature is so high as to dissociate many of the elements represented in the solar spectrum. If, as a paper published before the appearance of our Mount Wilson results indicates, he now believes that sun-spots and red-type stars are cooler than the reversing layer, he may have abandoned or modified his views regarding dissociation in spots.

PLATE III



TITANIUM FLUTINGS IN SPECTRA OF (1) SUN-SPOT AND (2) ARC-FLAME.

TITANIUM FLUTINGS IN SPOT SPECTRA

In our study of the spot spectra the titanium fluting of shortest wave-length which we have as yet been able to identify with certainty is that beginning at λ 5598.0. There seems to be some evidence for the presence of two more refrangible flutings at λ 5166 and λ 5449, but it is by no means conclusive. It would, in fact, be somewhat remarkable if these were present upon our plates, since they are much fainter in the titanium flame spectrum than the fluting at λ 5598, which is very faint and difficult in spots.[1]

As we go toward longer wave-lengths, the successive flutings become stronger in the spot spectrum, until in the deep red they form its most characteristic feature. The fluting at λ 7055 is especially noticeable, as it occurs in a region of the spectrum in which there are but few strong lines to interrupt its continuity. A comparison of this fluting in the spot and in the flame of the electric arc (negative copy) is given in Plate III.

The following tables contain the results of measures of the lines in the spot and the flame spectra, and a comparison of the two with each other and with the corresponding lines in the spectrum of the disk, where such exist. It has seemed desirable to give the evidence in full, not only on account of the importance of the question of the existence of the titanium flutings in spots, but also because of the almost equally important one of their existence in the spectrum of the disk. The results given here, however, are not to be regarded as complete, nor do the coincidences shown represent by any means the total contribution of the titanium flutings to the spot spectrum in this region. The flame spectrum of titanium in the red consists of a large number of flutings and bands, which overlie one another in an extremely complex fashion and contain a vast number of lines. The spectrum of the spot is similar, and a complete comparison of the two will involve a very large amount of measurement, and belongs rather to an exhaustive investigation of the spot spectrum than to the objects of this communication. Enough is given here, we believe, to indicate the nature of the results which a complete study would furnish.

[1] The head of this fluting has been measured upon numerous plates. Only two plates, however, and these taken under exceptionally good conditions, show the fine lines satisfactorily.

The method of procedure which we have followed has been to select the more important of the titanium flutings in the spectrum of the flame of the electric arc and to measure all lines within a considerable distance of their heads. The results are then compared with the lines measured in our photographs of spots. In order to show what proportion of the spot lines is due to titanium, a complete list is given for the regions investigated, except, of course, that the lines identified by Rowland are not included. The arrangement of the tables is as follows: The first two columns contain the wave-lengths and intensities of the lines measured in spots, the third and fourth columns the wave-lengths and intensities of lines given in Rowland's tables which may be coincident with the spot lines, and the fifth column the wave-lengths of the lines measured in the flame spectrum of titanium. We have assumed for the greater part of the spectrum the value of 0.05 tenth-meters as the largest discordance which may exist between lines coincident in spot, disk, and flame, our measures of the stronger lines identified by Rowland indicating ranges of about this amount. In the extreme red this value becomes slightly larger, the character of the lines preventing such accuracy of measurement as can be attained in the more refrangible regions.

An analysis of these tables gives the following results:

Total number of spot lines	234
Coincident with *Ti* lines	152

Thus about 65 per cent. of the total number of spot lines have corresponding lines in the band spectrum of titanium.

Number of spot lines with *Ti* coincidences which have possible coincidences in the disk spectrum		42
Identified by Rowland or not coincident with *Ti* lines	19	
Strong lines of *Ti* or other elements	6	25
		17
Number of lines in disk coincident with *Ti* lines		17
Number of spot lines with no *Ti* coincidences		82
Number of coincidences in disk for these		36

This summary leads to the important conclusion that 43 per cent. of the spot lines with no coincidences in the *Ti* band spectrum, and but 11 per cent. of those with such coincidences, have corresponding lines in the disk. Since the latter quantity is no larger than could

PLATE IV

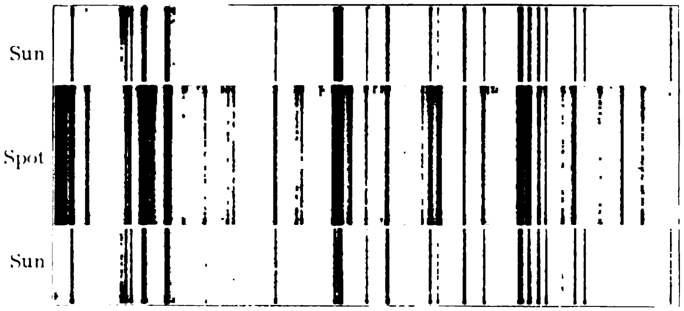


FIG. 1

Region λ 5040–λ 5090

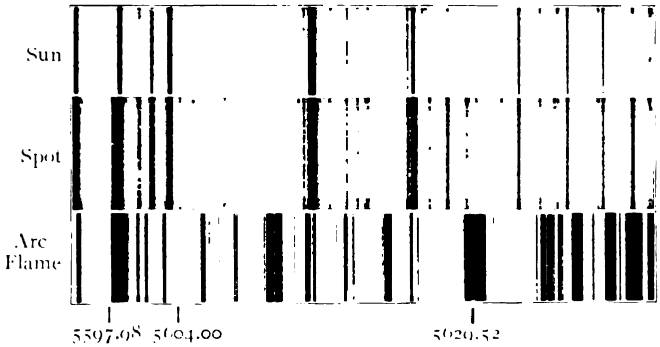


FIG. 2

Titanium Flutings in Spot and Arc Flame.

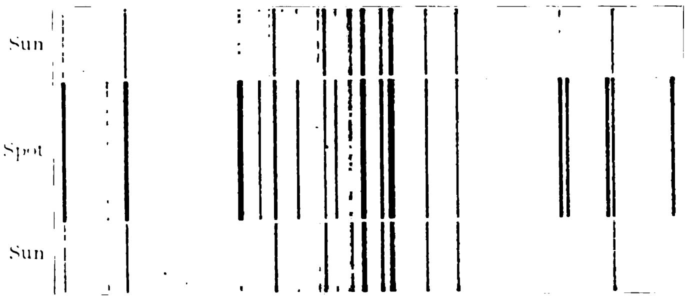


FIG. 3

Region λ 5680–λ 5740

PHOTOGRAPHS OF SUN-SPOT SPECTRA.

TABLE I

COMPARISON OF TITANIUM FLUTINGS WITH SPOT AND SOLAR SPECTRA

Spot		Sun		*Ti*	Notes
λ	Int.	λ	Int.	λ	
5597.98	1	...	.....	.99	1st head
5600.79	000	...	.....	.80	
5608.48	000	.52	000	.49	
5698.74	000	...	.....	.74	
5609.04	000	...	.....	.02	
5609.31	000	...	.....	.33	
5611.02	00	...	.....	.02	
* * * *	* *	* *	* * *	* *	
5629.52	0–1	...	.....	.52	2d head
* * * *	* *	* *	* * *	* *	
6158.84	0–1	.89	000	.84	1st head
6159.38	0	...	.....	.36	
6164.86	00	...	.....	.90	
6168.00	00–0	...	.....	.00	
6171.41	0	.44	0000	.40	
6172.18	1	.16	000	.16	
6174.41	1	...	......	.45	
6184.72	0	...	......	.67	
6187.82	0	...	......	.82	
6189.02	1–2	...	......	88.97	
6189.58	1	.59	0000 N	...	*V* 6189.58
6190.12	1	...	......	.11	
6192.42	0	...	......	.39	
6193.02	0–1	...	......	.01	
6193.56	0–1	...	......	.56	
6194.35	0	...	......	.30	
6195.31	00	...	......	.33	
6195.69	0–1	.66	0 N	...	
6197.29	0	...	......	.32	
6197.88	0–1	...	......	.91	
6198.68	1–2	...	......	.64	
6199.84	1	.72 .98	00 0000	.84	
6201.20	0–1	.18	000	.18	
6201.94	0	...	......	.94	
6202.58	1	...	......	.59	
6202.96	0	...	......	...	
6203.38	1	...	......	.38	
6206.60	1	...	......	.59	
6208.08	0–1	...	......	.08	
6208.65	0–1	...	......		
6209.17	1	...	......	08.97	*Ti* line double or triple
6209.55	0	...	......		
6210.18	2	...	......	.17	
6210.89	5	.90	000 N	...	*Ti* line at 6211.02, which is probably included in this
6211.88	0	...	......	.91	
6212.15	0	...	......	...	
6212.48	0	.48	0000	.45	
6214.07	2	.08	*V* 000	.03	
6214.43	0	...	......	...	*Cr* line at 6214.41
* * * *	* *	* *	* * *	* *	
6262.01	0	...	......	61.99	

TABLE I—*Continued*

Spot λ	Spot Int	Sun λ	Sun Int.	Ti λ	Notes
6262.52	0	...	......	.50	
6262.90	0	...	......	63.20	*Ti* line double
6263.45	0		......		
6263.99	1	...	......	.97	
6264.82	1	...	......	.84	
6266.14	2	...	......	.16	*V* 6266.54
66.51		.55	000	...	
6269.00	2	68.82	0000	...	
		69.08	V 000 N		
6274.86	4	.87	00	...	*V* 6274.89
6276.40	0	.45	0000 N	...	
6284.36	0	...	......	...	
6285.34	4	.38	V 00 N	...	
6286.17	1	...	......	...	
6286.98	0–1	87.01	0000 N	...	*V* 6287.02: *Cr* 6286.95
6289.23	00	...	......	...	
6289.70	2	.61	A (O) 1	...	Spot line broad
		.79	0000		
6292.11	0	...	......	.13	*V* line at 6292.09
6293.09	6	.03	*V* 000	.09	
		.17	A (O) 3		
6296.78	4–5	...	......	.82	
6297.25	0	...	......	...	
6300.58	0	.54	000	.62	
6300.93	0–1	.90	000	...	
6303.97	4	.98	000 N	.98	Strong line of *Ti*
6305.03	00	.07	0000 N	...	
6305.90	10	.88	0000	...	Spot line very broad. Due in part to strong *Ti* line 6305.77
				.77	
6307.21	0	...	......	...	*Cr* 6307.19
6307.59	0	...	......	.55	
6307.99	0	...	......	08.01	
6308.52	1	...	......	.47	
6309.72	0–1	...	......	...	*V* 6309.74
6311.06	1	.06	0000	...	
6312.46	3	.46	00 N	.46	Strong line of *Ti*
6313.18	1–2	...	......	.14	
6313.72	0–1	...	......	...	
6314.47	1	.45	A (O) 0	.42	
6316.62	1	.51	A(wv)0000N	.54	Spot line very broad
		.82	0000	...	
6322.02	0–1	...	......	...	
6323.53	1	...	......	.51	
6324.70	0	.71	A (O) 00	.74	*V* line at 6324.74
6325.39	2	.38	0000	.34	
6326.34	0–1	...	......	...	
6327.11	1	...	......	.13	
6331.80	1	...	......	...	
* * * *	* *	* *	* * *	* *	
6651.62	2	...	......	.56	1st head
6652.32	00–0	...	......	.30	
6653.19	00	.22	0000	.24	
6653.80	1	...	......	54.44	Broad patch in both spot and *Ti*
54.89					

TABLE I—*Continued*

Spot		Sun		*Ti*	Notes
λ	Int.	λ	Int.	λ	
6655.21	00	...	......	...	
6656.67	0–1	.62	0000	.67	
6657.25	0–1	.20	000 N	.27	
6657.99	0–1	...	......	.96	
6658.58	0	...	......	.54	
6659.20	0	.16	0000	...	
6660.78	0–1	.83	0000	...	
6661.37	1	.32	*Cr* 00	.32	
6662.08	00	62.01 ...	0000 N	61.99 ...	Broad in spot
6662.67	0	...	......	...	
6664.83	1	...	......	.79	
6665.55	1	...	......	.52	
6666.63 .77	0	... .78	...... 000 N	.70	
6667.24	00	...	......	...	
6668.03	0	...	......	.02	
6668.63	00	.64	000 N	...	
6669.06	0	.04	0000 N	.01	
* * * *	* *	* *	* * *	* *	
6681.11	1	...	......	.11	2d head
6681.65	00	...	......	.62	Practically a band in spot
6682.34	00	...	......	.30	
6682.91	00	...	......	...	
6683.63	00	...	......	.65	
6684.96	00	...	......	...	
6686.10	0–1	...	......	.11	
6687.78	1	.75	0000	...	
6688.28	00	...	......	...	
6689.06	0	...	......	...	
6689.59	00	...	......	...	
6690.14 91.44	1–2	...	......	...	A broad band in spot
6691.91	0	...	......	.95	
6692.83	00	...	......	.80	
6693.62	0	...	......	.61	
6694.58	00–0	...	......	.56	
6695.53	00	...	......	.51	
6697.19	00	...	......	...	
6698.12	00	...	......	.10	
6698.87	2	.91	0	.84	
6699.90	0	...	......	.92	
6700.36 6700.82	00 00	... ...	....	.64	*Ti* line double
6701.67	0–1	.62	000	.65	
6704.82	00	...	......	.86	
6705.85	0	.75 .94	0000 0000	.86	
6706.80	0	...	......	.81	
6707.39	00	...	..	...	
6708.13	4	.18	0000	...	*Ca* 6708.15
6709.23	0	.23	0000N	24	
6710.57	1	.57	0	...	
6711.49	0	.53	0000	.52	

TABLE I—*Continued*

Spot		Sun		*Ti*	Notes
λ	Int.	λ	Int.	λ	
6711.98	oo	...	......	...	
6712.57	oo	...	......	...	
6713.09	o	...	......	.14	
6714.30	I	...	......	.25	
6714.80	oo	...	......	.82	
6715.59	2	.64	I	.54	
6716.12	oo	...	......	.17	
6716.98	o	....	......	.95	
6718.49	oo	...	......	...	
6719.84	I–2	.88	ooo N	.86	Strong line of *Ti*
6720.49	oo	...	......	...	
6721.18	oo	...	......	.18	
6721.66	oo	...	......	.65	
6722.24	2	...	......	.24	
6722.75	oo	...	......	.78	
6723.74	oo–o	...	......	.76	
6724.20	oo–o	...	......	.19	
6724.60	oo	...	......	...	
6725.27	o	.22	oooo	...	
6725.89	o	...	......	.86	
6726.30	oo–o	...	......	.26	
6727.30	oo	...	......	...	
6728.53	I	...	......	.48	
6729.26	I	.27	oo	.22	
6730.05	oo	.00	ooo N	...	
6730.56	oo	.56	ooo N	.55	Fairly strong *Ti* line
6731.08 31.83	o	...	......	.56	Broad patch in spot
6731.96	..	...	......	.94	Bright streak in spot: dark space [in *Ti*
6732.83 34.00	I–2	..	......	... .06	Broad band
6735.31	oo	.28	oooo	...	
6735.79	o	...	......	.77	
6736.36	o	...	......	.23	Spot line is double: *Ti* line forms [one component
6737.28	o	...	......	.30	
6738.07	o	...	......	.07	
6738.70 39.79	2	... .78	...... o	.65 ...	Broad band in spot
6740.57	oo	...	......	...	
6740.82	o	...	.....	.83	
6741.72 42.44	2	..	......	42.12	
* * * *	* *	* *	* * *	* *	
7054.60	2	...	......	.65	Strong head in both spot and *Ti*
* * * *	* *	* *	* * * *	* *	
7069.32	o	.35	oooo N	...	
7069.73	oo	...	......	...	
7070.40	I–2	...	......	.37	
7071.35	I	...	......	.30	
7072.29	I–2	...	......	.27	Spot line includes solar 7072.13
7072.75	oo	.72	ooo	...	
7073.27	I	...	....	.29	

TABLE I—*Continued*

Spot		Sun		*Ti*	Notes
λ	Int.	λ	Int.	λ	
7074.18	00–0	...	......	.33	
7074.54	00	...	......		
7075.15	0	.19	0000	...	
7076.18	0	...	......	...	
7076.72	00–0	...	......	...	
7077.23	00	...	......	.15	Lines poor in both spot and *Ti*
7077.99	1–2	...	......	78.10	
7078.29	0		......		
7079.24	2	...	......	.26	
7080.47	1	...	......	.47	
7081.68	1	...	......	.69	
7082.96	1–2	...	......	.96	
7084.25	1–2	.24	000 N	.24	Fairly strong line of *Ti*
7086.66	00	.59	0000	...	
7087.90	1–2	...	......	88.00	2d head of fluting
7089.28	00	...	......	...	
7091.41	00	.43	0000	...	
7092.20	00	.22	000 N	...	
7092.85 93.65	2	...	......	93.3	Broad band in spot; very broad, hazy, and poor in *Ti*
7094.79	00	...	......	95.00	
7095.16	00		......		
7096.12	0	.14	000	...	
7096.63	0	.66	0000	...	
7097.50	0	...	......	...	
7099.19	0	.22	0000 N	...	
7099.83	1	.81	0000 N	.77	
7101.31	1	...	......	.35	
7102.63	1–2	.56	000 N	...	
7103.26	0	...	......	...	
7104.63	0	...	......	.65	
7105.02	00	...	......	...	
7106.36	1	.43	A ? 000Nd ?	.59	
7106.77	1	...	......		
7108.13	00	.19	0000 N	...	
7108.75	0	...	......	...	
7109.91	0	...	......	.87	
7110.66	00	.70	0000 N	...	
7111.75	1	.73	A 000 N	.73	
7112.87	00	...	......	...	
7113.91	00	.87	0000	...	
7116.29	0	...	......	...	
7117.34	1–2	...	......	.37	
7118.52	1–2	.55	A 1	...	Spot line probably double
7119.21	00	.25	0000	...	
7119.93	00	.96	000 N	20.00	
7120.16	00	...	......		
7120.43	00	...	......	...	
7121.21	0	...	......	.25	
* * * *	* *	* *	* * *	* *	
7125.89	2	...	......	.88	3d head of fluting

be accounted for by accidental coincidences within the limits of accuracy of the measures, the conclusion is justified that the titanium flutings are not present in the spectrum of the disk. This conclusion is greatly strengthened by the fact that of the nine heads of flutings which have been measured in spots and in the flame, and which are conspicuous as compared with the remainder of the lines of the band spectrum, only one has a possible counterpart in the spectrum of the disk. It seems certain, therefore, that the temperature of the reversing layer is normally too high to admit of the presence of the band spectrum of titanium.

SPARK[1] LINES IN THE RED

Though our comparison of spot lines with the spectra of the various elements is as yet very incomplete, and we have no plates extending to wave-lengths greater than λ 6700 for any except titanium, some results already found for the lines enhanced in the spark, and for a few lines strengthened in the flame spectrum, are of sufficient interest to warrant their inclusion here.

The following list includes the more important of the spark lines of iron, titanium, and nickel, and indicates their behavior in spots.[2] The amount of enhancement in the spark is given on a scale of 0 to 5.

[1] Since the beginning of this investigation, we have been troubled by a question of nomenclature. Lockyer has applied the appropriate name "enhanced line" to a line that is strengthened in the spark, as compared with the arc, and this designation is now generally used in the literature of spectroscopy. No suitable name has been suggested for lines that are strengthened in the flame of the arc, where the "enhanced lines" are weakened. In any event, confusion is likely to result from the fact that the lines which have always been regarded as characteristic of sun-spots—those which are more conspicuous in the spot than in the solar spectrum—are not "enhanced" lines. On the contrary, they are lines that are strengthened in the flame; the "enhanced" lines are weakened in spots. In the present paper we have thought best to adopt the old terms "spark line" and "flame line" for lines strengthened in the spark or flame, respectively. It is, of course, well understood that spark lines do not exist exclusively in the spark, or flame lines in the flame. The intensity of spark lines increases as the temperature rises, while that of the flame lines increases as the temperature falls.

[2] As the question of impurities has not yet been fully investigated, some of these lines may arise from foreign sources, though they would still seem to belong in the list of spark lines.

IRON

	Enhancem't in Spark	Intensity in Sun	Intensity in Spot	
6042.32	1	*Fe* 3	2	
6103.40 .51	1–2	*Fe* 4 — 1	4	
6147.95 48.04	3	— 2 *Fe* 3	3	
6149.46	3	— 2	0	
6238.60	3	— 2	0	
6247.77	5	— 2	00	
6315.52	1–2	*Fe* 2	1	
6380.96	1	*Fe* 4	1	Rowland's intensity too high. Complicated in spark by air line
6417.13	2	*Fe?* 1	00	
6420.17	1	*Fe* 4	2–3	
6456.60	5	— 3	0–1	
6516.31	1–2	— 2	00	
6569.46	1–2	*Fe* 5	4	Complicated in spark by air line
6627.80	1–2	*Fe?* 0	00	
6663.49 .70	1	— 1 *Fe* 3	3–4	

TITANIUM

6491.80	2	— 1	00–0	

NICKEL

6119.97	1	*Ni* 0	00	Difficult in spot on account of V 6119.74
6125.24	2	— 1	00–0	
6130.34	1	*Ni* 1	0	

The above list affords strong additional evidence for the conclusion that spark lines are weakened in the spectra of spots.

FLAME LINES IN THE RED

In view of the importance of accurate determinations of changes in line intensities in passing from the core of the electric arc to the flame, it has seemed desirable to take up this side of the investigation with a photometer especially adapted for the purpose. Accordingly, we have as yet made no systematic examination of the lines strengthened in the flame for the red region of the spectrum. A few cases, however, are so striking that they are deserving of comment.

The following table indicates the behavior in this regard of two lines of calcium and two lines of sodium. The amount of strengthening in the flame as compared with the core of the arc is on a scale of 0 to 5.

CALCIUM

	Strengthening in Flame	Intensity in Sun	Intensity in Spot
6573.03	5	*Ca?* 1	8
6708.18	4	0000	4*

* It is not certain that this line is due to calcium, as it appears strongly on plates of several other elements. It is in every case greatly strengthened in the flame.

SODIUM

6154.44	2	*Na* 2	5–6
6160.96	3	*Na* 3	5

The two calcium lines given above show the largest increase of intensity in passing from the core of the arc to the flame of any lines which we have encountered. The remarkable degree to which they are affected in spots also makes them conspicuous among the spot lines.

MOTION OF SPOT VAPORS IN THE LINE OF SIGHT

The importance of the question of the motion in the line of sight of the spot vapors as bearing on any theory of spot structure, is of course, very great, and has been kept in mind in the investigation of our observational material. In the method which we have adopted of photographing spot spectra it is necessary to make the exposures on spot and disk separately, occulting one while the other is being photographed. For this purpose an occulting bar is moved across the slit by means of a rack and pinion, as in most stellar spectrographs. Accordingly, the danger of errors arising from instrumental sources should not be great.

The study of the plates has led to the conclusion that there is as a rule very little motion in spot umbrae. Out of eighty plates of eleven spots only two gave any reasonably certain displacements of the spot lines, and even in these two cases the values were close to the limit of accuracy of the measures. In both instances the motion

was directed downward, and amounted to about 0.2 km a second. In one case, moreover, the motion was certainly temporary, since plates of the same spot taken on the following day gave no displacements whatever. The general conclusion, then, seems to be justified that the vapors forming the umbra of a well-developed spot are normally nearly at rest, with perhaps some presumption of a slow downward drift. This result is in agreement with that found by Mitchell from the study of a large number of spots during 1904–5. He says: "Line-shifts in the spot-spectrum, with the exception of those due to hydrogen, are very rare."[1]

EFFECT OF DENSITY

The suggestion has been made that the relative intensities of lines observed in spots and in the laboratory may be due to the increased density of the vapors producing them. To test this question, the following experiments have been made in our spectroscopic laboratory by Dr. Olmsted.

The spectrum of a 30-ampere arc, between iron poles, was compared with that of a 2-ampere arc, between carbon poles containing only a trace of iron. The changes in the relative intensities of the lines were similar to those observed in passing from the ordinary solar spectrum to that of sun-spots. Moreover, a 2-ampere arc between iron poles gave the same spectrum as the 2-ampere arc between carbon poles, with but little iron present.

A spark between iron poles, with no self-induction, was compared with a spark between one iron and one carbon pole, with self-induction to cut down the temperature. The changes of relative intensity were those observed in all other cases when the temperature is reduced. Two carbon poles, with a small amount of iron filings present, and two iron poles, both with self-induction, gave the same relative intensities of the lines.

Similar experiments were tried with manganese and calcium, in both the spark and the arc. In all cases the relative intensities of the lines seemed to depend simply upon the strength of the current, or the amount of self-induction, and to be entirely independent of the density of the radiating vapor.

[1] *Astrophysical Journal*, 22, 38, 1905.

THE TEMPERATURE OF SUN-SPOTS

The presence of the titanium flutings in spots, and their apparent absence from the ordinary solar spectrum, strongly confirm the view that the umbral vapors are cooler than those of the reversing layer. It is not yet certain that these flutings are due to the oxide, but they presumably represent a molecule that is dissociated at high temperatures. The fundamental differences between line and band spectra—for example, the fact that flutings are unaffected by pressure or by a magnetic field—are generally held to indicate that they arise from different aggregations; in short, that lines represent the atom, while bands represent the molecule.

In our laboratory experiments the strengthening of flame lines and the weakening of spark lines have always appeared to be associated with reduction of temperature. Crew's work has shown that rapid change of potential is an effective means of strengthening spark lines, but it almost certainly involves a momentary increase of temperature, and in any case cannot be considered the only possible mode of altering the relative intensities. The theoretical considerations so ably summarized by Crew in his recent address before the American Association[1] seem to favor the view that the relative intensities of spectral lines are more easily influenced by some electrical cause than by change of temperature. But how can the results of our experiments with so many different sources be accounted for in this way? We fail to see, for example, how electrical causes could have operated in our furnace, especially after the arc, which played on the *outer* walls of the carbon tube containing the metal, had been extinguished. It seems almost certain that in this case the relative intensities of the lines were determined by the temperature, or by chemical action, which might be a function of the temperature. The range of temperature obtainable in our furnace was hardly sufficient to produce unquestionable changes in the relative intensities of lines, though there seem to be some cases of this kind in the case of chromium. Pending the continuation of this work, we cannot claim to have done more than to prove that the changes of relative intensity from a 30-ampere to a 2-ampere arc, and from core of arc to flame, resemble

[1] *Science*, January 4, 1907.

those from core of arc to furnace. However, the flame is undoubtedly cooler than the core of the arc, and in the furnace the temperature was too low to melt titanium. Moreover, the well-known changes in the relative intensities of the calcium lines, in passing from the Bunsen burner to the oxyhydrogen flame, certainly indicate that temperature is quite as competent as change of potential to produce these phenomena.

The simplest way to account for the relative intensities of lines in the spectra of sun-spots and third-type stars is to assume that reduced temperature in these sources is the effective cause. For, on the one hand, the presence of the titanium flutings, which consistently rise and fall in intensity with the flame lines in all of our sources, and are absent from the solar spectrum, leaves little doubt that the vapors in sun-spots and third-type stars are cooler than the corresponding vapors in the reversing layer. On the other hand, laboratory experiments have shown that changes of temperature may produce, either directly or indirectly, just such spectral phenomena as those here involved. It therefore seems entirely unnecessary to assume that electrical phenomena, or other such causes, are at work, though their operation is not necessarily excluded.

Although we are thus inclined to regard the relative intensities of spot lines as resulting from reduced temperature, we by no means consider this cause competent to explain the many peculiarities exhibited by spot spectra. The existence of winged lines, for example, may depend upon the density and perhaps upon the level of the corresponding vapors. Further reference to this question is made below. A more complete investigation, however, will demand much work in the future.

STRATIFICATION OF THE VAPORS IN SUN-SPOTS

We now pass to the difficult task of examining a few of the complex details of spot spectra, for the purpose of interpreting them in accordance with some rational view of sun-spot structure. We are called upon to account for the following phenomena, among others perhaps equally important:

1. The gradual decrease, as we proceed toward shorter wave-lengths, in the amount by which spot lines are affected, and the close

agreement in the ultra-violet of the spot spectrum with the solar spectrum.

2. The presence in spots of the strong and sharply defined lines of the titanium line spectrum, together with the faint flutings of the band spectrum.

3. The fact that all of the sodium lines, about 80 per cent. of the calcium lines, and about 25 per cent. of the chromium, iron, and manganese lines are accompanied by wings, while none (or very few) of the lines of titanium or vanadium are thus affected.[1]

4. The reversals of spot lines observed by Young and Mitchell.

Although no complete discussion of these points can be attempted until more laboratory work has been done, their brief consideration at the present time may aid to clear the way for further investigations. The great similarity between the spot and solar spectrum in the ultra-violet may be regarded as the maximum development of a tendency which has already become very marked in the violet region. The gradual decrease in the amount by which titanium and vanadium spot lines are affected, as we pass from the yellow through the blue to the violet, and the decrease in the number of strengthened lines of iron in the blue and violet, are strong indications of such a tendency. The changes in the relative intensities of the lines observed in the laboratory show no such marked falling-off in the more refrangible region. For this reason it seems probable that many of the spot phenomena depend upon the level of the umbral vapors in the solar atmosphere.

In our previous paper we made the purely tentative hypothesis that the gradual weakening of the spot spectrum and its replacement by an almost unmodified solar spectrum in the ultra-violet might be attributed to the presence of the ordinary reversing layer over the umbral vapors. The absorption and scattering of the more refrangible spot radiations within this layer, and especially within the spot itself, would greatly decrease their intensity, while the superposed reversing layer would produce the ordinary solar spectrum. However, the difficulty of accounting for a condition of affairs in which the

[1] These figures were derived from a study of over 400 of the strengthened lines in the region from λ 4800 λ 7300. If wings could be seen on the fainter lines, these percentages would doubtless be modified.

reversing layer, undiminished in temperature (as indicated by the unchanged relative intensities of its lines), actually overlies the umbra and penumbra, is very great. It would seem that the temperature of vapors overlying a sun-spot must be lower than that of the corresponding vapors in the reversing layer above the photosphere, both because of the absence of strong convection currents, and because of the diminished radiation from the spot, due to the absorption and scattering of the radiation proceeding from below the umbra and the comparatively low temperature of the spot vapors. Our measures of the sun-spot lines, as explained elsewhere, show very few evidences of motion in the line of sight. In the instances when motion was detected it was directed downward. This result would be in harmony with our knowledge of other spot phenomena. It is a well-known fact, for example, that eruptions rarely or never occur in the umbra, but almost invariably at a point outside of the penumbra or in a "bridge." It would consequently appear improbable that convection currents, such as offer visible evidence of their presence in the photosphere, are to be regarded as existing in the umbra. If the umbral vapors overlie the photosphere, convection currents may possibly rise from the interior below them. Their effect upon the temperature of the region above the umbra, however, would certainly be greatly diminished by the strong absorption and scattering of the spot vapors.

It is thus difficult to see how the unchanged reversing layer can exist over sun-spots. How, then, are we to account for the presence of the ordinary solar spectrum in the ultra-violet region? We have shown in previous papers that the relative brightness of the umbra as compared with the photosphere is much smaller in the violet than in the red. At λ 4000, for example, the spot spectrum must be exposed about eleven times as long as the spectrum of the disk, in order to get a negative of equal intensity, while in the yellow an exposure six times as long will suffice to give such a result.[1] The long exposure required in the violet for the spot spectrum has suggested the possibility that two principal sources, from which a solar spectrum might be derived, must be taken into account:

1. Sunlight scattered in the earth's atmosphere.

[1] The ratios were incorrectly given in our previous paper.

2. Light from the photosphere, brought upon the slit by atmospheric disturbances during the exposure.

Sunlight scattered by small particles lying above the umbra (such as produce the solar spectrum observed in the inner corona), and diffuse light in the spectroscope, may also be involved, but probably in very small degree.

It is easy to determine the approximate intensity of the scattered light in our atmosphere. Photographs of the spectrum of the sky near the sun, taken for this purpose, showed that under the conditions existing at the time the sky spectrum at λ 4000 was about one-fortieth as bright as the spectrum of the center of the sun's disk. Our observations indicate the presence of certain spot lines, and consequently a very appreciable value for sun-spot radiation, much farther to the violet than λ 4000. At present we have a considerable number of lines in the ultra-violet reaching as far as λ 3662, and under the finest conditions of definition of the solar image this limit might perhaps be extended farther. In this region the intrinsic brightness of the sky would be appreciably greater than at λ 4000, and, in the longer exposure required to photograph the spot spectrum, the sky spectrum would appear on the plate with considerable intensity. In the less refrangible region of the visible spectrum, however, it would be much fainter, and perhaps hardly perceptible, partly because of its smaller intrinsic brightness, and partly because the exposure required for the spot is relatively much shorter than in the violet.

The effect of the photospheric light which enters the slit, because of the unsteadiness of the solar image, must always be appreciable in exposures of any considerable length. The rapid decrease in the exposure time required to photograph the spectrum of the umbra of a spot, when the definition of the sun's image becomes poor, is excellent evidence of the importance of this fact. Among other causes which would tend to introduce photospheric light into the spot spectrum would be astigmatism, due to distortion of the mirrors by heating; change of focal length during the exposure, resulting from the same cause; and unsteadiness of the spot image upon the slit, due to imperfect guiding.

The combined effect of these various causes would undoubtedly

account for the presence of an ordinary solar spectrum in the ultra-violet region of spot spectra. Whether its intensity would be as great as the observed intensity cannot be certainly determined until further quantitative studies have been made.[1]

In seeking to account for the phenomena enumerated (2) to (4), page 90, let us tentatively make the hypothesis that the minimum temperature and the maximum density of the spot vapors occur in the lower part of the umbra. We know little or nothing as to the nature of the radiation which proceeds from the photosphere or from any other source that may underlie the umbra. In any case, however, the intensity of this radiation must be greatly diminished by absorption and scattering in its transmission through the spot. The faintness of the wings accompanying so many of the lines, and the greater frequency of the winged lines in the less refrangible region, would seem to indicate that the radiation of the vapors which are dense enough to produce these wide wings must proceed from a very considerable depth, and thus be subject to strong absorption and scattering, which should be most marked in the more refrangible region. Again, we have the case of the titanium spectrum, in which the flutings are faint while the lines are strong and well-defined, in no instances (or very few) being accompanied by wings. The band spectrum, according to the generally accepted view, is due to the molecules, which will be most numerous in the region of lowest temperature. It should consequently be subject to more absorption and scattering than the line spectrum, and therefore be comparatively faint. Moreover, the flutings should rapidly grow fainter in the more refrangible region, which is the case.

The line spectrum, originating for the most part in a region of higher temperature, where the vapor is less dense, should give sharp lines, without wings. Comparing the titanium spectrum, therefore, with the spectrum of sodium, iron, or any other metal of low melting-point, we may say that the spot temperature is low enough to produce a band spectrum only in the case of so refractory a substance as titanium; whereas in the case of the other metals, the line spectrum is produced throughout the entire depth of the spot vapors.

[1] For an interesting discussion of these questions which has reached us since the above was put in type, see Newall, *Monthly Notices*, January 1907.

If this purely tentative hypothesis be sound, vanadium should behave much as titanium does, on account of its high melting-point. Hitherto, however, we have not been able to produce a vanadium band spectrum in the laboratory, or to identify it in spots. We are continuing our investigations on this subject in the hope of making the test.

The hypothesis that the temperature of the spot vapors is higher in their upper part provides the simplest way of accounting for the existence of bright reversals of some of the spot lines, observed by Young and Mitchell. The question whether the lines will be reversed or not depends upon the temperature and density gradients of the vapors, just as in the case of the electric arc. If a sufficient difference of temperature exists, and the upper vapors are dense enough, reversals will occur. It is evident, however, that this subject will require much further study, made with special reference to the individual peculiarities of the reversed lines.

JANUARY, 1907

ARBITRARY DISTRIBUTION OF LIGHT IN DISPERSION BANDS, AND ITS BEARING ON SPECTROSCOPY AND ASTROPHYSICS[1]

By W. H. JULIUS

In experimental spectroscopy, as well as in the application of its results to astrophysical problems, it is customary to draw conclusions, from the appearance and behavior of spectral lines, as to the temperature, density, and motion of gases in or near the source of light. These conclusions must in many cases be entirely wrong, if the origin of the dark lines is exclusively sought in absorption, and that of the bright ones exclusively in selective emission, without taking into account the fact that the distribution of light in the spectrum is also dependent on the anomalous dispersion of the rays in the absorbing medium.

It is not in exceptional cases only that this influence makes itself felt. Of the vapors of many metals it is already known that they bring about anomalous dispersion with those kinds of light that belong to the neighborhood of several of their absorption lines.[2] In all these cases the appearance of the absorption lines must to a greater or less extent be modified by the above-mentioned influence, since the mass of vapor traversed by the light is never quite homogeneous. Hence it is necessary to investigate the effect of dispersion on spectral lines separately; we must try to distinguish it entirely from the phenomena of pure emission and absorption.

The previously described experiments with a long sodium flame,[3] in which a beam of white light alternately traveled along different paths through that flame, constitute a first attempt in this direction. With these relative displacements of beam and flame the rays of the anomalously dispersed light were much more bent, on account of the

[1] The main part of this paper was communicated at the meeting on September 29, 1906, of the Royal Academy of Sciences of Amsterdam.

[2] After Wood, Lummer and Pringsheim, Ebert, especially Puccianti has investigated the anomalous dispersion of various metallic vapors. In *Nuovo Cimento*, (5) **9**, 303, 1905, Puccianti describes over a hundred lines showing the phenomenon.

[3] W. H. Julius, "Dispersion Bands in Absorption Spectra," *Proc. Roy. Acad. Amst.*, **7**, 134–140, 1904; *Astrophysical Journal*, **21**, 271, 1905.

uneven distribution of the sodium vapor, than the other rays of the spectrum; absorption and emission changed relatively little. The result was that the distribution of the light in the neighborhood of D_1 and D_2 could be made very strongly asymmetrical, which could easily be explained in all details as the result of curvature of the rays. The existence of "dispersion bands" was thus proved beyond doubt.

But the pure effect of emission and absorption was not absolutely constant in these experiments, and only conjectures could be made concerning the density of the sodium vapor in the different parts of the flame. Moreover, the whirling ascent of the hot gases caused all rays, also those which suffered no anomalous dispersion, to deviate sensibly from the straight line, so that the phenomena were too complicated and variable to show the effect of dispersion strictly separated from that of emission and absorption. So our object was to obtain a mass of vapor as homogeneous as possible and, besides, an arrangement that would allow us to bring about arbitrarily, in this vapor, local differences of density in such a manner that the average density was not materially altered. The absorbing power might then be regarded as constant. At the same time it would be desirable to investigate the vapor at a relatively low temperature, so that its emission spectrum did not have to be reckoned with.

In a series of fine investigations on the refractive power and the fluorescence of sodium vapor, R. W. Wood[1] caused the vapor to be developed in an electrically heated vacuum tube. It appeared possible, by adjusting the current, to keep the density of the vapor very constant. Availing myself of this experience, I made the following arrangement for the investigation of dispersion bands.

APPARATUS

NN′ (see Fig. 1) is a nickel tube of 60 cm length, 5.5 cm diameter, and 0.07 cm thickness. Its middle part, having a length of 30 cm, is placed inside an electrical furnace of Heraeus (pattern *E* 3). Over its extremities covers are placed, the edges of which fit into circular rims soldered to the tube, which consequently shut air-tight when the rims are filled with cement. When the furnace is in action, a steady current of water, passing through the two mantles *M* and *M′*, keeps

[1] *Phil. Mag.*, (6) **3**, 128; **6**, 362, 1903.

the ends of the tube cool. Each of the two caps has a rectangular plate-glass window, and also, on both sides of this, openings a and b (b' and a'), placed diametrically opposite to each other and provided with short brass tubes, the purpose of which will appear presently.

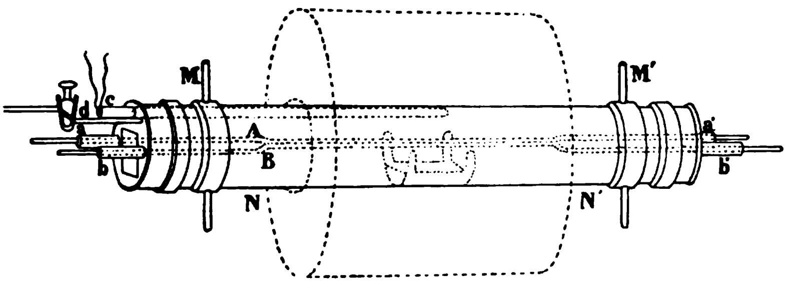


FIG. 1

Moreover, in one of the two caps (see also Fig. 2) two other short tubes c and d are fastened in openings; through c the porcelain tube of a Le Chatelier pyrometer is fitted air-tight, while on d a glass cock with mercury lock is cemented, leading to a manometer and a Geryk air-pump. As soon as the sodium (a carefully cleaned piece of about 7 grams) had been pushed to the middle of the tube in a small nickel dish provided with elastic rings, the tube was immediately closed and exhausted.

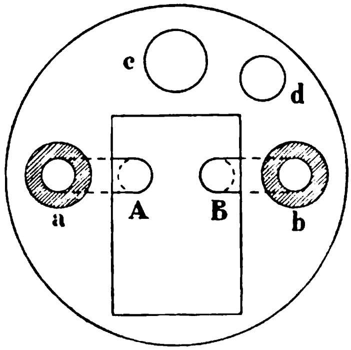


FIG. 2

We shall now describe the arrangement by which arbitrary inequalities in the density distribution were produced inside the mass of vapor. It consists of two nickel tubes A and B of 0.5 cm diameter, leading from a to a' and from b to b', and so bent that in the heated middle part of the wide tube they run parallel over a length of 30 cm at a distance of only 0.8 cm. In the four openings of the caps, A and B are fastened air-tight by means of rubber packing. This kind of connection leaves some play, so that by temperature differences

between the wide and the narrow tubes these latter need not alter their shape through tension. At the same time the rubber insulates A and B electrically from NN'. The four ends of the narrow tubes which project are kept cool by mantles with running water (these are not represented in the figure).

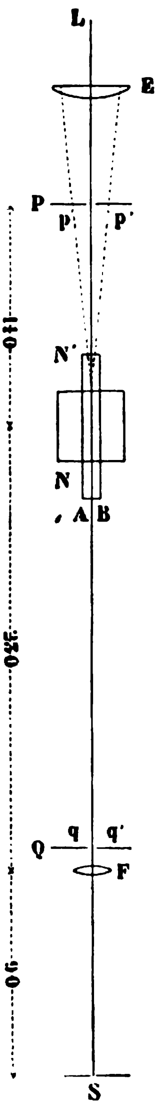


FIG. 3

If an electric current is now passed through A or B, the temperature of this tube rises a little above that of its surroundings; if an air-current is passed through it, the temperature falls a little below that of its surroundings. The intensities of the currents, and consequently the differences of temperature, can in either case be easily regulated and kept constant for a long time.

Fig. 3 gives a sketch of the whole arrangement. The light of the positive carbon L is concentrated by the lens E on a screen Q having a slit-shaped aperture of adjustable breadth. The lens F forms in the plane of the slit S of the spectrograph a sharp image of the diaphragm P. The optical axis of the two lenses passes through the middle of the tube containing the sodium vapor, exactly between the two small tubes A and B.

If now the opening in the diaphragm P has the shape of a vertical narrow slit, and if its image falls exactly on the slit of the spectrograph, then the continuous spectrum of the arc-light appears with great brightness. If the tube NN' is not heated, D_1 and D_2 are seen as extremely fine dark lines, attributed to absorption by the sodium, which is always present in the neighborhood of the carbons. In order that this phenomenon might always be present in the field of view of the spectrograph as a comparison spectrum, also when the tube is heated, a small totally reflecting prism was placed before part of the slit S, to which part of the principal beam of light was led by a simple combination of lenses and mirrors without passing the electric furnace. Thus the

unmodified spectrum of the source is also seen on each photograph that was taken.

The spectral arrangement used consists of a plane diffraction grating 10 cm in diameter (ruled surface 8 by 5 cm) with 14,436 lines to the inch, and two silvered mirrors of Zeiss; the collimator mirror has a focal length of 150 cm, the other, of 250 cm. Most of the work was done in the second spectrum.

When heating the sodium for the first time a pretty large quantity of gas (according to Wood, hydrogen) escaped from it, which of course was pumped off. After the apparatus had been operated a couple of times, the tension within the tube remained for weeks less than 1 mm of mercury; also during the heating, which, in the experiments described in this paper, never went beyond 450°. The inner wall of NN', and also the small tubes A and B, are after a short time covered with a layer of condensed sodium, which favors the homogeneous development of the vapor in subsequent heatings. It is remarkable that scarcely any sodium condenses on the parts of the tube that project from the furnace, so that the windows also remain perfectly clear. The density of saturated sodium vapor at temperatures between 368° and 420° has been experimentally determined by F. B. Jewett.[1] He gives the following table:

Temperature	Density	Temperature	Density
368°	0.00000009	395°	0.00000270
373	0.00000020	400	0.00000350
376	0.00000035	406	0.00000480
380	0.00000043	408	0.00000543
385	0.00000103	412	0.00000590
387	0.00000135	418	0.00000714
390	0.00000160	420	0 00000750

These densities are of the same order of magnitude as those of mercury vapor between 70° and 120°. At 387° the density of saturated sodium vapor is about one-thousandth of that of the atmospheric air at 0° and 76 cm.

OBSERVATIONS

If we now regulate the intensity of the current in the furnace in such a manner that the thermo-couple indicates a steady temperature

[1] "A New Method of Determining the Vapor-Density of Metallic Vapors, and an Experimental Application to the Cases of Sodium and Mercury," *Phil. Mag.*, (6) **4**, 546, 1902.

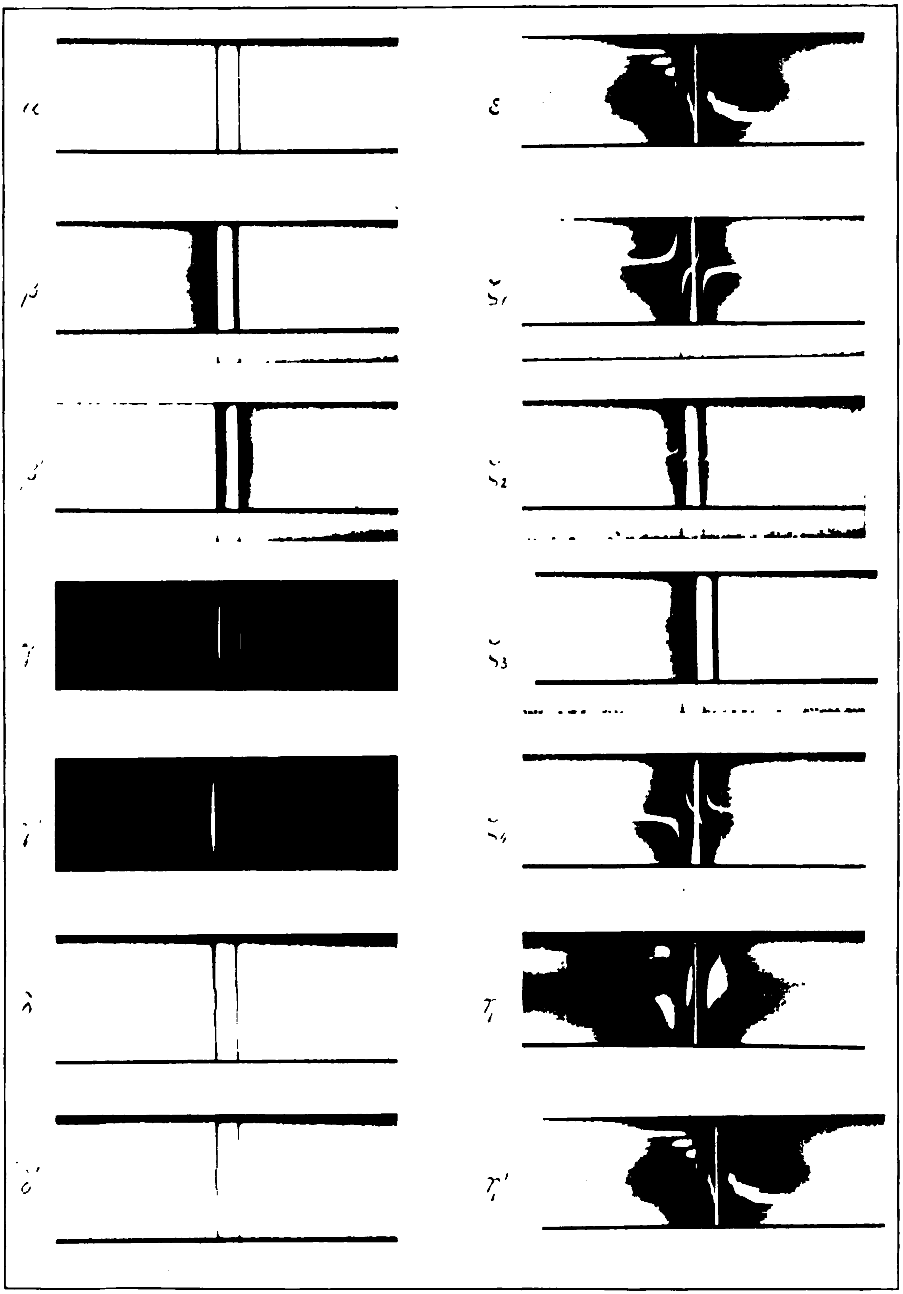
(in many of our experiments 390°), then the density of the vapor is not everywhere the same within the tube, for the temperature falls from the middle toward the ends; but since the surfaces of equal temperature are practically perpendicular to the beam of light, all rays pass nearly rectilinearly through the vapor. Accordingly the spectrum is only little changed; the two D lines have become somewhat stronger, which we shall, for the present, ascribe to absorption by the sodium vapor in the tube.

We now blow a feeble current of air through the tube A, which thus is slightly cooled, so that sodium condenses on it, the vapor-density in its neighborhood diminishing. We soon see the sodium lines broaden considerably. This cannot be the consequence of increased absorption, since the average vapor-density has decreased a little. The reason is that rays of light with very great refractive indices are now bent toward q' (Fig. 3), and rays with very small indices toward q; hence in the image of the slit P which is formed on S, rays belonging to regions on both sides of the D lines no longer occur, while yet this image remains perfectly sharp, since the course of all other rays of the spectrum has not been perceptibly altered. If now at the same time the tube B is heated by a current of, say, 20 amperes, by which the density-gradient in the space between the tubes is increased, the breadth of the lines becomes distinctly greater still. The heat generated in the tube by the current is about 1 calorie per second; it is, however, for the greater part conducted away to the cooled ends of the tube, so that the rise of temperature can only be small.

By switching a current key and a cock, A and B can be made to suddenly exchange parts, so that A is heated, B cooled. The dark bands then shrink, pass into sharp D lines and then expand again, until, after a few minutes, they have recovered their former breadth.

The lines in the transition stage are fine and sharp, however, only if the temperature of the furnace is very constant. If it rises or sinks, the minimal breadth appears to be not so small. In this case, however, there certainly exist currents in the mass of vapor which cause the distribution of density to be less regular. When, therefore, A and B being at equal temperatures, we still sometimes see the sodium lines slightly broadened, it stands to reason to attribute this also to refraction in such accidental irregularities.

PLATE V



That spectral lines possess some breadth is commonly ascribed either to motion of the light-emitting and absorbing molecules in the line of sight, or to changes in the vibrational period of the electrons by the collisions of the molecules. We now have a third cause—anomalous dispersion in the absorbing medium. The whole series of phenomena observed in our sodium tube corroborates the opinion that this latter cause must in many cases be regarded as by far the most important. It will appear that this conclusion holds not only for dark, but also for bright spectral lines.

If the slit in the diaphragm P is made much broader toward p', this has no influence on the spectrum as long as A and B are at the surrounding temperature. The D lines appear narrow, as in α, Plate V. If A is now cooled below this temperature, and B is raised above it, the dark D lines broaden only in the direction of the shorter wave-lengths, while at the side of the longer wave-lengths the intensity of the light is even increased. Indeed, the deficiency of these longer waves, which has been observed in the case of the *narrow* slit in P, is now overcompensated by the anomalously bent rays coming from the broad radiating field p' and finding their way through the slit Q to S. The resulting aspect is shown in β on the plate.

The spectrum β passes into β' when the temperature difference between A and B is made to change its sign, or also when the original temperature difference is maintained, but the slit in P is made much wider toward p instead of toward p'; for with both alterations the rays of longer and those of shorter wave-lengths than D_1 and D_2 only exchange parts.

A small shifting of the diaphragm P in the direction toward p' (starting from the conditions fulfilled when taking β) brings the image of the screen p upon the slit S, and thus prevents all the light not undergoing anomalous dispersion from reaching S. This causes the spectrum γ to appear, which makes the impression of an emission spectrum of sodium with slightly shifted lines, although it is evidently only due to rays from the field p' which have undergone anomalous dispersion in the vapor.

In a similar way the pseudo-emission spectrum γ' is obtained by shifting the diaphragm a little, starting from the conditions that gave β'.

The cases β and β' may be combined by using a diaphragm *P* with an opening of the shape of Fig. 4. When the slit *S* occupies the position of the dotted line in the image of this opening, then, if *A* is cooled, we shall have the conditions of β in the upper and lower parts of the spectrum, and the conditions of β' in the middle part. The resulting combination of the spectra β and β' may be easily imagined, and has not, therefore, been reproduced. But it is of some interest to notice the appearance of the same combination when the density-

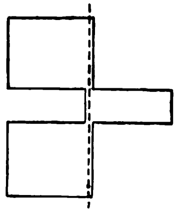


Fig. 4

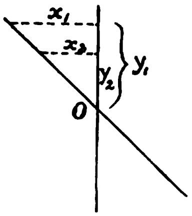


FIG. 5

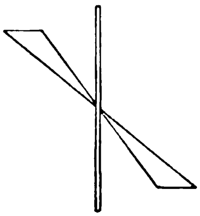


FIG. 6

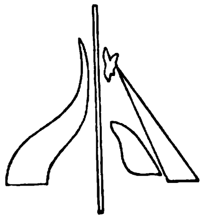


FIG. 7

gradient is made much smaller than it was when taking β and β'; for now we get δ and, after reversing the gradient, δ'. The line-shifting here produced has, of course, nothing to do with Doppler's principle. The two photographs further prove that even these narrow lines are almost totally due to anomalous dispersion instead of to absorption; indeed, the real, straight absorption line must be common to the three sections, and we see that there is scarcely any room left for it.

(On several of the photographs a few narrow bright lines appear;

they are emission lines of the arc, belong to the extreme violet of the third spectrum, and bear no relation to the phenomena with which we are concerned. The line a little to the right of D, for instance, is probably the calcium line λ 3933.83, for $3933.83 \times \frac{3}{2} = 5900.74$.)

Let us now return to the diaphragm P with a narrow slit placed in the optical axis. (A piece of glass coated with tin-foil, in which a slit was cut out, was generally used.) The spectrum then shows broad bands when there is a sufficient density-gradient between A and B. If an opening is cut in the tin-foil beside the slit, a group of rays of definite refractivities (and consequently also of definite wave-lengths) is given an opportunity to reach S through Q, and a bright spot is formed in the dark band, the shape of which depends on the shape of the opening in the tin foil, but is by no means identical with it. Thus, for instance, the spectrum ϵ shows the effect of a series of rectangular openings in the screens p and p'.

The law connecting the form of bright areas in the dispersion bands with the shape of openings in the screen is not very simple, because it depends on the configuration of the surfaces of equal density in the space between the tubes A and B. Some idea of the connection may be got if we simplify the problem by supposing those surfaces to be parallel planes, perpendicular to the plane containing the axes of A and B. The latter plane may cut the slit P in the point O and the slit S in O'. We shall take O as the origin of rectangular co-ordinates x (horizontal) and y (vertical), by which the points in the plane of the screens p and p' may be determined. Points in the image on S may be designated by x' and y' with respect to O'.

Now let us suppose a pin-hole xy to be made in p'. Light coming from this point will be focused by the lens F at a point $x'y'$ beside the slit S, provided it has not deviated in the sodium vapor. It does not get into the spectrograph. But the rays undergoing anomalous dispersion will spread out nearly horizontally; the lens F unites them in a continuous series of points having about the same y', but various values of x'. Only those for which $x' = 0$ enter into the spectrograph. If in the spectrum the middle of one of the sodium lines be called O'', the co-ordinates of the bright spot, produced in the dark dispersion band by the beam that entered, will be y'' (pro-

portional to y') and z, the abscissa z depending on the wave-length λ of that beam.

The connection between this wave-length and the abscissa x of the hole is given by the dispersion-curve of the sodium vapor. Indeed, we can easily prove that x is proportional to $n-1$, the factor only depending on linear dimensions of the arrangement, and on the density-gradient of the vapor.[1] So, for a given x, n may be computed; the corresponding λ is taken from the dispersion curve, and in the spectrum we have $z=\lambda-\lambda_D$. The ordinate y'' is derived from y by merely introducing focal distances. We shall thus have expressed the co-ordinates of the bright spot in terms of the co-ordinates of the pin-hole.

The following instance may serve to elucidate the connection between corresponding figures in the plane P and in the spectrum, without calculation.

Instead of the pin-hole we make a second straight slit in the diaphragm, cutting the first one obliquely in O (Fig. 5). Now all positive and negative values of x, and therefore of $n-1$, are represented each of them belonging to a separate value of y which is proportional to it:

$$y_1 : y_2 = x_1 : x_2 = (n_1 - 1) : (n_2 - 1) .$$

As the ordinates y'' in the spectrum are proportional to y, we have also

$$y_1'' : y_2'' = (n_1 - 1) : (n_2 - 1) .$$

At the same time

$$z_1 : z_2 = (\lambda_1 - \lambda_D) : (\lambda_2 - \lambda_D) .$$

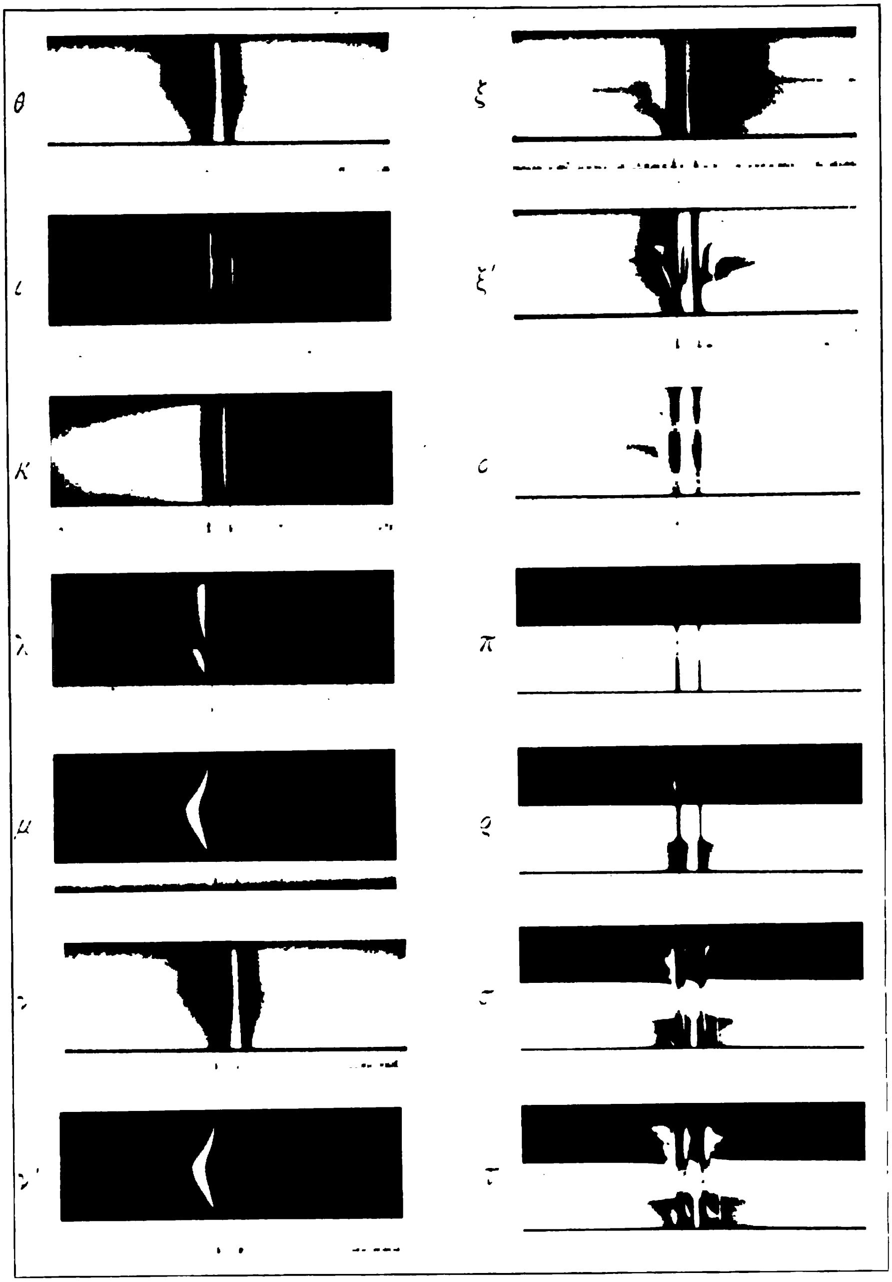
The bright curve in the spectrum, therefore, is the dispersion-curve itself, with the point $n=1$, $\lambda=\lambda_D$ taken for the origin of co-ordinates.

The spectrum ζ_1 of Plate V realizes this case. It has been obtained by using a diaphragm with an opening of the shape of Fig. 6. The width of the oblique slit was enlarged toward the ends in order to increase the luminosity of the ascending and descending branches of

[1] From the equations (1) and (2) on pages 106 and 107 follows immediately

$$x = dlR\frac{d\Delta}{ds} = dl\frac{d\Delta}{ds} \cdot \frac{1}{\Delta}(n-1) .$$

PLATE VI



the curve. When the electric current and the air current through the tubes are diminished, the figure shrinks to ζ_2; when they are stopped, we return to α; reversing the gradient makes the spectrum proceed through ζ_3 to ζ_4.

Having thus experimentally found the relation between the two figures for a simple case, it is not difficult to design for any desired distribution of light the shape of the required opening in the diaphragm. The flower η, for instance, requires the diaphragm represented in Fig. 7; by reversing the gradient the image η passes into η'.

Thus we possess the means for arbitrarily producing all stages of enhancement, wingedness, reversal, shifting, duplication, ramification of bright or dark spectral lines, and it seems possible faithfully to reproduce all phenomena observed in this respect in the spectra of sun-spots, faculae, flocculi, or prominences. On Plate VI a number of arbitrary distributions of light have been collected. They were all produced in sodium vapor of 390° on the average. In θ on the dark dispersion band D_2 a bright double line is seen, reminding us of the spectrum of the calcium flocculi described by Hale. In the same negative D_1 also shows a fine double line which, I fear, will be invisible in the reproduction. The spectrum ι is not unlike that of a prominence taken with the tangential slit; κ reminds us of certain star spectra; etc. The photographs π, ρ, σ imitate the development of a prominence and a sun-spot spectrum: π represents the spectrum of the quiet solar limb with radially placed slit; in ρ a prominence appears and a spot with phenomena of reversal; σ shows all of this in a stronger degree. If now the density-gradient is made to change sign, the image first shrinks again to π, after which it expands to τ, in a certain sense the inversion of σ.

The striking spectacle of these phenomena, the gradual changes of which admit of perfect control, is only poorly reproduced by the photographs.

Plate VII shows on a slightly larger scale some photographs taken in the third spectrum with sodium vapor of only 320°. The density of the saturated vapor at this temperature is unknown. If the temperature-density-curve found by Jewett is extended beyond the observations so as to be in harmony with the shape of the better-known curve of mercury, we may infer that at 320° the density will

probably be inferior to 0.00000003. The density-gradient produced by cooling or heating our tubes must have been of the order of magnitude 0.00000001 in these experiments. The diaphragm used was the same as that which served with δ and δ'. When taking υ and ϕ, the slit S occupied the position of the dotted line in the image of the opening, Fig. 4; with χ the slit was a little to the left; with ψ and ω a little to the right. We see from these photographs that the real absorption lines of the sodium vapor must have been excessively narrow; indeed, it is dubious whether they can be distinguished at all and the distribution of the light seems to be wholly governed by anomalous dispersion.

THE RELATION BETWEEN THE CURVATURE OF THE RAYS AND THE DENSITY-GRADIENT

The question arises whether it is *probable* that circumstances such as were realized in our experiments are also met with in nature, or in ordinary spectroscopical investigations undertaken with entirely different purposes.

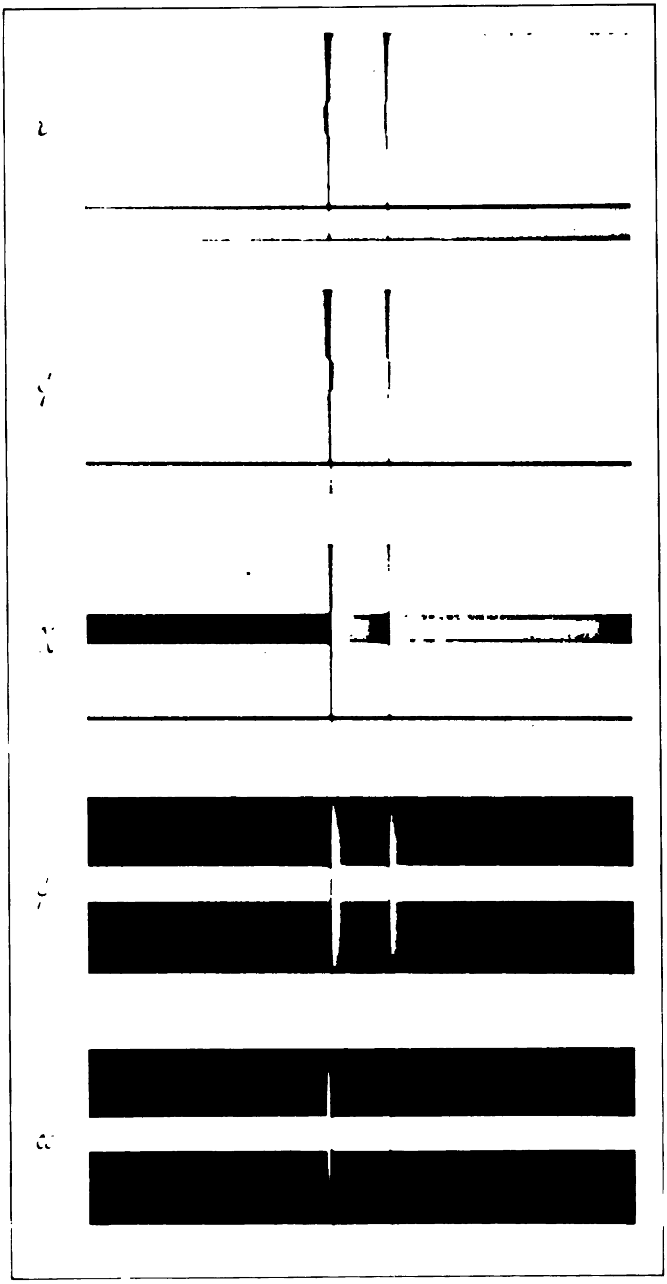
We remark, in the first place, that curiously shaped diaphragm openings are not absolutely essential for the production of phenomena as those described above. If, for instance, our source of light had a constant, say circular, shape; if, on the other hand, the direction and magnitude of the density-gradient in our tube had not been so regular, but very different in various places of the field reproduced by the lens F, then the D lines would also have shown all sorts of excrescences, now determined by the configuration of the density distribution.

In the second place, we will try to form some idea of the quantitative relations.

The radius of curvature ρ of the path of the most deviated rays occurring in our photographs may be easily estimated from the distance d of the diaphragm to the middle of the furnace, the distance x of one of the most distant diaphragm openings to the optical axis, and the length l of the space in which the incurvation of the rays is brought about. For

$$\rho : l = d : x \,. \tag{1}$$

PLATE VII



Putting $x=1$ cm, $d=110$ cm, $l=27$ cm, this gives $\rho=3000$ cm. The average density Δ of the sodium vapor was in this case about one one-thousandth of that of the atmospheric air.

Let us see how ρ changes with the density-gradient. We always have

$$\rho=\frac{n}{n'},$$

if n represents the local index of refraction of the medium for the ray under consideration and $n'=\frac{dn}{ds}$ the change of this index per centimeter in the direction of the center of curvature. We have approximately for a given kind of light

$$\frac{n-1}{\Delta}=\text{constant}=R,$$

$$n=R\Delta+1,$$

$$n'=\frac{dn}{ds}=R\frac{d\Delta}{ds}.$$

From this follows

$$\rho=\frac{R\Delta+1}{R\frac{d\Delta}{ds}};$$

but since for rarefied gases n differs little from unity, even for the anomalously dispersed rays which we consider, $R\Delta$ may be neglected with regard to 1 and we may write

$$\rho=\frac{1}{R\frac{d\Delta}{ds}}. \tag{2}$$

For every kind of light ρ is consequently inversely proportional to the density-gradient of the vapor in the direction perpendicular to that of propagation.

An estimate of the magnitude of the density-gradient existing, in our experiments, between A and B may be obtained in two ways. It may be inferred either from the difference of temperature produced, or from formula (2). The temperature difference between A and B would have been pretty easy to determine thermo-electrically; up to the present, however, I have had no opportunity to make the

necessary arrangement. Besides, the relation between the density distribution in the space traversed by the rays, and the temperatures of A and B, cannot be so very simple, since we have to deal, not with two parallel planes, but with tubes, from which, moreover, many drops of liquid sodium hang.

The second method at once gives an average value of $\frac{d\Delta}{ds}$ for the space traversed by the rays. It requires a knowledge of $R=\frac{n-1}{\Delta}$ for a kind of ray for which in our experiments also ρ has been determined.

Now, Wood[1] gives a table for the values of n for rays from the immediate vicinity of the D lines. These data, however, refer to saturated sodium vapor of 644°; but we may deduce from them the values of n for vapor of 390° by means of the table which he gives in his paper on page 317.

For, when we heat from 389° to 508°, the refractive power of the vapor (measured by the number of passing interference fringes of helium light $\lambda=5875$) becomes $\frac{98}{9}=11$ times greater, and at further heating from 506° to 644° again $\frac{50}{4}=12.5$ times greater (now found by interference measurement with light from the mercury line $\lambda=5461$); hence from 390° to 644° the refractive power increases in ratio of 1 to $11\times12.5=137$.

Since now for rays situated at 0.4 Ångström unit from the D lines[2] we have $n-1=\pm0.36$ (as the average of three values taken from Wood's table on page 319), we ought to have with sodium vapor at 390° for the same kind of rays

$$n-1=\frac{0.36}{137}=0.0026\ .$$

The density Δ at 390° is, according to Jewett, 0.0000016, whence

$$R=\frac{n-1}{\Delta}=\frac{0.0026}{0.0000016}=1600\ .$$

[1] *Phil. Mag.*, (6) **8**, 310, 1904.

[2] The spectrum ζ_1 in our plate shows that the extremities of the peaks correspond pretty well to light of this wave-length; for they approach the D lines to a distance which certainly is no more than one-fifteenth of the distance of the D lines which amounts to 6 Ångström units. For these rays the opening of the diaphragm was 1 cm distant from the optical axis.

Then from formula (2) follows

$$\frac{d\Delta}{ds}=\frac{1}{R\rho}=\frac{1}{1600\times3000}=0.0000002\ .$$

DISPERSION BANDS IN THE SPECTRA OF TERRESTRIAL SOURCES

It is very probable that, when metals evaporate in the electric arc, values of the density-gradient are found in the neighborhood of the carbons that are more than a thousand times greater than the feeble density-gradient in our tube with rarefied sodium vapor.[1]

The radius of curvature will, therefore, in these cases be over a thousand times smaller than 30 meters, and so may be no more than a few centimeters or even less. A short path through the vapor mass is then already sufficient to alter the direction of certain rays very perceptibly.

If now an image of the carbon points is produced on the slit of a spectroscope, then this is a *pure* image only as far as it is formed by rays that have been little refracted in the arc, but the rays which undergo anomalous dispersion do not contribute to it. Light of this latter kind, coming from the crater, may be lacking in the image of the crater, and, on the other hand, penetrate the slit between the images of the carbon points. Thus, in ordinary spectroscopic observations, broadening, not only of absorption lines, but also of emission lines, must often to a considerable extent be attributed to anomalous dispersion.

When we bear this in mind, many until now mysterious phenomena will find a ready explanation. So, for instance, the fact that Liveing and Dewar[2] saw the sodium lines strongly broadened each time when vapor was vividly developed after bringing in fresh material, but saw them become narrower again when the mass came to rest, although the density of the vapor did not diminish. If by pumping nitrogen into the evaporated space the pressure was gradu-

[1] If, for instance, we put the vapor-density of the metal in the crater, where it boils, at 0.001, the density of the vapor outside the arc at a distance of 1 cm from the crater at 0.00001, then we have already an average gradient 5000 times as large as that used in our experiments.

[2] "On the Reversal of the Lines of Metallic Vapors," *Proc. R. S.*, **27**, 132–136; **28**, 367–372, 1878–1879.

ally increased, the lines remained sharp; but if the pressure was suddenly released, they were broadened. All this becomes clear as soon as one has recognized in the lines dispersion bands, which must be broad when the density of the absorbing vapor is irregular, but narrow, even with dense vapor, if only the vapor is evenly spread through the space.

Another instance. According to the investigations of Kayser and Runge, the lines belonging to the second secondary series in the spectra of magnesium, calcium, cadmium, zinc, mercury, are always hazy toward the red and are sharply bordered toward the violet; whereas lines belonging to the first secondary series or to other series are often distinctly more widened toward the violet. With regard to the spectrum of magnesium they say:

> Auffallend ist bei mehreren Linien, die wir nach Roth verbreitert gefunden haben, dass sie im ROWLAND'schen Atlas ganz scharf sind, und dann stets etwas kleinere Wellenlänge haben. So haben wir 4703.33, ROWLAND 4703.17; wir 5528.75, ROWLAND 5528.62. Unschärfe nach Roth verleitet ja leicht der Linie grössere Wellenlänge zuzuschreiben; so gross kann aber der Fehler nicht sein, denn die ROWLAND'sche Ablesung liegt ganz ausserhalb des Randes unserer Linie. Wir wissen daher nicht, woher diese Differenz rührt.[1]

Kayser has later[2] given an explanation of this fact, based on a combination of reversal with asymmetrical widening; but a more probable solution is, in my opinion, obtained when we regard the widened serial lines partly as dispersion bands.

If we assume that, when we proceed from the positive carbon point, which emits the brightest light, to the middle of the arc, the number of the particles associated with the second secondary series decreases, then rays coming from the crater, whose wave-length is slightly greater than that of the said serial lines, will be curved so as to turn their concave side to the carbon point. Their origin is erroneously supposed to be in the prolongation of their final direction, so they *seem* to come from the arc, and we believe we see light emitted by the vapor, in which light different wave-lengths occur, all greater than the exact wave-length of the serial lines. The observed displaced lines of the second secondary series are consequently comparable to apparent emission lines of the spectrum γ of Plate V.

[1] Kayser und Runge, *Über die Spektren der Elemente*, 4, 13.

[2] Kayser, *Handbuch der Spektroskopie*, 2, 366.

In this explanation things have been represented as if the light of these serial lines had to be *exclusively* attributed to anomalous dispersion. Probably, however, in the majority of cases, emission proper will indeed perceptibly contribute to the formation of the line; the sharp edge must then appear in the exact place belonging to the particular wave-length.

How can we now explain that lines of other series are diffuse at the opposite side? Also this may perhaps be explained as the result of anomalous dispersion, if we assume that of the emission centers of these other series the density *increases* when we move away from the positive carbon point. In this case, namely, the rays originating in the crater, which are concave toward the carbon point and consequently seem to come from the arc, possess shorter wave-lengths than the serial lines; i. e., the serial lines appear widened toward the violet. This supposition is not unlikely. For the positive and negative atomic ions which, according to Stark's theory, are formed in the arc by the impact of negative electronic ions, move in opposite directions under the influence of the electric field; hence the density-gradients will have opposite signs for the two kinds. Series whose lines are diffuse toward the red, and series whose lines flow out toward the violet, would, according to this conception, belong to, or be produced by, ions of opposite signs—a conclusion which at all events deserves nearer investigation.

The examples given may suffice to show that it is necessary systematically to investigate to what extent the already known spectral phenomena may be the result of anomalous dispersion. A number of cases in which the hitherto neglected principle of ray-curving has undoubtedly been at the root of the matter are found in Kayser's handbook, **2**, 292–298, 304, 306, 348–351, 359–361, 366.

DISPERSION BANDS IN THE SPECTRA OF CELESTIAL BODIES

Since almost any peculiarity in the appearance of spectral lines may be explained by anomalous dispersion, if only we are at liberty to assume the required density distributions, we must ask, when applying this principle to astrophysical phenomena: Can the values of the density-gradient for the different absorbing gases in celestial bodies really be such that the rays are sufficiently curved to exert

such a distinct influence on the distribution of light in the spectrum?

In former communications[1] I showed that the sun, for instance, may be conceived as a gaseous body, the constituents of which are intimately mixed, since all luminous phenomena giving the impression as if the substances occurring in the sun were separated, may be brought about in such a gaseous mixture by anomalous dispersion. We will now try to prove, not only that this *may* be the case, but that it *must* be so on account of the most likely distribution of density.

Let us put the density of our atmosphere at the surface of the earth at 0.001293. At a height of 1050 cm it is smaller by $\frac{1}{760}$ of this amount, so that the vertical density gradient is

$$\frac{0.001293}{1050 \times 760} = 16 \times 10^{-10}.$$

The horizontal gradients occurring in the vicinity of depressions are much smaller; even during storms they are only about one one-thousandth of the said value.[2] Over small distances the density-gradient in the atmosphere may of course occasionally be larger, through local heating or other causes.

Similar considerations applied to the sun, *mutatis mutandis*, cannot, however, lead to a reliable estimate of the density-gradients there occurring. A principal reason why this is for the present impossible is found in our inadequate knowledge of the magnitude of the influence, exerted by *radiation-pressure* on the distribution of matter in the sun. If there were no radiation-pressure, we might presuppose, as is always done, that at the level of the photosphere gravitation is twenty-eight times as great as on the earth; but it is counteracted by radiation-pressure to a degree, dependent on the size of the particles; for some particles it may even be entirely abolished. The radial density-gradient must, therefore, in any case be much smaller than one might be inclined to calculate on the basis of gravitational action only.

[1] *Proc. Roy. Academy Amsterdam*, **2**, 575; **4**, 195; **5**, 162, 589, and 662; **6**, 270; **8**, 134, 140, and 323. *Astrophysical Journal*, **12**, 185–200; **15**, 28–37; **18**, 50–64; **21**, 271–291. *Physikalische Zeitschrift*, **4**, 85–90; 132–136; **6**, 239–248. A sketch of a solar theory, in which refraction and dispersion have been considered, is to be found in the *Revue générale de sciences*, **15**, 480–495, 1904.

[2] Arrhenius, *Lehrbuch der kosmischen Physik*, p. 676.

Fortunately we possess another means for determining the radial density-gradient in the photosphere, at any rate as far as the order of magnitude is concerned. According to Schmidt's theory, the photosphere is nothing but a critical sphere the radius of which is equal to the radius of curvature of luminous rays whose path is horizontal at a point of its surface. This radius of curvature is consequently $\rho=7\times10^{10}$ cm, a value which we may introduce into the expression for the density-gradient:

$$\frac{d\Delta}{ds}=\frac{1}{R\rho}\,.$$

The refractive equivalent R for rays that undergo no anomalous dispersion varies with different substances, to be sure; but in an approximate calculation we may put $R=0.5$. Then at the height of the critical sphere we shall have

$$\frac{d\Delta}{ds}=\frac{1}{0.5\times7\times10^{10}}=0.29\times10^{-10}\,,$$

(this is 50 times less than the density-gradient in our atmosphere). All arguments supporting Schmidt's explanation of the sun's limb are at the same time in favor of this estimate of the radial density-gradient in the gaseous mixture. It should be observed, on the other hand, that, when things are considered from other points of view than from Schmidt's theory, this density-gradient appears by no means improbably large. Yet gradients of this order of magnitude will produce ray-curving in a degree amply sufficient for giving rise to very conspicuous dispersion phenomena, as we shall see presently. If, therefore, arguments are found for assuming larger density-gradients, our explanations will thereby only be corroborated.

Let us now consider rays that do undergo anomalous dispersion. In order that light, the wave-length of which differs but very little from that of one of the sodium lines, may seem to come from points situated some seconds of arc outside the sun's limb, the radius of curvature of such anomalously bent rays need only be slightly smaller than 3×10^{10} cm. Let us put, for instance,

$$\rho'=6\times10^{10}\text{ cm}\,.$$

If we further assume that of the kind of light under consideration the wave-length is 0.4 Ångström units greater than that of D_1, then

for this kind of light $R' = 1600$, as may be derived from the observations of Wood and of Jewett;[1] we thus find for the density-gradient of the sodium vapor,

$$\frac{d\Delta'}{ds} = \frac{1}{R'\rho'} = \frac{1}{1600 \times 6 \times 10^{10}} = 0.0001 \times 10^{-10},$$

a quantity 2900 times smaller than the density-gradient of the gaseous mixture.

Hence if only one three-thousandth part of the gaseous mixture consists of sodium vapor, then, on account of the assumed radial density-gradient of the mixture, the critical sphere (or the photosphere) will already seem to be surrounded by a "chromosphere" of light, this light having a striking resemblance with sodium light. This kind of light has, so to say, its own critical sphere which is larger than the critical sphere of the light not anomalously refracted. If the percentage of sodium were larger, the "sodium chromosphere" would appear higher.

It is customary to draw conclusions from the size of the chromospheric and flash crescents, observed during a total eclipse with the prismatic camera, as to the *height* to which various vapors occur in the solar atmosphere. According to us, this is an unjustified conclusion. On the other hand, it will be possible to derive from these observations data concerning *the ratio in which these substances are present in the gaseous mixture*, provided that the dispersion-curves of the metallic vapors, at known densities, shall first have been investigated in the laboratory.

Until now we have dealt only with the normal radial density-gradient. By convection and vortex motion, however, irregularities in the density distribution arise, with gradients of various direction and magnitude. And since on the sun the resultant of gravitation and radiation-pressure is relatively small, there the irregular density-gradients may reach values that approach the radial gradient sooner than on the earth, or may be occasionally larger.

The incurvation of the rays in these irregularities must produce capriciously shaped sodium prominences, the size of which depends, among other causes, on the percentage of sodium vapor in the gaseous mixture.

So the large hydrogen and calcium prominences prove that rela-

[1] See page 108.

tively much hydrogen and calcium vapor is present in the outer parts of the sun; but perhaps even an amount of a few per cent. would already suffice to account for the phenomena.[1]

If we justly supposed that non-radially directed density-gradients are of frequent occurrence in the sun, and there disturb the general radial gradient much more than on the earth, then not only rays from the marginal region, but also rays from the other parts of the solar disk, must sensibly deviate from the straight line. Chiefly concerned are, of course, the rays that undergo anomalous dispersion. ***Every absorption line of the solar spectrum must consequently be enveloped in a dispersion band.***

To be sure, absorption lines of elements which in the gaseous mixture occur only in a highly rarefied condition, present themselves as almost sharp lines, since for these substances all density-gradients are much smaller than for the chief constituents, and so the curvature of the rays from the vicinity of these lines becomes imperceptible. Also some lines of strongly represented elements may appear sharp, since not all lines of the same element, with given density, cause anomalous dispersion in the same degree. Perhaps there are even absorption lines which under no condition give rise to this phenomenon; though this would be rather improbable from the point of view of the theory of light.

Be this as it may, the limitations mentioned do not invalidate our principal conclusion: that the general interpretation of the solar spectrum has to be modified. We are obliged to see in Fraunhofer's lines not only absorption lines, as Kirchhoff does, but chiefly dispersion bands (or dispersion lines). And that refraction has a preponderant influence also on the distribution of light in stellar spectra cannot be doubted either.

We must become familiar with the idea that in the neighborhood of the celestial bodies the rays of light are in general curved, and that consequently the whole interstellar space is filled with ***nonhomogeneous radiation fields***[2] of different structure for the various kinds of light.

[1] This result would be in accordance with a hypothesis of Schmidt (*Physikalische Zeitschrift*, **4**, 232 and 341), according to which the chief constituent of the solar atmosphere would be a very light gas, until now unknown.

[2] "Das ungleichmässige Strahlungsfeld und die Dispersionsbanden," *Physikalische Zeitschrift*, **6**, 239–248, 1905.

STUDIES IN SENSITOMETRY I.

THE DAYLIGHT SENSITOMETRY OF PHOTOGRAPHIC PLATES, AND A SUGGESTED STANDARD DISPERSION-PIECE

By ROBERT JAMES WALLACE

INTRODUCTORY

The universal adaptation of the modern dry plate, and the varying demands which are made upon its service, have resulted in an increase of knowledge relative to the imperfections of the photographic plate as a means of recording anything save the actual form of the object photographed. In many cases even that is doubtful. These imperfections have compelled the "testing" of the various plates by many individuals, the object of such tests being principally the determination of the relative color-sensitiveness and comparative speed.

Many methods have been suggested for this purpose, the enumeration of which needs find no place here. It is sufficient to say that methods depending for their results upon the use of colored glasses and pigments are now generally recognized as incomplete, and as leading to erroneous conclusions where the work is in any degree quantitative. What one desires to know is the sensitiveness of the plate to pure color, not admixture, because, if one knows this sensitiveness, it is a comparatively easy matter to calculate the action of mixtures. For example, a patch of red-pigment-stained paper may be photographed, and a strong impression of the same developed upon a plate; but it does not follow that, because such an impression is obtained, the plate is "red-sensitive." For although the patch reflects red, it also, in less degree, reflects all other hues of the spectrum; and the developed impression is just as likely to be due to the combined action of such other hues, as to the red, when we take into consideration the fact that the plate is relatively many times more sensitive to those hues which are secondary in reflection.

Discarding these various makeshifts, a great number of photographic workers have of late acquired various forms and types of spectroscopes, and have literally flooded the journals devoted to that

subject with all sorts and conditions of spectra. This, while to be welcomed as a move in the right direction, is yet liable to give rise to many very grave errors in interpretation. The gravity of these errors has been commented upon by sundry writers at various times, and a brief notice was given to the subject by the present writer in a former paper.[1] It may, however, contribute to the clearness of this whole subject if such errors are described here at somewhat greater length.

The possession of a "spectroscope" does not imply results of value unless its possessor understands his instrument, and is acquainted with the laws of light and color. Much excellent material is readily available, and there is little excuse for the heterogeneous results which are unhappily so common in photography, in which almost every worker appears to be a law unto himself.

The three constants which govern the definition of color are hue, purity, and luminosity. By "hue" is meant what is ordinarily termed "color;" for when we speak of an object as having such and such a color, we are referring to its hue. The next constant, purity, concerns the admixture of the color with other colors, or with white light; while luminosity refers to the brightness of the hue under consideration. Of these three constants the photographic plate is chiefly concerned with the last.

The entire value of the spectrum for this class of investigative work lies in the fact that in it we obtain a standard of pure color, from the verdict of which there is no appeal, and to which everything colored must inevitably be referred. But there are many widely different forms of spectroscopes available, from the small direct-vision prismatic instruments to the concave diffraction grating, each of which has its own particular value for different lines of work, but which are, generally speaking, ill suited for sensitometric work in pure photography.

PRISMATIC AND DIFFRACTION SPECTRA

First dividing the subject into its two great classes of prismatic and diffraction spectra, let us consider each separately. In the first instance we are dealing with a spectrum formed by the passage of light through a prism (or prisms), as the name implies. In the

[1] *Astrophysical Journal*, **22**, 153, 1905, and **24**, 268, 1906.

direct-vision instrument, of which the Browning may be taken as a type, we have an element of dense flint glass combined with an element of crown glass. In instruments of angular deviation type, we have generally an element of flint glass alone, used on account of the greater dispersion obtainable from a glass of comparatively high refractive index. In all prismatic spectra the error arising from irrationality is the most readily noticeable. The abnormality in the hues by reason of the unequal distribution of intensity (luminosity) is not so apparent, however, although rendering the results by one prism not comparable with those from another. A very serious cause of error lies in the fact that the absorption of the glass composing the prism has a strong influence upon the results; generally speaking, the higher the refractive index of the glass employed, the greater the absorption. Again, two prisms of identical refractive index may give photographic results diametrically opposite to each other, because of varying absorption in the prisms themselves, aside from density; for example, two prisms could have identical refractive indices, and yet one be composed of colorless glass, while the other was composed of gray or blue glass.

CONVERSION FORMULAE

Various formulae have been advanced from time to time, designed to bring those discordant results into harmony with one another; but unfortunately they do not satisfy the conditions demanded in quantitative plate-testing. They are all principally concerned with the dispersion of the spectrum, and not with its relative luminosity. What one wants to know is not merely whether or not a plate is sensitive to red, but in what degree that sensitiveness exists. All plates are sensitive to the least refrangible hues, if they get sufficient exposure, but that plate which requires relatively the shortest exposure, other things being equal, is the best plate for work in that region; or, in other words, that plate which will show the greatest extent of the spectrum with normal exposure is the best plate for all round work.

The formula most commonly in use by photographic workers is $\frac{a}{b}m = n$, where a = prismatic dispersion, b = normal dispersion, m =

density of prismatic spectrum, and n = density of normal spectrum; hence $am = bn$.

In order that we may clearly understand the value of this formula, measurement was made of the prismatic spectrum ($\mu_D = 1.6994$) shown in Fig. 1*a* (Plate VIII) and its curve plotted in the usual manner (Fig. 2). A wave-length scale being then prepared, this curve was reduced by means of the above formula, the result of which is shown in Fig. 3, *a*. On the same scale are plotted the measurements from a spectrum

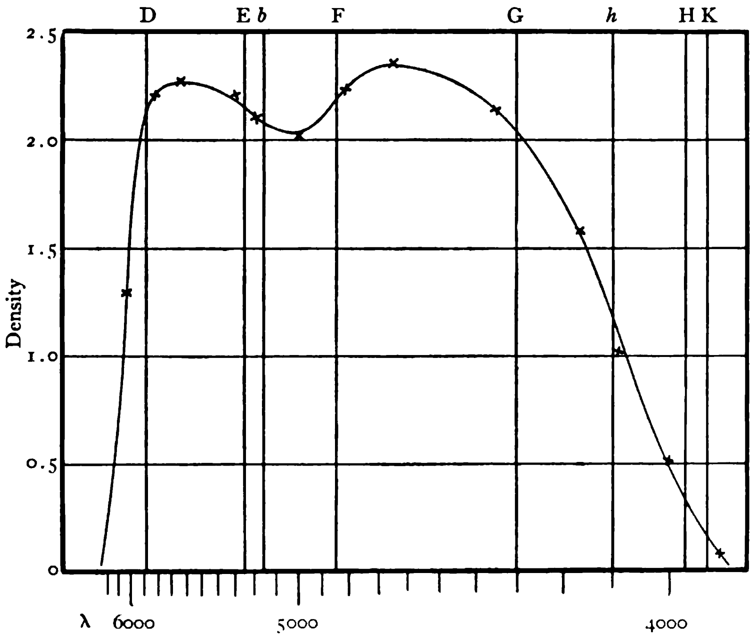


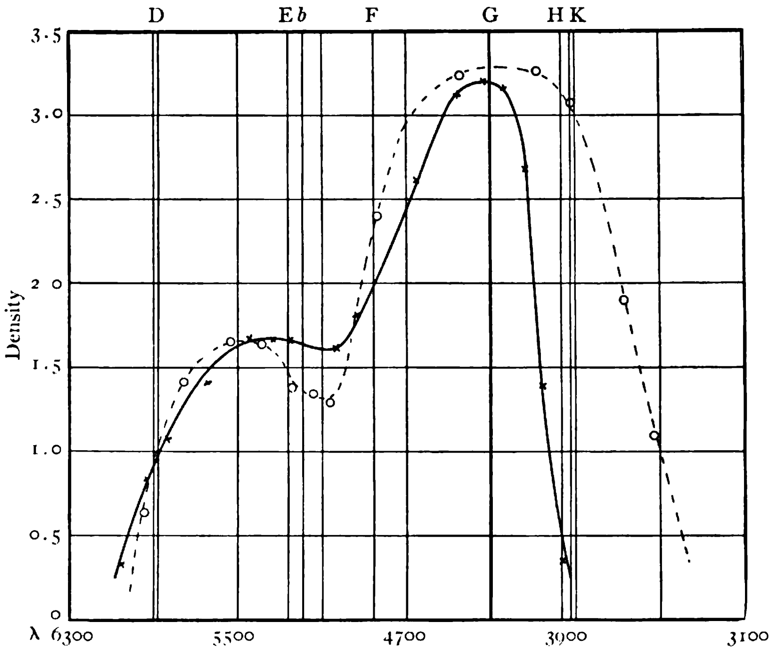
FIG. 2

negative obtained by a replica-grating upon a similar plate, the value of whose region of highest density was practically identical with that of the reconstructed curve of the prismatic spectrum. The woeful lack of agreement is strongly in evidence. Not only is the reconstructed curve deficient in the ultra-violet, but the maximum of sensitiveness is seen to be shifted bodily toward the red end. Further words are unnecessary on this point. What is wanted is a formula which will take into consideration the loss in luminosity by absorption

and the shift due to density of the material used in the construction of the prism.[1]

REFLECTION DIFFRACTION GRATING

Turning now to diffraction spectra, it is well known that the speculum metal on which the original grating is ruled possesses in itself a selective absorption which again varies with different "meltings," and which influences the distribution of color-intensity throughout the spectrum. As the grating ages it becomes tarnished by exposure



(a. continuous line)

FIG. 3

to the air and the various fumes of the laboratory, and this tarnish is in itself a strong factor in unequal color distribution. Again, the nature of the groove made by the cutting diamond not only determines the distribution of spectral intensity, but influences also the luminosity of the individual hue, amounting in exceptional cases even to abnormality, so that the spectra of no two gratings are defi-

[1] A search for a formula fulfilling the requirements specified is now in progress with prospect of a successful result.

nitely comparable one with the other, in so far as spectral luminosity is concerned.

REPLICA-GRATING

It may be objected that several of the complaints just cited do not amount to much in practical photographic sensitometry. Granted that this is so, they are disadvantages and have been treated as such. Not all have been mentioned, however, but merely *those which can be remedied by the adoption of the replica-grating as a standard dispersion-piece for investigative work in sensitometry*, when used without the addition of a prism.

Inasmuch as anything is a "standard" if we know what it is, we may begin first with the material of which the replica-grating is manufactured. We have a definite compound, collodion, resulting from the mixture of amyl acetate and pyroxylin, which is always prepared in the same way. The replicas themselves are composed of the same amount of solution, dried under similar conditions, and give a film of the same thickness. These replicas are made from the same original, and are therefore practically identical, while the grating from which they are made gives a fairly even distribution of light throughout the various orders. These films are mounted upon glass of similar thickness, quality, absorption, and refractive index.

There is no possibility of surface oxidation of the replica, nor does selective absorption enter into the account[1] for all the methods necessary to a complete test of photographic plates.

Inasmuch as the distribution of intensity is greatly dependent upon the shape of the groove made by the cutting diamond, it may be argued that equally minute differences in the grooves of the replica-grating would have the same effect. While this is undoubtedly true, yet, as a matter of fact, such differences, although looked for, have not yet been detected. In a spectro-photometric examination of a number of replicas, all made from the same original, at different times throughout the course of five years, which had been prepared under temperatures varying about 8° C., the results were gratifyingly exact. The method of manufacture, however, would indicate such results, when we consider that the shrinkage in the drying would be

[1] *Astrophysical Journal*, **22**, 129, 1905.

identical, provided the conditions and materials were similar. Obviously the same argument applies to the method of mounting.

While it is not claimed that the replica-grating is perfectly suited for all classes of work, yet it is believed that its adoption in sensitometry would avoid the great lack of accord between the results of one worker in photography and that of another, with the unprofitable discussion which inevitably ensues.

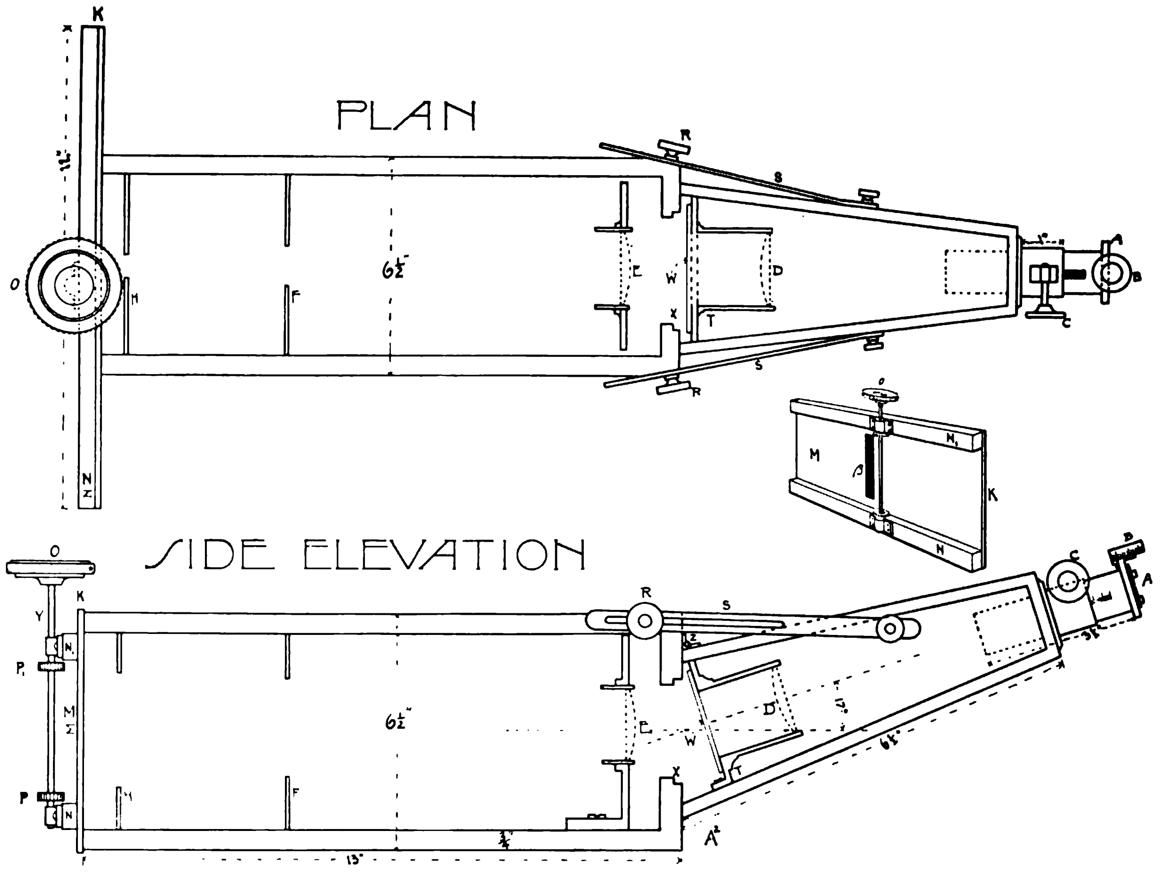
After many experiments, and consultation with authoritative scientists, the writer offers this form of grating to the photographic investigator for adoption as a standard dispersion-piece in sensitometry, in the hope of establishing uniformity in photographic results. In order, furthermore, that this standard may be disseminated widely and be of universal application, the writer has decided to present to each known investigator in photography of any nationality, who may apply, one of these standard replicas of a size suited to his needs.

It may be argued that no replica-grating can be compared in defining power with an original ruling. Argument upon this point is unnecessary, inasmuch as what is wanted for photographic investigation is not critical definition of spectral *lines*, but the correct definition of spectral *hues*. In most of the negatives the Fraunhofer lines are purposely obliterated, because they interfere with the measurement. It is, however, now a matter of common knowledge that good-quality replica-gratings leave little, if anything, to be desired on the score of definition, and, except for the spectroscopy of position, even those of secondary quality define *far in excess* of the requirements of the work in hand.

REPLICA GRATING SPECTROGRAPH

The form of spectrograph suggested for use with the replica has been modeled along lines somewhat similar to an instrument devised by Baker,[1] but possesses several modifications. Its plan may be easily understood by reference to the drawing (Fig. 4). Simplicity combined with rigidity was the principal aim in the construction of the instrument. Lenses of greater focal length could be used for the formation of a longer spectrum without any difference ensuing save

[1] *Journal Royal Photographic Society*, 46, 161, 1906.



A, slit with graduated head B.
D, E, Achromatic plano-convex lenses of 12 inches focus.
W Replica Grating which may be either in position as shown, or at X when the wedge-frame with ½-inch base is inserted at A².

F, H. Diaphragms.
K Metal slide for plate-holder.
N N_1, Metal guides for plate-holder which slides between them at M.
P, P_1, Pinions engaging in rack on plate-holder.
β. Aperture admitting spectrum to photographic plate.

in the length of exposure time, but the dimensions of the instrument would thereby be increased. With the specifications given the spectrum measures 6.2 cm from λ 6900 to λ 3550 (B-N), which is of good measurable length. For special examination of the red and infra-red end a narrow brass wedge-frame is inserted at A^2, which changes the angle of the collimator and brings the C line (λ 6563) in the center of the plate. To enter here into a discussion of the resolving power of the instrument is unnecessary when we take into consideration the work for which it is intended. The spectrograph may, if furnished with slit and lenses of good quality, be used for a very high grade of spectroscopic work. For visual observation an eyepiece can be held by means of an adapter at the plane of the plate.

The spectrograph should occupy a definite permanent position in the laboratory, with the collimator pointing to the northern sky always at the same angle. This latter point is provided for in the construction of the instrument. The width of the slit should remain constant, and all the light reaching it should pass through thin milk glass or other diffusing medium, free from selective absorption. Exposures for the determination of selective sensitiveness should not be made unless the altitude of the sun is greater than 15°. The length of exposure which constitutes the beginning of the series varies with the speed of the plate—i. e., longer with a "slow" than with a "fast" plate. In the case of the Seed "27" plate (as indicative of fast plates) the exposures found most suitable run as follows: 2, 5, 15, 30, 60 seconds, 2, 4, 8 minutes; these eight exposures, together with two others yet to be described, occupying the entire $3\frac{1}{4}\times4\frac{1}{4}$ plate. With a "slow" plate the first two exposures are omitted. Obviously it would not be advisable to adhere closely to those times when using sky light under *extreme* meteorological conditions such as exceptionally bright on one day and raining on the next.

PHOTOMETER

It may be proper at this place to consider the instrument constructed for the measurement of photographic densities.

A very complete bibliography and discussion relative to photometers suited to this class of work are given by Mees and Sheppard

in their paper "Sensitometric Investigations."[1] Following this, the writer had constructed a Hüfner spectrophotometer; but, after experimenting some considerable time with this form of instrument, it was discarded in favor of a modified Brace prism instrument, as the permanent line dividing the two fields under comparison was found very objectionable, and prevented a match of as high a degree of accuracy as if the shades actually adjoined one another.

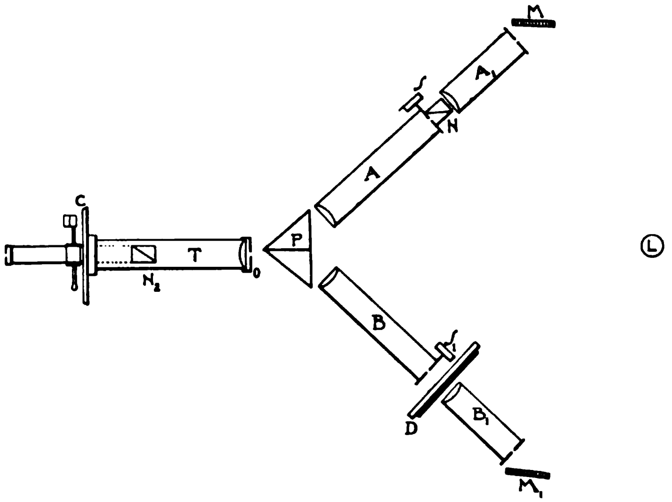


FIG. 5

In Brace's original instrument[2] measurement of differing intensities is made by varying the slit-width on one of the collimators, the readings taken being in terms of the screw-pitch (or slit-width), the optical value of which may be obtained by interpolation upon a scale derived from a previous calibration by means of a rotating sector-disk. In order to make the instrument more particularly suitable for the measurement of photographic plates, a number of changes and additions were made, which will now be described.

Immediately in front of, and in contact with, one of the collimator slits A (Fig. 5), a Nicol prism N_1 was mounted; while in front of that, and in line with its axis, a supplementary collimator A_1 is carried by a

[1] *Journal Royal Photographic Society*, **44**, 200, 1904.

[2] D. B. Brace, "On a New System for Spectral Photometric Work," *Astrophysical Journal*, **11**, 6, 1900.

rigid supporting-piece. Collimator B was also furnished with a supplementary collimator B_1, the function of each being the delivery of a beam of parallel light to their respective slits S S_1.

An analyzing Nicol N_2 is carried in the telescope tube T (which has been lengthened in order that the rectangular diaphragm in front of the objective O might be in distinct focus). The angle of rotation of the analyzer is read upon the graduated circle C. Beyond each of the two collimators A_1 B_1 two mirrors M M_1 are fed from 150 c. p. incandescent lamp at L.

The plate whose opacity is to be measured is held in a special carrier D between collimators B B_1, where, by means of a sliding-piece, the differing opacities are brought successively into position in front of the bilateral slit.

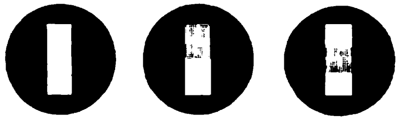
The dispersion-piece employed is the now well-known Brace prism P, which is made up of two equal 30° flint prisms of refractive index 1.64822 for D, and carries on one of its inner surfaces a deposited silver strip 5 mm in width, the two prisms being cemented together.[1] When first constructed, this cementing medium was alpha-bromo-naphthalin, which possesses a refractive index very close to that of the glass used. Constant trouble was, however, experienced on account of the volatile nature of this medium and the difficulty of sealing it in and eventually the prism was taken apart and recemented with Canada balsam. On account of the difference between the refractive index of the balsam and that of the glass, there is always present a small amount of reflected light; but as this light is proportional to the intensity of the incident light, it introduces no error in the readings worthy of any consideration, and is visible only when measuring very low densities.[2]

In adjusting the instrument for use, the prism table is raised until the beam from collimater B passes slightly below the center of the prism, and the field presented when viewed through the observation tube T (which carries no eye-lens, being pierced only with a 2.5 mm

[1] This prism was ground and polished by Mr. O. L. Petitdidier, of Chicago, to whom thanks are due for its optical excellence.

[2] Experiments are at present under way toward the adjustment of a balsam or non-volatile cementing medium of similar refractive index to the glass employed in the prism.

circular opening) is an illuminated rectangle, which is of even brightness throughout (*A*, Fig. 6), when all adjustments are made, and the light is equally intense from either collimator. Should one beam be possessed of greater intensity, the field will show two squares of differing brightness (*B*, Fig. 6). This arrangement has been found more satisfactory in practice than that usually employed, viz., when the light enters the prism centrally and the field is shown crossed by the image of the silver strip, as in *C*, Fig. 6.[1]



A B C

FIG. 6

When beginning a series of measures upon photographic plates, the slits on their respective collimators are first opened to approximately the same width, the greatest opacity in the plate to be measured is run into position, and a rough trial match is made, the object simply being the assurance of sufficient slit-width to give light enough for the lowest measure without running too close to the extinction point (zero). In practice it is not deemed advisable to read an opacity requiring a smaller mean angular measure than 2°.0 (=approximately 3.0 units of Hurter and Driffield).

The mirrors are carefully adjusted to reflect their light centrally through each collimator. Then, while the eye observes the interface of the prism through the telescope tube, and with the analyzer set at 90°, slit S is altered slightly in width until an exact match is obtained between the two halves of the field, which is indicated by the *absolute disappearance of the dividing line.* From now on until the measurement of the plate is completed, neither slits, light, nor mirrors should be moved or altered.

The varying opacities are now slid successively into position in front of slit S_1, and the analyzer rotated until a match with each is secured. As the zero of the analyzer circle indicates the point of extinction, the formula for the expression of the luminous intensity is $\sin^2 \theta$, where θ = the angle of the analyzer, while the degree of

[1] It would probably serve the purpose better if the silver strip covered the entire lower (or upper) half of the interface. The beam of parallel light from the collimators could then pass centrally through the prism, and the single dividing line would fall in the center of the field of view.

blackening when represented in Hurter and Driffield density units $=\Delta \log \sin^2$ from the "fog value."[1] In practice, readings made from both sides of the extinction point furnish a mean which eliminates any error due to the false position of the zero.

The method of recording the measures obtained is shown in Table I, which presents the density measurements of Plate 2 "*lower*" (see page 137).

TABLE I

No.	Angle		Mean	Log Sin²	Δ Log Sin² (= Density)
	Above	Below			
Fog	76.5	72.8	74.8	9.9691	
1	66.0	67.5	66.8	9.9268	0.0423
2	55.9	54.5	55.3	9.8299	.1392
3	42.0	43.5	42.8	9.6644	.3047
4	32.5	35.1	33.8	9.4906	.4785
5	25.1	28.0	26.6	9.3220	.6671
6	20.6	23.0	21.8	9.1396	.8295
7	15.5	18.5	17.0	8.9318	1.0373
8	12.7	15.6	14.2	8.7794	1.1897
9	9.0	12.8	11.4	8.5918	1.3773

As an example of the agreement in the measures of different observers a large number of settings were made by Messrs. Parkhurst and Jordan and the writer, upon the same opacities, A, B, the results of which are given in Table II.

TABLE II

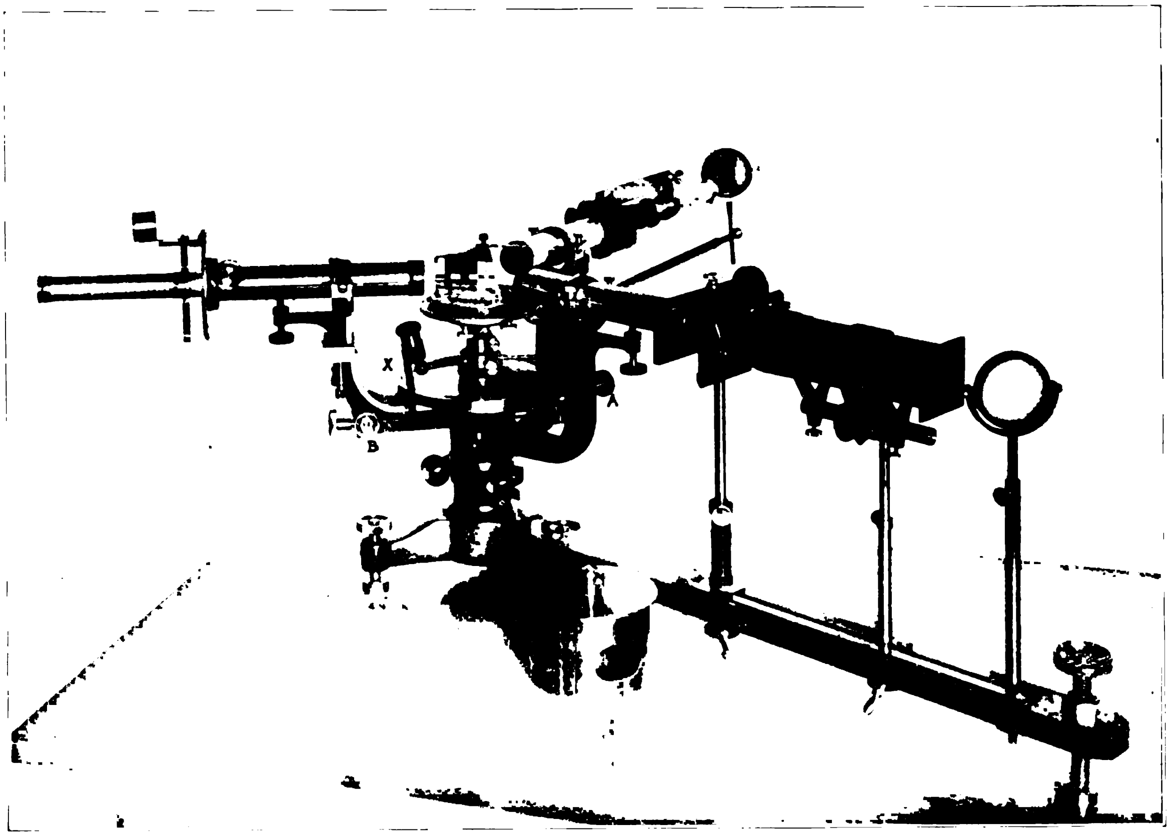
Observer	Mean Angular Measures		Log Sin² θ (= Density)	
	A	B	A	B
J	14°05	9°205	8.7674	8.4076
P	13.93	9.250	8.7642	8.4122
W	14.05	9.220	8.7674	8.4090

(Mean of 12 settings for each value) Difference: Density A = .0032
Probable error of average ± .0009 Density B = .0046

It is well known that in visual photometry the position of the star relative to the comparison light exercises an influence upon the measures, and for this reason it was deemed advisable to test the

[1] A table was constructed giving the value of $\log \sin^2$ from 0° to 90° in tenths of a degree, thus enabling rapid work.

PLATE IX



SPECTROPHOTOMETER FOR MEASUREMENT OF PHOTOGRAPHIC OPACITIES.
($\frac{1}{8}$ NATURAL SIZE).

match obtained in the spectro-photometer with reference to the vertical and horizontal positions. As it was not practical to arrange the instrument to show the two squares in a horizontal plane, all measures were made by alteration in the position of the observer. Professor Barnard and Mr. Parkhurst kindly made the necessary settings, and from a mean of ten in each position, for each observer, the net result obtained was not above the error of observation. The instrument may therefore be regarded as free from error in this regard.

In using the instrument as a spectrometer, special fronts have been constructed for holding color-cells, etc., while the records are made from readings on the divided circle *X*, which is carefully graduated on silver, and reads with two verniers direct to 20″. The prism-table is also graduated. Collimators *A* and *B*, together with the observing telescope *T*, rotate around the optical center of the instrument and are furnished with clamping screws. *T* and *B* are also equipped with tangent slow-motion screws for delicate adjustments. It is, however, not advisable to disarrange the instrument when set up and adjusted for photometry, but to make use instead of separate instruments for different lines of work. The aperture is 25 mm. with a focal length of 200 mm. Plate IX is made from a photograph of the completed instrument.[1]

Another important point regarding this instrument is the ability to displace the two spectra horizontally relative to each other, so that the red of one spectrum is in juxtaposition with any hue in the other by movement of the single slow-motion screw shown at *A*, Plate IX, while direct measurement may be made in any region of the matched spectra by movement of the slow-motion screw *B*.

The spectrophotometer as just described was constructed for various lines of work requiring critical measurement, but such an instrument is by no means essential. The extremely simple and ingenious arrangement devised by Pfund[2] should be well able to meet all of the requirements in ordinary density measures.

[1] Originally the photometer was a three-arm spectroscope constructed by Gaertner, of Chicago, with his usual skill. The alterations necessary to convert it into its present form were made by the writer in the instrument shop of the observatory.

[2] *Johns Hopkins University Circular*, **4**, 20, 1906.

INFLUENCE OF LIGHT IN SENSITOMETRY

The next point in order of importance is the nature of the light used for the determination of selective spectral sensitiveness, and upon this point there seems to be as great a diversity in modern usage as there is in the spectroscope employed. It is conceded on every hand that daylight is the illuminant *par excellence*, but the impossibility of obtaining such light, constant in intensity and quality, has led to the substitution of almost every known source of illumination.

If the question were one which concerned only the integrated luminosity, the difficulties could be much more readily overcome. But unfortunately the distribution of spectral intensity is a more potent factor. One has but to compare the spectra of the various sources, even roughly, to find that they present no agreement among themselves. Some are deficient in the red rays, while others are deficient in the violet (Plate VIII).

In the comparison of the acetylene flame (*a*), the spectroscope was arranged with a Hüfner-Albrecht rhomb immediately in front of and in contact with the slit-jaws. One of the rhomb surfaces was illuminated by a beam of diffused daylight, while the remaining incident surface received a beam from the diaphragmed acetylene flame. The distance of the burner from the slit was altered until the spectra appeared visually equal in the green. Exposures were then made upon a Cramer isochromatic plate for varying lengths of time, the daylight and acetylene spectra impressing themselves simultaneously.

In the comparison of the candle (*b*), benzine (*c*), and *Mg* (*d*) flames, and the incandescent electric light (*e*), the replica-grating spectrograph was used without the rhomb, daylight exposures being made at the beginning and end of each series.

The great lack of ultra-violet in *a*, even with extreme overexposure, is readily observed, together with the strong action of the yellow-green, which with a suitable plate would be shown as extending with increasing action into the red. In *d* this effect is reversed, and the maximum action is shown to lie in the ultra-violet (as is also the case in the use of the electric arc-light); *b* and *c* are very similar to each other, and show the characteristic drop in the ultra-violet, with the corresponding increase at the red end.

PLATE VIII

Diffraction (Fig. 1) and Prismatic (Fig. 1a) Daylight Spectrum on Cramer Trichromatic Plate

FIG. 1

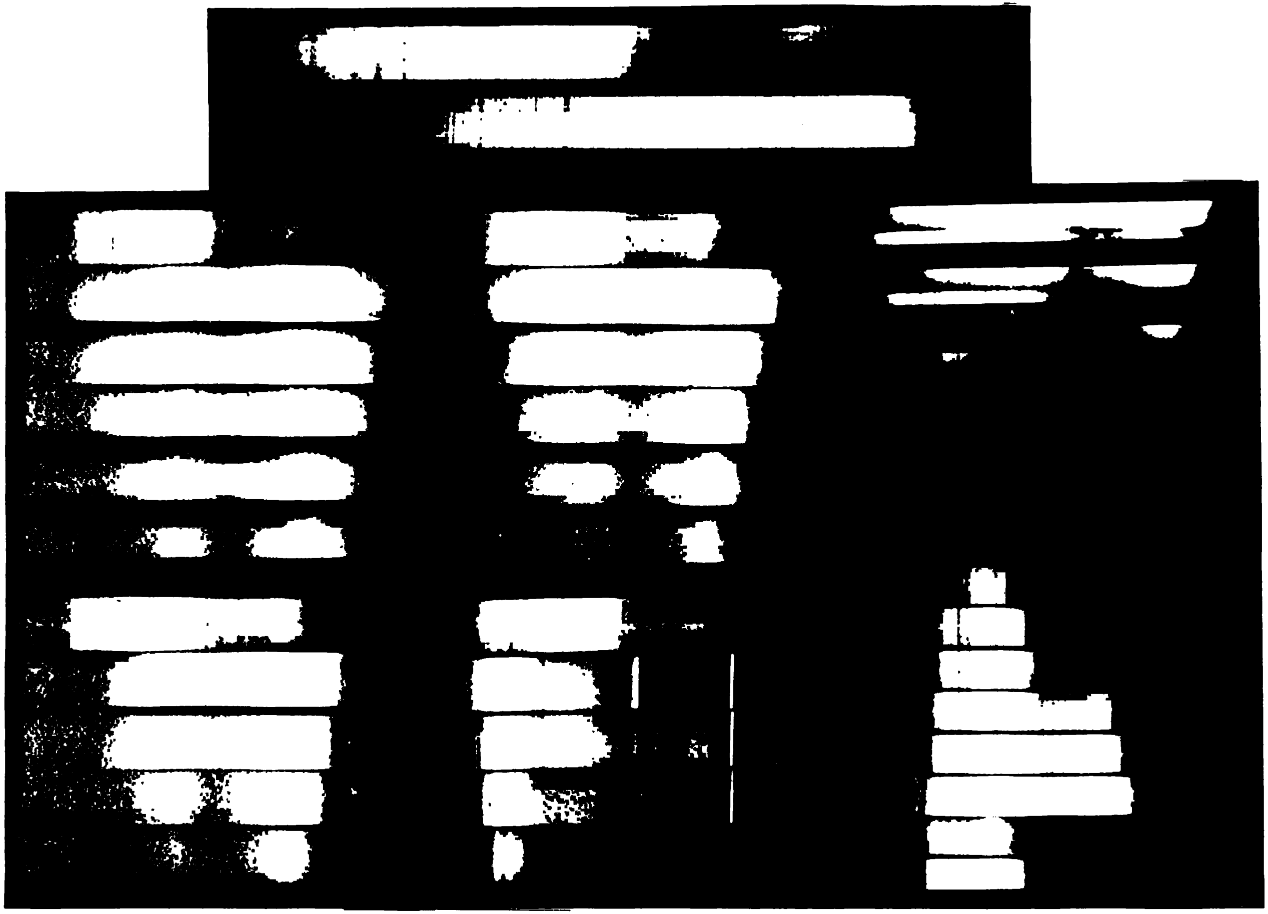
FIG. 1a

b

c

a

f



COMPARISON OF LIGHT-SOURCES USED IN SENSITOMETRY

b. Candle Flame. *e.* Incandescent Electric Light. *a.* Acetylene Flame.
c. Scheiner Benzine Flame. *d.* Magnesium Flame. *f.* Cramer "Inst. Iso." Plate.

Notwithstanding that this is a point to which many workers have directed attention, yet unfortunately in the majority of cases those same workers continue their use and publish spectral comparisons of plates made with the same light-source which they condemn, and for which they give no correction factor. It is obvious that the relative selective sensitiveness of two plates determined by a light vastly different from daylight cannot furnish any reliable quantitative information regarding the true values of the plates, unless the artificial light be accurately calibrated in terms of daylight (by photographic means), and a formula derived from such calibration which may be used as a correction factor.

Efforts have been made to calibrate a light to the spectral value of daylight, and though several approximations have been arrived at, yet we are still far from a satisfactory conclusion. The latest and best work in this direction is due to Mees and Sheppard,[1] who have suggested as a standard, acetylene gas, burning under constant pressure, and with special care as to its purification, etc., to insure constancy in the luminous intensity. This latter point presents no especial difficulty. Inasmuch as the flame of acetylene gas is greatly deficient in the violet end of the spectrum, they devised a compensating color-filter to correct this deficiency, whose action may be briefly explained by stating that it was intended to absorb proportionately the excess from the least refrangible end of the spectrum. While this combination was undoubtedly an improvement, yet it was by no means satisfactory, and that this was recognized by these careful investigators themselves is proved by the introduction of still another make of filter in a later publication.[2] This latter filter can, however, still be considered as no more than an approximation, which is indeed what these workers themselves term it.[3]

There are, however, numerous opportunities for the use of a standard artificial light, in which the difference in spectral distribution from daylight does not enter greatly into consideration, and for such the acetylene "standard" of Mees and Sheppard offers decided

[1] *Journal Royal Photographic Society*, **44**, 293, Nov. 1904.

[2] *Ibid.*, **46**, 114, 1906.

[3] *British Journal of Photography*, **53**, 707, 1906.

advantages. The writer has made use of a somewhat similar arrangement with most satisfactory and encouraging results.

The question now is: Does an approximation so arrived at offer any advantages over diffused daylight, if used under certain conditions, when applied to the determination of selective sensitiveness? Briefly, I hope to show that it does not.

DAYLIGHT (VARIATION IN COLOR)

It has been many times stated that daylight was utterly unsuited for sensitometric tests, because of (1) the difference in the intensity of the various hues as the slit is illuminated by white cloud or blue sky, and (2) the variation in general brightness-intensity. Experiments were made to determine the actual difference in the first instance, by using the replica-grating spectrograph as just specified. The instrument was so arranged that the axial line of the collimator pointed directly to the zenith. Immediately over it was held a Zeiss apochromatic lens of 314 mm (12.4 in.) focal length, which formed an image of the cloud upon the slit-plate. Several exposures were made on different dates, and on various makes of plates, only such days being chosen as presented well-defined cumulus clouds in a clear and intensely blue sky. Care was taken that the slit was entirely filled with the cloud-light or blue sky, as the occasion demanded and exposures were made to immediately follow one another, the exposure times being from 5 seconds to 1½ minutes, and on the same plate

The results were exceedingly interesting, the negatives from the blue sky showing, as was to be expected, an absorption of the complementary hues at the least refrangible end. This absorption was, however, but slight in general. Those negatives whose timing showed the greatest contrast-difference were selected for measurement, together with a plate exposed to the spectrum from the same sky, but with the lens removed and a sheet of ground glass interposed in its place.

Density measurement of these negatives gives the results as detailed in Table III.

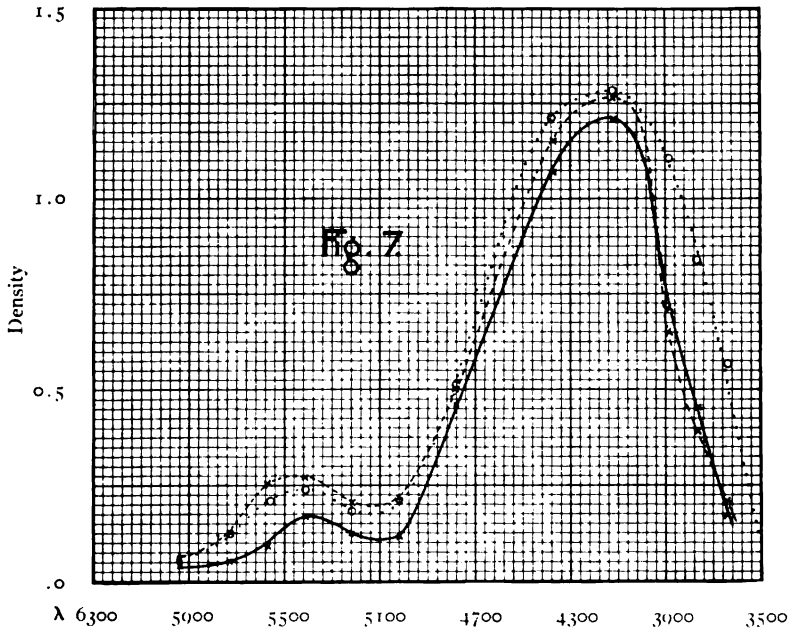
When plotted, these results give the curves of Fig. 7.

It will be noted that the difference between the secondary maxima in the yellow-green, when expressed in density, =0.11, and this amount would be still further reduced if the "blue sky" negative had

TABLE III

Wave-Length	Density (Blue Sky)	(White Cloud)	(Ground Glass)
3640	0.1750	0.2122	0.5731
3760	.4536	.3956	.8401
3880	.7106	.6498	1.1121
4120	1.2124	1.2638	1.2861
4370	1.0702	1.1486	1.2182
4780	0.4592	0.4992	0.5112
5020	.1172	.2234	.2194
5220	.1218	.2012	.1896
5410	.1636	.2739	.2400
5570	.0952	.2619	.2109
5730	.0494	.1261	.1253
5940	.0382	.0676	.0666

been exposed for a slightly greater length of time. The strong absorption in the ultra-violet from λ 3400 to λ 3700 in the "cloud" and



Blue sky, continuous line.
White cloud, ×- - -×- - -×
Ground glass, o · · · · · o · · · · · o

"sky" negatives is, of course, due to reflection and absorption by the component parts of the lens-system. On the other hand, a

change in altitude of the observer would show a still greater difference, the "blue" of the sky becoming more intense as the altitude increased, and necessitating an increase in the exposure time.

Comparison of these results, together with comparison of exposures made when the collimator formed an angle of 25° with the plane of the horizon (which of course showed considerably less difference), indicates that daylight from a low angle, when properly diffused, is a sufficiently reliable guide for practical tests in selective sensitiveness. The second objection will be dealt with presently.

HURTER AND DRIFFIELD'S INVESTIGATIONS

To correctly understand and appreciate the argument advanced for the use of daylight as a standard in plate-testing necessitates a fairly clear understanding of the work of Hurter and Driffield, whose "Photo-chemical Investigations"[1] first raised photography from mere rule-of-thumb practice, and placed it upon a definite basis of scientific fact. They discovered and enunciated the laws governing the action of light and development, and furnished a terminology which is not likely to be supplanted. Unfortunately, for some unknown reason the important results of these eminent workers are but little known in America, and one may therefore be pardoned for briefly capitulating those points which bear directly upon the present paper.

The first great distinction made by them is in the definition of the terms "opacity" and "density," which are in ordinary use, synonomous. *Opacity* is defined as representing merely the optical property of the reduced silver in the negative to impede the passage of light; *transparency* is therefore the inverse of this. *Density*, on the other hand, is a physical measure of the amount of silver reduced in the film, and is expressed as the logarithm of the opacity, thus

$$D = \log O = \log \frac{I_1}{I},$$

where I = the intensity of the light transmitted, I_1 = the intensity of the incident light, and O = the opacity. This distinction between opacity and density must be firmly fixed in the mind.

If a plate be impressed with a series of different accurate exposures

[1] *Jour. Soc. of Chem. Industry*, May 31, 1890.

increasing in geometrical progression, as 1, 2, 4, 8 . . . . 256, and developed, and the resulting scale of opacities be measured, it will be found, that if the logarithms of these opacities are plotted (densities), there will result a characteristic curve. The central portion of this is practically a straight line, and throughout this straight portion the deposits of reduced silver (blackening) will increase in arithmetrical progression, as 1, 2, 3, 4, ... 9; that is, there is a definite logarithmic relationship between the amount of light acting and the action itself. The density unit is the density of a deposit which transmits the tenth part of the incident light.

The enunciation of the law of "constant density ratios" provoked considerable controversy from photographic workers in general; but, while the investigators' conclusions were disputed, they advanced to the support of their statements definite scientific proofs which confirmed them. It was found that the relation existing between the amount of light and the density-ratios is fixed and unalterable by the constitution of the developer, or time of development; the *opacity* ratios are, however, altered. For example, suppose that a plate exposed to light for a definite length of time behind a revolving sector-disk with graduated apertures be cut into two portions, and each portion be then developed for a different length of time, we should obtain as a result negatives which differed greatly in their appearance one from another; that is, that one which had received the shortest time of development would be what is termed a "thin" negative, while that receiving the longest time of development would be what is usually termed "contrasty." Yet the ratio existing between the densities would be identical, although the opacity-ratio varied, the increase in development causing the various densities to grow, but in such a manner that they would still bear the same ratio to one another. When the opacity-ratio is the same as the ratio of exposure, the negative is the true inverse of the original. The determination of the characteristic curve shows that a plate has considerable "latitude" in exposure,[1] so that negatives developed together which had received greatly different exposures would yield identical prints, provided that the exposures lie within the straight portion of the curve.

[1] "Latitude" is defined as the ratio of the exposure at which over-exposure commences to that at which under-exposure commences.

On the other hand, in the case of two negatives developed for a precisely similar length of time, but with one exposure double that of the other, we have the opacity-ratios constant, while the densities vary. In other words, the extra exposure simply means the addition of an equal amount of deposit on the varying densities composing the negative, and merely affects the time required in printing. To such an extent is it possible to vary the exposure time (with constant development) that increasing exposures of from *one to sixteen times* normal will produce identical prints, the only real difference between them being the time occupied in printing. The negatives, however, ***appear*** vastly different from one another. This fact was exceedingly well illustrated recently in a photographic magazine.[1]

VARIATION OF MEAN INTENSITY IN DAYLIGHT

With this explanation we may now consider the second objection to the use of daylight, namely, the variation in the mean intensity. It will readily be perceived that this intensity-variation of course amounts to nothing more than an alteration in the length of exposure, and therefore comes under the jurisdiction of the law expressed in the preceding paragraph. In order, not only to test the validity of the law, but also to obtain a personal measurement of the photographic light-change during the course of a few hours, the following exposures were made with the revolving sector-disk,[2] under conditions as specified. The record detailed in Table IV is from the laboratory notebook.

All four plates were developed together in the same tray with rodinal 1 : 24, for two minutes, at a temperature of 17° C., and, when fixed and dried, were carefully measured, and the densities plotted. The resulting curves are shown in Fig. 8, and it will be readily seen that they bear out very exactly the theoretical requirement that they lie parallel to one another. Their density-ratios vary, but their

[1] F. Dundas Todd, "Development, Scientific and Practical," *Photo-Beacon*, **16**, 300, 1904.

[2] The sector-disk exposure machine was constructed somewhat similar to the arrangement of Mees and Sheppard (*Journal Royal Photographic Society*, July 1, 1904, p. 229), but fitted with a removable cap pierced with a 4.0 mm circular aperture and covered with ground glass for use with daylight. When working with the constant acetylene light, the cap is removed and the burner instantly placed in position.

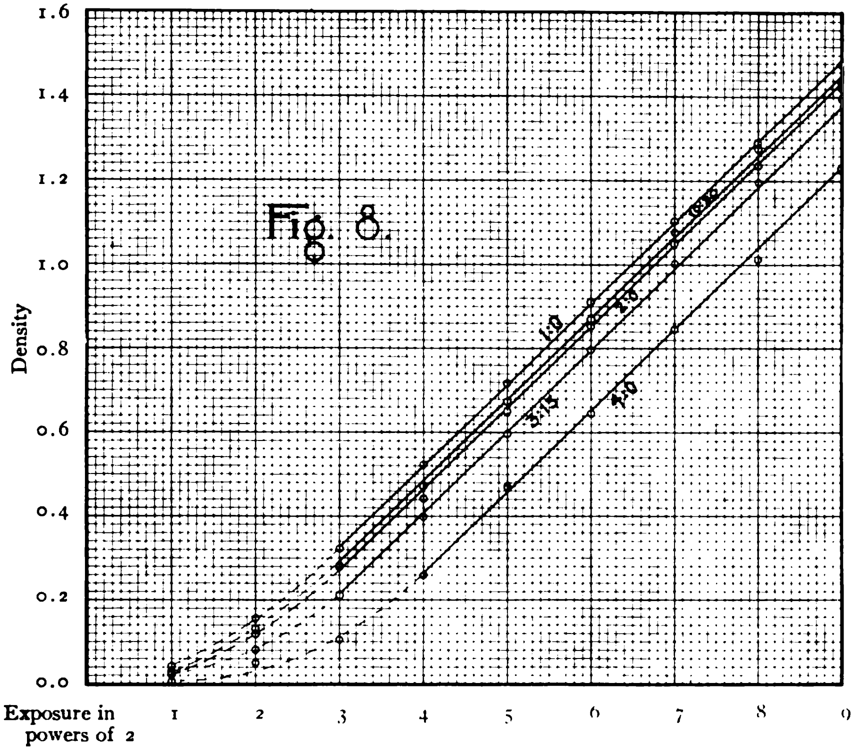
TABLE IV

Sky bright and sunny. Plate, Seed "27" Emul. 11168; Oct. 12, 1906

Plate No.	Time of Exposure	Position on Plate	Length of Exposure	Remarks
1........	10 A. M.	Upper half	3 min.	Blue with white clouds
1........	10:30 A. M.	Lower half	"	Blue with white clouds
2........	11:15 A. M.	Upper half	"	Slightly brighter
2........	12 M.	Lower half	"	Slightly brighter
3........	1 P. M.	Upper half	"	Still brighter
3........	2 P. M.	Lower half	"	Intensity about the same
4........	3:15 P. M.	Upper half	"	Slightly duller
4........	4 P. M.	Lower half	"	Light much weaker

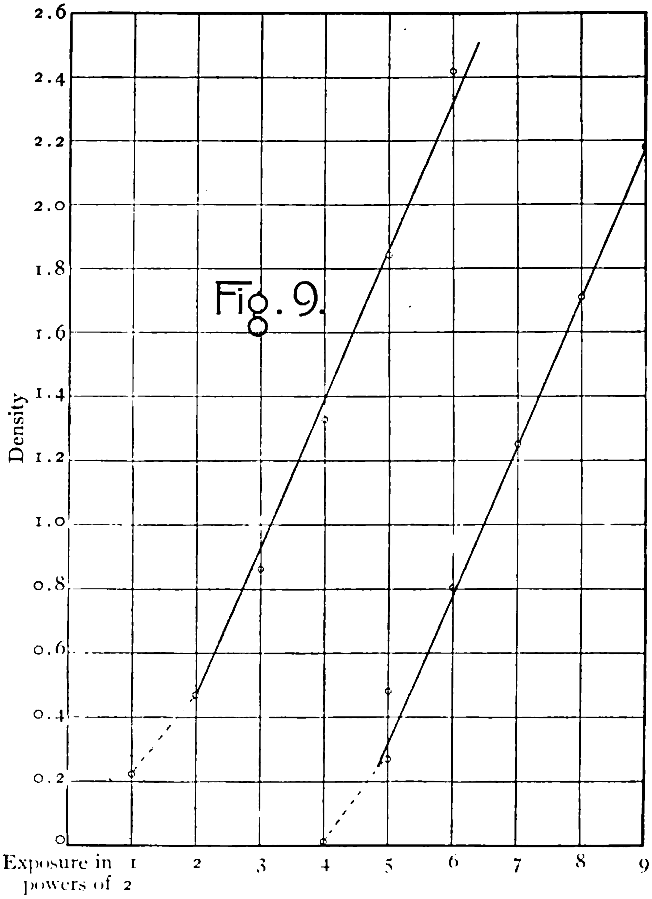
Two exposures on each plate. All four plates cut from one large plate.

opacity-ratios are constant. Between 1 P. M. and 4 P. M. there is an indicated difference in light-action of $2^{1.3} = 2.5$ times, which allows for considerable fluctuation in light-value. But that this is by no



means a limiting value is shown by the parallelism of the curves in Fig. 9, which represent a difference in light-action of $2^{3.3}$ or 9.8 times. There is no difficulty in obtaining any amount of corroborative data in this connection.

It should be remembered, that with varied exposure and equal time of development, we obtain, as the exposure is increased, an addition



of an equal density (or fog) to the complete negative; but this increase of density does not in any way alter the opacity-ratios existing between the series of exposures on the same plate (throughout the straight portion of the curve), such opacity being governed by the development:

there is no alteration in the gradation. Hence the curves of two differently exposed spectra would, as a whole, be parallel to each other, although the height of the ordinates (densities) would vary.

IRRATIONALITY OF PLATE CURVES

To be able to refer to the speed of any particular plate as possessing a definite numerical value presents advantages which cannot be disputed. But if such a numerical value is based upon some source of selective radiation which differs from daylight, such as a candle in the case of Hurter and Driffield, the screened acetylene light of Mees and Sheppard, or the benzine lamp of Eder,[1] then the comparative speed-values obtained for "ordinary" and orthochromatic plates, are ***certain*** to be unreliable to a greater or less extent, dependent upon the closeness of the approximation of the artificial standard to daylight, and they must therefore be accepted provisionally.

A method commonly in use in testing the speed of one plate against another, is to expose the two plates to identical amounts of the same light-action, and then develop them together in the same tray for a similar length of time, and compare the resulting negatives. Provided that both plates have a similar composition, the method cannot be objected to; but when the plates in question are possessed of a different chemical constitution (the consideration of orthochromatic plates being laid aside for the moment), such a method is very liable to lead in many cases to most erroneous conclusions.

If two pairs of differently constituted plates (A, A_1, and B, B_1) be exposed simultaneously to the same light-action, and then developed together, giving one pair double the length of time of the other, as $AB=3$ minutes, and $A_1B_1=6$ minutes, it is very common to find, that with the first pair where A possesses a greater density than B, in the case of the second pair with the lengthened development, the effect would be entirely reversed and B_1 will have a greater density than A_1. The following series of plates was therefore prepared: a Seed "27" and a Cramer "instantaneous isochromatic" were exposed to precisely the same amount of light-action behind the revolving sector-disk, and then cut into eight strips each in the

[1] Eder and Valenta, *Beitrage zur Photochemie* (French translation by Belin, entitled *Système de sensitométrie*).

dark-room. All of these strips began development together in the same bath at the same time, but, at stated intervals, each pair of plates ("27" and "Iso.") was removed from the developer together and

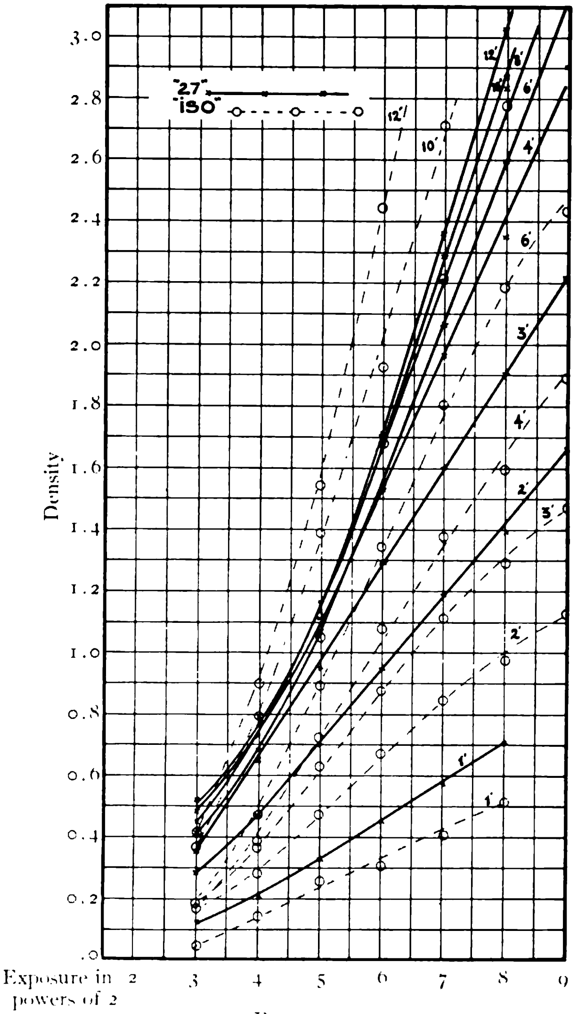


FIG. 10

passed directly into the fixing bath. From the measurement of these negatives the curves shown in Fig. 10 were plotted, and serve well to illustrate this phase of plate-action.

It will be noted that, with but one exception (8 minutes), no single pair of plates gives curves which are parallel to themselves. The gradation of the two plates is entirely different. In the present instance the Cramer plate is known to be slightly slower than the Seed, and hence it will be perfectly correct for the curve of the former to lie lower than that of the latter; but it will be seen that, if we measure the distance apart of these curves representing the various development times, it is a constantly decreasing quantity up to a certain point, namely, that at which the slope of the straight portion of the curves are similar; and from that point on, the conditions are absolutely reversed, and the isochromatic plate acquires a greater density than the "27," and appears the faster plate of the two. Under the method of equal time of development, therefore, no true deduction could be made regarding the relative speed of these plates from any of the eight pairs of curves shown.

It has been shown by Abney[1] that a change in the gradation-curve takes place when the exposure is to light of differing wave-length, the curve becoming steeper; that is, the contrasts are more marked as the wave-length increases. In the present instance with normal development the curve of the "Iso" plate is less steep than its accompanying "27" plate, although it is sensitive farther toward the less refrangible end of the spectrum. It would appear, therefore, that the change in the slope of the curve is due mainly to the constitutional (chemical) difference between the two plates, which leads to a difference in the velocity of the chemical reaction in ordinary development.

In order, then, to obtain a direct comparison between two plates, it is necessary *not* to develop for precisely similar lengths of *time*, but for precisely *similar amounts of development-action*—i. e., reduction product; under which circumstance the gradation-curves will lie parallel to one another. In trichromatic work, where different plates are used and the development is for equal times, as is very commonly the case, this change in the gradation-curve due to constitutional plate-difference must therefore be as carefully guarded against as is the change in gradation due to difference in wave-length, if the true color-value of the object be seriously considered.

[1] "On the Variation in Gradation of a Developed Photographic Image," *Proc. R. S.*, **68**, 300, 1901.

DEVELOPMENT FACTOR

In the development of either the spectral records or the sector-disk exposures it will be evident, therefore, that the duration of development is of considerable importance. Hurter and Driffield have named the amount of development received by a plate the "development factor" (γ). This factor may be calculated from their formula

$$\gamma=\frac{D_2-D_1}{\log E_2-\log E_1},$$

where D_1 and D_2 are two densities selected from the straight portion of the curve, lying as far apart as possible, and E_1E_2 = the relative exposure times for the densities considered. It was shown by these workers that when $\gamma=1.0$ the negative is the true inverse of the original; when greater than 1.0, the contrasts of the original are increased; while if less than 1.0, they are diminished. In testing by diffused daylight it is not possible to obtain the values of E_1E_2 expressed in c.m.s. (candle-meter-seconds); nor for practical results is it essential. If one takes instead the ratio of the light-apertures in the revolving sector-plate, results of sufficient accuracy may be readily secured.

In the sector-disk made by the writer the apertures were cut in brass with much care, and the edges beveled. Yet, notwithstanding all efforts to the contrary, the error on the smaller apertures was considerable. This error may be noted by comparison with the theoretical ratio, thus:

Theoretical ratio=	1,	2,	4,	8,	16,	32,	64,	128,	256
True ratio=	1.04,	2.03,	4.06,	7.94,	15.83,	31.84,	63.8,	127.6,	256.0[1]

When it is borne in mind that in the everyday practical sensitometry of photographic plates use is made of those obtainable commercially, and not of an article specially coated on an accurate surface, it will readily be appreciated that the use of the theoretical aperture-ratios is well within the limits of "plate error;" to make use (except for special work) of the true ratio is an unnecessary refinement.

The value of γ may also be obtained graphically by drawing a line parallel to the straight portion of the characteristic curve, start-

[1] My best thanks are due to Professor Raymond Burnham, of the Armour Institute, Chicago, for the measurement of the disk apertures.

ing from the point 1.0 in the log E scale; for, as expressed by Hurter and Driffield, $\gamma = \tan\,\theta$, where θ is the angle of inclination of the curve from the horizontal base-line. The value is read directly from the scale of density-ordinates. In practice the writer finds it convenient to shift this 1.0 point two divisions to the left along the abscissa scale, and thus economize space.

The constant t_{γ_1} (the time necessary for any plate to reach a development factor of 1.0) advanced by Mees and Sheppard, which is of great value in the indication of the development speed of various plates, may be determined by the method of Driffield for determining

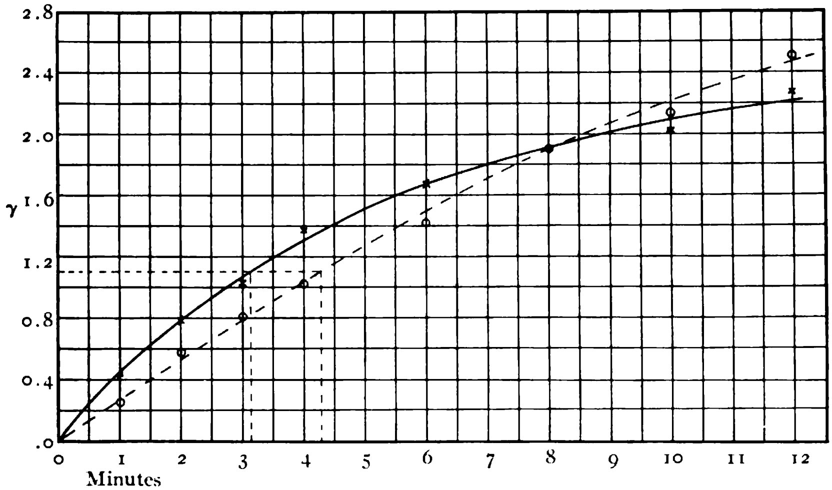


FIG. 11

the "control factor."[1] The development factors (γ), extracted as already described, are now plotted with the time of development as abscissae, and the two development factors as ordinates, which together with the zero point determine the curve. Take, in illustration, the Seed "27" and Cramer "instantaneous isochromatic" plates (Fig. 11), which were exposed behind the sector-disk and developed at a temperature of 17° C. for from 1 to 12 minutes, respectively. From the replotted curves in Fig. 11 the development time necessary to reach $\gamma_{1.1}$ may be read off directly as indicated—viz., $3^{m}\ 10^{s}$ and $4^{m}\ 15^{s}$, respectively.

[1] *Photographic Journal*, 43, 17, January 1903.

RELATIVE SPEED

The method now advanced by the writer consists in selecting some one plate whose quality and general behavior present a reliable uniformity against which all other plates may be compared. In the Seed "27" Gilt Edge we have a plate which may fairly be considered as filling the requirements, because, in spite of the fact that occasionally it has suffered a slight drop in speed, it is characterized by a remarkable uniformity.

Briefly, the speed of a plate is required to be known. It is cut in such a manner that it, together with a "27," may lie in the holder and be exposed at the same time behind the revolving sector-disk to the same light-intensity. After exposure, each plate is cut into two strips, and all are developed at the one time and at constant temperature, being removed from the developer in pairs after the lapse of 4 and 8 minutes, respectively, and then fixed, washed, dried, and measured.

From these measurements is extracted γ, and the time necessary to reach, say, $\gamma = 1.0$, is read off directly, and used as a time-factor for the development of another pair of exposures upon two more of the same plates developed at a similar temperature.

The measurement of this second pair of plates will in turn give curves which lie parallel to one another, and from which the relative speed may be obtained.

One measures, therefore, the distance apart of the curves (horizontally), and, remembering that the exposure increase for each step rises in powers of two, the difference in speed between the two plates for a given intensity of light of similar spectral composition will be two, raised to a power the value of which will be determined by the distance measured; e. g., the mean distance apart of the two curves in Fig. 9 is 3.2, then $2^{3.2}$ is the difference in speed; or, the exposure time would have to be increased 9.19 times in order to obtain similar density.

In this method of relative speed determination there is the extra work entailed by the measurement of the "27" plate with every determination, but such work really amounts to very little in actual time, and has the added advantage in the use of daylight in place of some artificial "standard" of more or less doubtful value.

Experimentally the writer has not been able to so accurately expose and develop a third pair of plates as to have the plotted results actually superpose, the difference from a mean curve being ± 0.02 of a density unit in the most favorable instances, and running up as high as ± 0.05 in exceptional instances. Further discussion upon this and kindred points is reserved for a following paper.

COLOR-SENSITIVENESS (χ)

When Hurter and Driffield advanced their epoch-making methods for the sensitometry of photographic plates, the use of a candle in this connection was allowable, because at that time the orthochromatic plate was but little used and less generally understood. At the present writing there is scarcely a manufacturer of photographic plates throughout the world who does not prepare one or more brands of color-corrected plates, and it is merely a question of a very brief time until the use of the orthochromatic plate will be imperative for everything save the photography of black and white.

When we consider the color-sensitive plates of the present day as a whole, there are four points which strike even the casual observer as characteristic: (1) the *strong* sensitiveness to the blue-violet; (2) the secondary sensitiveness to the yellow-green; (3) the *low* sensitiveness to the blue-green; and (4) the lack of sensitiveness to the red. It is evident that these values should be definitely known, and that, whatever method is adopted for their estimation, it should be comprehensive enough to thoroughly differentiate them.

Mees and Sheppard have proposed the constant χ as representing the ratio of the inertia of the blue-sensitiveness to the inertia of the yellow-sensitiveness. Their method of determining this value (which is an improvement upon the system of Eder) consists in exposing a plate to their screened acetylene light behind the sector-disk, which plate is still further screened by the interposition of a color-filter transmitting only the red light to λ 5900 (A–D).[1] Another plate is exposed in the same manner, but with the interposition of a green

[1] The spectral transmission value of this color-filter should be very carefully determined, as it is composed of rose bengal and tartrazine. This latter dye even in concentration transmits the ultra-violet at λ 3700. The rose bengal of course transmits the violet very fully.

filter transmitting light from λ 5900 to λ 5000 (D–$b\frac{1}{2}$F), while a third plate is exposed through a blue filter transmitting light from λ 5000 on ($b\frac{1}{2}$F+). The densities of these plates are then measured, and the ratio of the inertias is obtained.

In the opinion of the writer, the division of the spectrum into three parts furnishes altogether insufficient information for either the scientist, plate-maker, trichromatic worker, or student of orthochromatism.

A plate exposed through the red color-filter may give a very high value, and thus indicate a red-sensitiveness which does not exist, the action being due entirely to the orange at, say, λ 6000, to determine which reference *must* be made to the spectrum. A somewhat different criticism applies to the results obtained by exposure through the green filter, whose transmission ends in the region of photographic low-sensitiveness in the blue-green. The elimination of this insensitive gap (or results tending to such elimination) is of considerable importance in practical plate-making, and therefore the relative values of plates for this region should be definitely recorded; the division of the spectrum at this point by the green and blue filters makes such determination impossible.

The method, however, has a certain broad value for the estimation of sensitiveness when required for use with wide-banded color-filters, such as are generally used in trichromatic work; but it is unquestionably true that it cannot compare in quantitative estimation with a series of daylight spectrum negatives where the action of the plate for every wave-length of light is definitely apparent. As any system of sensitometry to be popular must be rendered as simple as is consistent with definiteness, then what could be easier than to quote the density-readings at, say, six points[1] of the spectrum measured, if any further numerical evaluation be required?

DEVELOPMENT OF SPECTRA

In the development of the spectrum plates obtained by exposure in the spectrograph as described, special care must be taken to hold as constants (*a*) the constitution of the developer; (*b*) the tempera-

[1] The six points referred to may be at $\lambda\lambda$ 3800, 4100, 5100, 5500, 5900, 6100, for all of the ordinary orthochromatic plates. In cases of special red-sensitiveness then the density-value of a seventh point may be added.

ture of the developer; (*c*) the time of development (as determined by the γt curve and corresponding sector-strip). Due attention to these points will result in negatives directly comparable with one another.

MEASUREMENT AND INTERPRETATION OF SPECTRA

In a series of spectrum exposures upon two different plates, that one of each is selected for measurement whose region of maximum opacity corresponds approximately to a density of 2.5. This is very readily selected by comparison with a standard density plate upon which the measured densities have been plainly marked.[1] Now, it makes absolutely no difference whether this spectral maximum lies in the yellow or in the violet. The only thing to took for is *a* maximum of 2.5.[2]

In the practice of the writer this is still more readily determined by setting the analyzer circle of the spectro-photometer at 3°, and moving the spectrum plate in front of the collimator slit until we arrive at that one whose maximum opacity approximately equalizes the field in the viewing telescope; then that spectrum is marked for measurement. With a similar procedure on other plates we obtain spectra which may be compared directly with one another, because, generally speaking, they represent as their maximum an opacity of action of 256 light-ratio units, under identical conditions of development.

Using a narrow slit in the spectrophotometer, the spectrum selected may now be measured, and its curve plotted in the usual manner, with the densities expressed as ordinates and the wavelengths as abscissae; or, if preferred, the ordinates may read light-unit ratios, the values being obtained by interpolation upon the curve already obtained from the corresponding sector-disk negative previously exposed and developed at a similar temperature, and taking the aperture ratios as units. This method was advanced by the writer in a former paper,[3] and serves the very useful purpose of indi-

[1] Such a plate may be obtained from a Scheiner or a Hurter and Driffield sector-disk.

[2] Because 2.5 is conveniently the highest allowable density for reliable direct measurements.

[3] "Preliminary Note on Orthochromatic Plates," *Astrophysical Journal*, **22**, 153, 1905. It should be mentioned, however, that the values given in this former paper were from visual *estimates*, in place of the present *measures*, although the following paper (*ibid.*, **22**, 350, 1905) confirms by experiment the values first derived.

cating at a glance the ratio of opacity to exposure for differing wave-lengths.

In such a spectrum record we obtain a quantitative estimate of the plate under test. Aside from the spectrum selected for measurement, we see at a glance the true region of maximum sensitiveness as evidenced in the shorter exposures. The growth of density in the least refrangible region with increase of exposure is readily marked, and its relation to the blue-sensitiveness may be easily approximated when the exposure time is known. If, for example, the spectrum selected for measurement as having normal exposure be the resultant of an exposure of a minutes, then the value x of the light-units acting at any point on some other exposure b may be expressed as $\frac{b}{a}c = x$, where c is the value of the light-units corresponding to an exposure a.

Furthermore, we do not have to depend upon the impress of a glass scale (for example) in order to record spectral position, into which there enters an element of uncertainty consequent upon the accidental displacement of either that or the dispersion-piece. On looking at a print from such a record, the observer has absolutely no means of knowing whether the scale is in true position or not, or to what extent the negative may be overexposed. On the other hand, the daylight record leaves no element of uncertainty, because the Fraunhofer lines indicate at a glance the exact wave-lengths, and also serve to show over- or under-exposure.

In all of the spectra exposed above normal there is present an amount of "fog" which arises from the "spreading" of the light at the region of maximum sensitiveness, and interior reflection in the spectrograph; furthermore, the overlapping ultra-violet of the second-order spectrum is apt to lead to false conclusions in the estimation of color-sensitiveness. For that reason the remaining two exposures are made after introducing the wedge between the collimator tube and the front board of the camera, the increase in the angle of incidence thus causing a corresponding displacement of the spectrum on the plate. An ammonium picrate color-filter is then introduced in front of the slit to completely absorb the overlapping violet. These latter two exposures are necessary to the correct appreciation of the actual extent of sensitiveness in the red. The appearance presented

by such a negative plate as has been described is shown in *f*, Plate VIII.

It should be definitely understood that this suggested method of daylight sensitometry is advanced as a practical everyday means of arriving at reliably comparable results suited to the requirements, not only of the general worker in photography, but also of those who are making use of the photographic plate in obtaining records of scientific value. No one is more conscious than the author that it contains some points which may in time be improved upon, but it at least serves the useful purpose of definitely pointing out in connected form the greater number of pitfalls and inaccuracies which beset the path of sensitometry, and further indicates a means of obtaining exceedingly good results with the minimum of time and equipment. It will be obvious that the method is primarily suited to those who are *users* of plates, rather than to those whose work tends principally toward the *manufacture* of the material. For this latter class, however, there is no reason why the method may not be extended to embrace the requirements suited to their needs.

SUMMARY

We may summarize the foregoing and tabulate the entire process as follows:

1. The advancement of the replica-grating as a standard dispersion-piece, together with a simple form of spectrograph suited to its use.

2. A suggested method of daylight sensitometry (making use, as far as possible, of the laws discovered by Hurter and Driffield), of which the following is a résumé:

a) Exposure of one $2\times4\frac{1}{4}$ plate scored down the back, but not broken through, together with one $2\times4\frac{1}{4}$ Seed "27" plate, for, say, two minutes, in the sector-disk machine.

b) The scored plate is broken through into two secondary slips, and all four plates are now developed (preferably together) with a constant developer, for a constant length of time, and at a constant temperature, with the exception of one of the secondary slips which remains in the developer for exactly double that of the others.

c) Measurement of the density strips and extraction of γ, t_{γ_n} and latitude.

d) Exposure of a second pair of $2 \times 4\frac{1}{4}$ plates, and development for the time necessary to obtain equal amounts of development action as found from *d*, retaining composition of developer and temperature as constants. Measurement of same and extraction of speed-ratio.

e) Exposure of one $3\frac{1}{4} \times 4\frac{1}{4}$ plate to a series of eight exposures in the spectrograph varying from two seconds to eight minutes, and two further exposures on the same plate with the collimator wedge in position and through the ammonium picrate screen.

f) Measurement of selected spectrum for quantitative color estimation.

YERKES OBSERVATORY,
January 24, 1907

Reviews

Introduction to Astronomy. By Forest R. Moulton. New York: The Macmillan Co., 1906. Pp. 557, with 201 figures and 4 star maps. $1.25.

The appearance of an introductory treatise on astronomy by Professor Moulton is of interest to all readers of science, and of special interest to teachers of astronomy. Professor Moulton's point of view is his own, in many ways unlike that of the textbooks in general use. Although the order and emphasis of presentation may be sometimes criticized, there can be no question that the book is throughout suggestive and stimulating.

The introductory chapter is a new feature. It offers a preliminary outline of the whole subject, the author holding that a brief survey of the science is desirable, in order that main lines may not be lost sight of in details. It contains a good statement of the method of scientific inquiry, and an excellent presentation of the steps of evolution in astronomy.

The chapter on co-ordinates gives good suggestions for approximate naked-eye use of co-ordinates; and, indeed, throughout the treatise the examples combine practical relations with common-sense theory to an unusual degree. Professor Moulton does not eschew philosophy and psychology in the problems he considers. In his discussion of time he sets forth its psychological aspect quite fully.

The author gives an early chapter to constellations, and he combines with the description of each constellation the leading facts regarding proper motion, parallax, variability or spectroscopic binaries, which the group presents. For an understanding of these terms, so far in advance of the stage reached, the introductory outline provides a sufficient basis.

Another novel feature is the treatment of earth and planets in one group, rather than the ordinary separate treatment of the earth's motions followed by that of the planets. Although logical, it may be questioned whether this method would be equally clear and satisfactory for the average student. Further, in the case of the earth's motions, a larger familiarity with the actual phenomena of the sky is desirable, before the chapter is reached in which they are treated. The degree of ignorance which the average young person carries in regard to the rising and setting of the

celestial bodies and other fundamental phenomena is amazing. It has been my custom to ask a series of questions bearing upon these immediate facts of observation, before taking up the study with a class of beginners. I find a large measure, not only of ignorance, but of confused misapprehension of what is to be seen in the sky.

The historical view of the planetary theory is clearly and concisely given, with unnecessary details wisely omitted. The application of the heliocentric theory to the observed motions might well be more fully developed. Few students could pass from theory to observation without further guidance than the text offers. These problems of planetary position are more immediately connected with the early development of the science than any others, and they are within the firmer grasp of the average beginner than the dynamic questions to which Professor Moulton devotes more attention. He has before now given expression to his preference for dynamic relations in instruction as against those of position, but a longer training is necessary to clear thinking in these lines than the beginner usually possesses.

Professor Moulton describes the work of Kepler and Newton in an admirable manner—clear, concise, and interesting. The presentation of perturbations, always difficult to lay before a class with clearness and brevity, is extremely well put, with due precautions regarding ultimate conclusions. In the chapters on the solar system there is much to commend. Here, as elsewhere, the author gives very simple and illuminating illustrations, as, for instance, of the parallax of the sun, the uncertainty in the determination of comets' orbits, the theory of light production and absorption. Perhaps undue time is afforded to the zodiacal light and the gegenschein, to the curtailment, for instance, of methods of finding the solar parallax. The chapter on comets and meteors closes with a good statement of the investigations now needed to establish a better knowledge of these bodies and their relations.

The placing of the chapter on the constitution of the sun is another unusual feature. It is postponed to the close of the section devoted to the solar system, standing between it and the consideration of the stellar system. This is well chosen, in view of the great prominence now given to the sun as a star. The brief description of the spectroheliograph, with a clear and simple figure, contained in this chapter, will be welcomed by teachers of astrophysics.

Chapter xv, devoted to theories of evolution, is full of matter. The value of a theory in itself as a stimulus to investigation is dwelt upon. Darwin's tidal theory is fairly criticized. The planetesimal theory of

Chamberlin and Moulton—called in the text the spiral nebula theory, to distinguish it from the Laplacian theory—is given in considerable detail. If the theory holds its ground in the future, the space given to it in the treatise will not prove excessive. But if it fails to make a good showing, later editions will curtail its limits. It may be questioned whether in a textbook for beginners so large a share of attention should be given to a theory which has not yet undergone general discussion by the world of astronomers. But its deep interest and its rich suggestiveness lead one to overlook the propriety of its setting.

The illustrations are generally good. Some of the figures might, to advantage, be somewhat larger. The fine photographs of stars and nebulae obtained in recent years at our leading observatories are well reproduced. The star maps, bound with the text, will prove convenient for class use.

MARY W. WHITNEY

Sternverzeichnis enthaltend alle Sterne bis zur 6.5ten Grösse für das Jahr 1900. Bearbeitet auf Grund der genauen Kataloge und zusammengestellt von J. und R. AMBRONN. Mit einen erläuternden Vorwort versehen und herausgegeben von DR. L. AMBRONN. Berlin: Julius Springer, 1907. Pp. 183. M. 10; interleaved for notes, M. 12.

This compact catalogue, containing in 158 pages the positions of 7796 stars of magnitude 6.5 and brighter, cannot fail to be of the greatest service to astronomers generally. The arrangement of the work is particularly to be commended, and makes its use rapid and very satisfactory. In addition to a current number, there is given the name of constellation, with Bayer's letter and Flamsteed's number (if any); the magnitude (for northern stars from the Potsdam photometric Durchmusterung; for the southern stars from the "Uranometria" of Pritchard and the Harvard photometries); the right ascension to tenths of seconds of time, and the declination to seconds of arc, with annual variations; the reference to the catalogue from which the position is taken, and the *B. D.* number; and finally a column of remarks. Foot-notes refer to stars interesting by reason of duplicity or variability, and give the individual names of the brightest stars. The letter *c*, conspicuously placed in the A. R. column, indicates that the proper motion is given in the list of *Eigenbewegungen* occupying 18 pages at the end of the catalogue.

The precision of the places is entirely adequate for all spectroscopic and photometric purposes, and the reference to the original catalogue makes a greater precision immediately available where necessary.

Professor Ambronn is to be congratulated in having in the persons of his wife and son able coadjutors, who performed the great part of the reductions and prepared the manuscript for publication.

The book will be needed in every observatory and by all field astronomers and there will be few teachers of astronomy who could not use it at times to great advantage.

E. B. F.

THE

ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXV — APRIL 1907 — NUMBER 3

THE FLUORESCENT AND ABSORPTION SPECTRA OF ANTHRACENE AND PHENANTHRENE VAPORS

By T. SIDNEY ELSTON

I. INTRODUCTION

The fluorescence of vapors was first observed by Lommel[1] for iodine vapor in 1883. A few years later Ramsay and Young[2] and E. Wiedemann[3] found that some liquid solutions, when raised above their critical temperatures, fluoresce. Later, Wiedemann and Schmidt[4] discovered a long list of fluorescent organic vapors, among which were anthracene and phenanthrene. They found that these vapors under the action of sunlight or light from a carbon arc fluoresced an intense blue, the fluorescent light lying, as a whole, on the red side of the region of maximum absorption. In 1896 they[5] discovered also that the metallic vapors of sodium and potassium are fluorescent. Quite recently Professor R. W. Wood[6] has made an extensive study of the fluorescence of sodium vapor. Hartley[7] has shown that mercury vapor also is fluorescent under certain conditions.

With the exception of sodium vapor, none of these vapors had been investigated extensively for its fluorescence; and yet it seemed

[1] *Wied. Ann.*, **19**, 356, 1883.

[2] *Chem. News*, **54**, 203, 1886.

[3] *Wied. Ann.*, **41**, 209, 1890.

[4] *Ibid.*, **56**, 18, 1895.

[5] *Ibid.*, **57**, 447, 1896.

[6] *Phil. Mag.*, **10**, 513, 1905.

[7] *Proc. R. S.*, **76** A, 428, 1905.

desirable that such work should be done, for the sake of the aid which it might afford for the solution of the problem of luminescence, and ultimately of the problem of the nature of matter. In the present investigation the primary object was to study in more detail the fluorescence of anthracene vapor, one of the very strongly fluorescent organic vapors. Later a similar study was made of phenanthrene because of the close resemblance between its fluorescent spectrum and that of anthracene, its isomer.

II. THE FLUORESCENT SPECTRUM OF ANTHRACENE VAPOR

a) *Method and apparatus.*—The fluorescence of the anthracene vapor was first studied by means of a small quartz spectrograph. The pure anthracene used was obtained from the commercial product by distilling twice from an excess of caustic potash, and then crystallizing from benzene under the action of the sunlight. A few centigrams of the crystals were inclosed in a crown-glass bulb (5 to 8 cm in diameter), which was then evacuated and sealed.

The bulb was placed in a heating apparatus arranged as shown in Fig. 1. It was hung in a wire cage between the two side tubes, T_1 and T_2, in such a manner that the light employed to excite fluorescence, after passage through the condensing lens, L, and reflection from the mirror, M, downward through the mica window, W, was focused at the center of the bulb.

The tube, T_1, contained a rectangular opening at its inner end and a glass window near its outer end. When the bulb was heated and the anthracene made to fluoresce, the fluorescent light was examined by the spectrograph through this tube.

The larger tube, T_2, provided with a cap over its outer end, served a double rôle: first, as an observing tube for adjusting the apparatus so as properly to focus the light in the bulb; and, secondly, as a dark background against which to observe the fluorescence through the other tube, T_1.

Care was always taken to adjust the bulb so that the cone of exciting light should be reflected from the bottom of the bulb straight back along its path. This was done in order to get rid, as far as possible, of the light scattered by reflection from the sides of the bulb. It was found, when bulbs of the proper size and shape were adjusted

in this way, that an exposure of three hours, made for scattered light alone, gave a very faint photograph as compared with that of the fluorescent and scattered light combined.

Various sources of violet and ultra-violet light were tried—viz., the sun, the carbon and zinc arcs in air, the mercury, silver, and lead arcs *in vacuo*, and the cadmium, aluminium, copper, zinc, and magnesium sparks. Of these the carbon arc produced the most

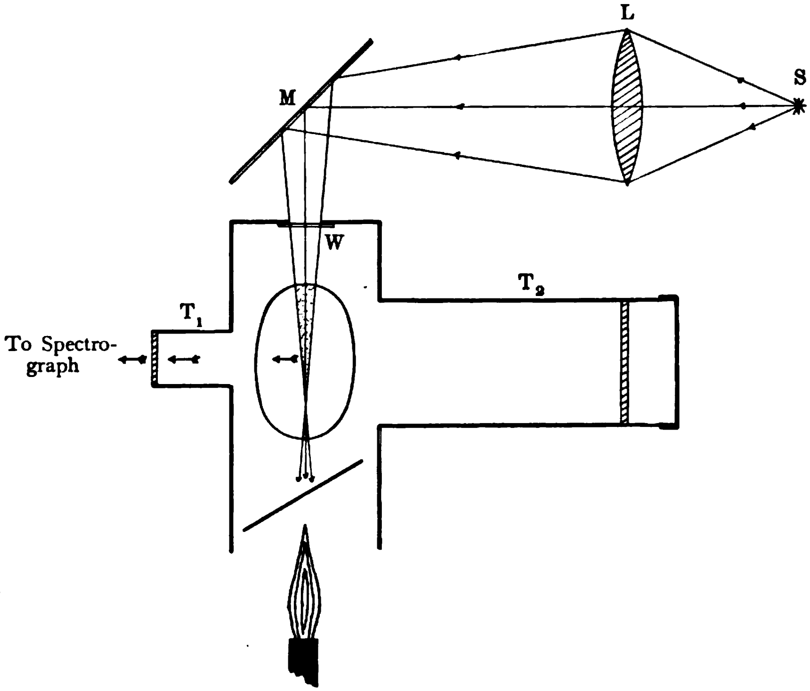


FIG. 1

intense fluorescence in the anthracene vapor, and so was employed almost exclusively. It was found that the fluorescence was due more to the light from the incandescent gases between the carbons than to the incandescent carbons themselves. Its intensity increased or decreased with the intensity of the group of three cyanogen bands in the violet.

b) The fluorescence.—When the bulb was heated until the anthracene began to vaporize, the path of the arc-light through it was

marked by a cone of illuminated particles which scattered partially polarized white light in all directions. Gradually this disappeared as the anthracene became completely vaporized, and was replaced by a cone of brilliant blue fluorescent light in which no polarization could be detected. Examined photographically with the quartz spectrograph, it gave a spectrum (Plate X, 1*a*) which extended continuously from 365 $\mu\mu$ to 470 $\mu\mu$, with its regions of maximum intensity located at 390, 415, and 432 $\mu\mu$. The same spectrum was also obtained when sunlight, or the light from any of the other sources was employed as the exciting light.

The range of the apparatus was from λ 325 to 540 $\mu\mu$, the limit in the ultra-violet being set by the crown glass of the bulb which is opaque to radiation of wave-length shorter than 325 $\mu\mu$.

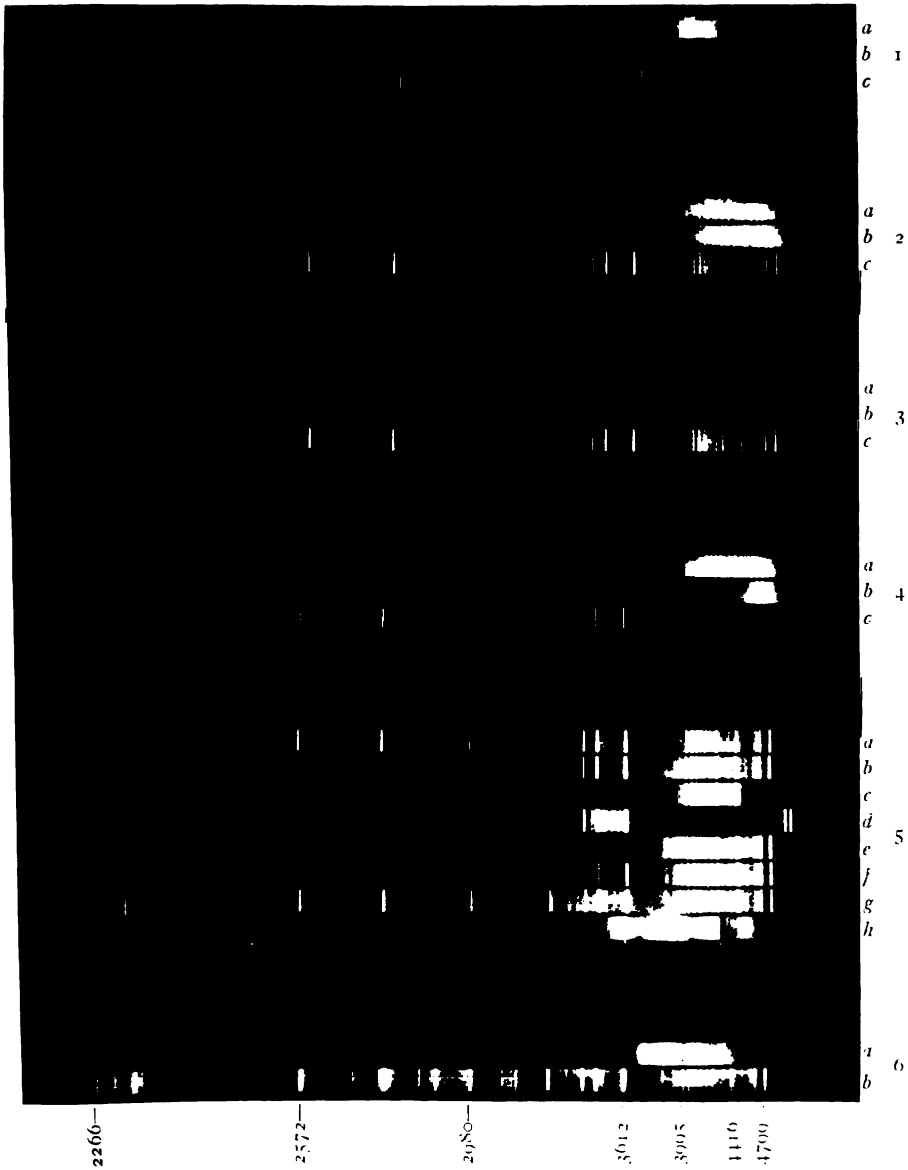
The photograph did not show the slightest evidence of lines in the fluorescent spectrum. This, it was thought, might be due to the small resolving power of the quartz spectrograph in the region of the fluorescence; so a photograph was taken with a special, three-crown-prism spectrograph, designed by Professor Wood for his work on sodium vapor. The resolving power of this instrument was about fifteen times that of the quartz spectrograph; but in this case also the photograph failed to show any lines in the spectrum. The fluorescence was not intense enough to warrant the use of instruments of higher resolving power. It seems highly probable, however, that the fluorescent spectrum of anthracene vapor is not resolvable into lines like that of sodium vapor, but resembles, instead, the banded spectra of fluorescent liquids and solutions.

III. THE EFFECT OF INCLOSING A FOREIGN GAS WITH THE ANTHRACENE

To determine what effect upon the fluorescence the presence of another gas would have, the anthracene was inclosed in the bulb in turn with various gases at atmospheric pressure. It was found that some gases had practically no effect, while others extinguished the fluorescence more or less completely. The results are given in the accompanying table.

In the case of the gases listed in the first column of the table the quality of the fluorescence was unaffected by their introduction into

PLATE X



FLUORESCENT AND ABSORPTION SPECTRA OF ANTHRACENE AND PHENANTHRENE VAPORS

1. *a*, Fluorescence of Anthracene; *b*, Diffused Light; *c*, Cadmium Spark.
2. *a*, Nernst Filament; *b*, Absorption of Anthracene; *c*, Cadmium Spark.
3. *a*, Nernst Filament; *b*, Absorption of Anthracene (dense vapor); *c*, Cadmium Spark.
4. *a*. Nernst Filament; *b*, Absorption. Commercial Anthracene; *c*, Cadmium Spark.
5. *a*, *g*, Cadmium Spark; *h*, Carbon Arc Spectrum; *d*, Solution A as screen; *c*. Solution B; *e*, Solution C; *b*, Crown-Glass Plate as screen.
6. *a*, Fluorescence of Phenanthrene; *b*, Cadmium Spark.

FLUORESCENCE OF ANTHRACENE VAPOR

Not Affected by	Extinguished by	Weakened by
Nitrogen Hydrogen Illuminating gas Carbon monoxide Carbon dioxide Mercury vapor	Cyanogen Chlorine Sulphur dioxide Oxygen	Air

the bulb. A slight decrease in intensity, however, was observed, due, probably, to the higher pressures in the bulb after the gases were introduced (see § IV). So far as could be ascertained, none of these gases reacts chemically upon the anthracene vapor within the range of temperatures set by the apparatus.

In the case of the gases listed in the second column the effect was practically to extinguish the fluorescence. Only the faintest trace, if any, of fluorescence could be observed when they were mixed with the anthracene. For these gases it was found that chemical action began either as soon as the temperature was raised to a point where the anthracene was vaporized (this was the case with cyanogen and chlorine), or at a temperature not much higher (this was the case with sulphur dioxide and oxgyen). It seemed highly probable from this that the extinction of the fluorescence was due to some sort of chemical influence.

That the chemical influence, if such it be, which affected the fluorescence, need not be such as to produce a permanent chemical change in the anthracene was shown in the following way: A bulb containing anthracene and oxygen was placed in an air-bath with a similar bulb containing only anthracene. The temperature was then raised just high enough to obtain strong fluorescence in the second bulb, but not high enough to cause any visible chemical reaction between the oxygen and the anthracene in the first bulb. The first bulb was tested for fluorescence and found not to show the slightest traces. It was then removed and the oxygen pumped out, the crystallized anthracene being left behind, and the bulb again placed in the bath and tested. The fluorescence in this bulb was now found to be as brilliant as in the other one. To find out if some of the anthracene had not been acted upon and a consequent change in

volume of the oxygen produced, a bulb was used which terminated in a long capillary tube. The oxygen gas and anthracene crystals were inclosed at atmospheric pressure and the bulb sealed. Then, after subjecting the bulb to the same heating as before, the end of the capillary tube was broken off under mercury, and the oxgyen again brought to atmospheric pressure. Not the slightest change in its volume was observed. If any chemical action had occurred, it was evidently of such a nature as to produce compensating changes in the volume of the reacting substances. As this is highly improbable, we are led to conclude that no reaction took place between the anthracene and the oxygen, but that whatever mutual action occurred produced effects which lasted only while the given conditions of temperature and mutual contact were maintained. During this time practically all of the anthracene molecules must have been affected, for the extinction of the fluorescence was complete throughout the vapor.

The foregoing facts may be explained by assuming that, when anthracene vapor is mixed with oxygen at a temperature not much below that at which the two react chemically, there is a preliminary grouping of the oxygen molecules about the anthracene molecules, or vice versa, and that, while thus associated and before they rush together into closer contact to form a new chemical compound, the mutual forces are such as to prevent the anthracene molecules from fluorescing, but are not great enough to bring about a reaction. As thus considered, the phenomenon is simply the first stage of a chemical reaction.

The effect of oxygen upon the anthracene vapor is in many ways similar to that of many liquid solvents upon the fluorescent substances dissolved in them.[1]

When the vapor was mixed with air, the fluorescence was much weakened, becoming almost too weak to be seen when air at atmospheric pressure or greater was used. This weakening was easily traceable to the oxygen contained in the air.

It seems highly probable from the foregoing facts that the presence of foreign molecules, heavy or light, among the fluorescing anthracene molecules does not have any appreciable effect upon the

[1] Kauffmann and Beiswenger, *Zeitschrift fur phys. Chem.*, 50, 350, 1904.

fluorescence at ordinary pressures, except in those cases where a chemical change, either incipient or permanent, occurs.

IV. THE EFFECT OF PRESSURE

In the experiment with the illuminating gas a thick-walled glass bulb was employed, and the pressure was varied from 1 to 12 atmospheres. The visible fluorescence became less and less intense as the pressure was increased, but was still perceptible at the highest pressure. So far as this was tried with the other gases, the same results were obtained, an increase of pressure in each case causing a diminution in the intensity of the fluorescence. The change in intensity was not very marked until the pressure exceeded 1 atmosphere. At all pressures below this the intensity of the fluorescence was practically independent of the pressure.

The quality of the fluorescence was not affected by the pressure; the same spectrum was obtained at high as at low pressures.

V. THE EFFECT OF TEMPERATURE

The temperature of the bulb was varied from the temperature at which the vapor began to fluoresce (the boiling-point of anthracene is 351° C.) to the temperature at which the glass began to soften (the melting-point of the glass bulb is about 1000° C.). As the temperature was raised the intensity of the fluorescence diminished. This was the case whether the bulb contained the pure anthracene alone or anthracene mixed with an inert gas. Of course, as the temperature was raised, the pressure of the inclosed vapor was increased; so the effect may have been due to this.

VI. THE EFFECT OF VARYING THE DENSITY OF THE VAPOR

When the amount of anthracene inclosed in the bulb was gradually increased, and the vapor density thus correspondingly increased, it was found that the cone of visible fluorescence, which at first extended completely through the bulb from the point where the exciting light entered to the point where it left, was foreshortened in the direction of its length, until finally it was shrunk down almost to the surface where the exciting light entered the bulb. At the same time the intrinsic brightness of the fluorescence was diminished, due, undoubtedly, to the consequent increase in pressure.

The foreshortening of the cone of fluorescence means, of course, that the particular radiation which excites the visible fluorescence is completely absorbed out of the incident light before this penetrates very far into a bulb filled with dense anthracene vapor.

The quality of the fluorescence was not affected, as the spectrum photographs for bulbs containing different amounts of anthracene showed.

VII. THE ABSORPTION SPECTRUM

To obtain the absorption spectrum of the vapor, it was found convenient to use a long glass tube in which to inclose the anthracene. This was placed inside a cylindrical iron tube with a glass window at each end, as shown in Fig. 2.

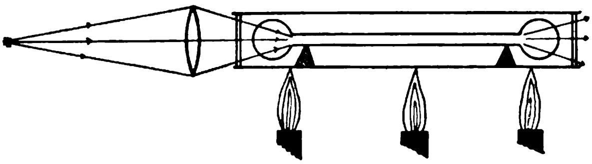


FIG. 2

The light from a Nernst filament was focused in the tube at one end, and, on emerging at the other end, was examined photographically with the quartz spectrograph.

The photograph (Plate X, *3b*) was taken when the anthracene vapor in the tube was quite dense. It shows that the pure dense vapor absorbs continuously from λ 425 to 325 $\mu\mu$, the limit set by the crown glass. When the tubes containing less anthracene, or the bulbs employed in getting fluorescence, were used, the absorption did not extend so far down toward longer wave-lengths (Plate X, *2b*).

The absorption of the vapor was also studied with a 12.5-foot concave Rowland grating, but with the same result. There was no evidence of lines in either case; the absorption consisted of a continuous region between λ 325 $\mu\mu$ and about 425 $\mu\mu$.

Purified anthracene was used in obtaining the absorption spectra just described. When the commercial anthracene (Eimer & Amend, 90 per cent.) was used, the absorption spectrum obtained (Plate X,

4*b*) was found to be the same as for the pure anthracene, with the exception that there was a narrow band at wave-length 452 $\mu\mu$.

VIII. THE RELATION BETWEEN THE EXCITING LIGHT AND THE FLUORESCENCE—STOKES'S LAW

1. *The carbon arc as source.*—In the earlier stages of the investigation (when the fluorescent spectrum was excited by the light from a carbon-arc lamp and the photograph taken showed that it consisted of three bands which were somewhat similarly spaced, but farther toward the red, as compared with the three intense cyanogen bands of the arc in that region of the spectrum), it was thought probable that the three cyanogen bands had each excited its own fluorescent band in the anthracene vapor, thus giving rise to the three bands found in the fluorescent spectrum. The fact, that sunlight excited the same bands seemed to make this doubtful; but it was not until the work, now to be described, was done that it was shown that, in all probability, the entire fluorescence of the vapor excited by light from the carbon arc was largely due to the strong lines in the head of the middle one of the three cyanogen bands, the one whose head lies at wave-length 388.3 $\mu\mu$.

Two methods were employed in determining the approximate wave-length of the exciting light.

a) The first to be tried was what might be called the "method of exclusion." It consisted in screening off different portions of the spectrum in turn, by means of properly chosen absorbing solutions, and observing the effect upon the fluorescence. By referring to Dr. Uhler's photographs of the absorption spectra of various solutions, the following solutions were chosen and found to answer the purpose:

A. Nitroso-dimethyl-aniline solution, 2 parts by volume of saturated solution to 3 parts of water.

B. Potassium permanganate solution, 1 part by volume of saturated solution to 50 parts of water.

C. Naphthol disulfonic acid solution, 2 parts by volume of saturated solution to 5 parts of water.

These solutions were introduced into a glass cell (1 cm thick) which was placed in the path of the exciting light between the condensing lens (see Fig. 1) and the bulb.

When solution A was employed as an absorbing screen, the anthracene vapor failed to fluoresce. Now, this solution has a strong absorption band extending from 375 to 480 $\mu\mu$ (see Plate X, 5*d*), while it transmits quite freely on both sides of the band, within the limits set by the glass. It is evident from this that the light which causes the fluorescence in the vapor is located somewhere in the region 375–480 $\mu\mu$. It is in this region that two of the cyanogen bands are located; the third lies just outside of it toward the ultra-violet.

The solution B is transparent to light of wave-length 395–465 $\mu\mu$, but is quite opaque to the rest of the spectrum in the given region. When this solution was substituted in the cell for solution A, no fluorescence could be observed. This shows that the light producing the fluorescence does not lie between 395 and 465 $\mu\mu$, and therefore must lie either in the region 465–480 $\mu\mu$, or in the region 375–395 $\mu\mu$, or in both. That it does not lie in the region 465–480 $\mu\mu$ is probable from the fact that the carbon arc spectrum is relatively weak in this region. We are then led to conclude that the exciting light lies somewhere between the wave-lengths 375 and 395 $\mu\mu$. It is in this region that the strong lines in the middle one of the three cyanogen bands are located.

b) A confirmation of the foregoing results was obtained in the following way: A slit, two condensing lenses, and a large crown-glass prism were arranged so as to give a comparatively pure and bright spectrum of the arc. A slitted screen, with the bulb containing the anthracene vapor behind it, was then passed from point to point along through the spectrum. Starting at the red end there was no visible evidence of fluorescence in the vapor until the bulb was brought into that part of the spectrum corresponding to wave-lengths 390–375 $\mu\mu$, when it began to fluoresce with the characteristic brilliant blue color, the cone of fluorescence extending entirely through the bulb. When moved on into the region of the cyanogen band whose head is at 359 $\mu\mu$, a dim fluorescence was observed which penetrated only a few millimeters into the bulb. It had played small part, therefore, in the photographs of the fluorescent spectrum previously taken, where the collimator of the spectrograph had been directed toward the center of the bulb.

The slitted screen was now adjusted so as to allow only the light

near the head of the cyanogen band at 388.3 $\mu\mu$ to enter the bulb. A photograph of the fluorescence produced was taken which gave the same bright bands and continuous regions shown in the former photographs (see Plate X, 1*a*).

The slit was then moved so as to admit to the bulb the head of the cyanogen band at 359 $\mu\mu$, and a photograph of the fluorescence was taken. It showed that the spectrum was the same as that excited by the other cyanogen band.

The relative spectral positions of the fluorescence and the exciting lights for the two experiments just described are shown in Fig. 3. The block E_1 represents the spectral width and location of the exciting light in the first experiment, and F_1 the general form of distribution of intensity in the fluorescence which it excited in the anthracene vapor, while E_2 and F_2 represent the corresponding quantities in the second experiment. The two curves have been deduced from the photographs and indicate that the fluorescence in the two cases was the same in every way except in absolute intensity.

2. *The zinc arc and magnesium spark as sources.*—To obtain further evidence of the relation between the fluorescence and the wave-length of the light which excites it, two sources were employed which gave regions of exciting light quite widely separated. The first was the zinc arc, which gives an isolated group of very intense lines in the region 328–335 $\mu\mu$, just within the ultra-violet limit set by the absorption of the crown-glass bulb; the second was the magnesium spark, which gives an intense isolated triplet in the region 383–385 $\mu\mu$, not very far from the limit toward the red beyond which the light ceases to have power to excite fluorescence in the anthracene vapor. This limit was earlier found to be in the neighborhood of wave-length 400 $\mu\mu$.

Between 325 and 400 $\mu\mu$ the arc-spectrum of zinc contains, besides the group just mentioned, only three very faint lines whose aggregate intensity is less than 1 per cent. of that of the group. If the zinc arc is used, therefore, to excite the fluorescence in the anthracene, the exciting light is practically all located in the region 328–335 $\mu\mu$. When this was done (the zinc arc being substituted for the carbon arc) and the fluorescence produced in the bulb was photographed, it was found to give the same fluorescent spectrum as the carbon

arc. The block E_3 and the curve F_3 in Fig. 3 represent respectively the exciting light and the fluorescence for this case.

In the case of the magnesium spark there are a few faint lines in the region 325–400 $\mu\mu$ besides the triplet mentioned, but their aggre-

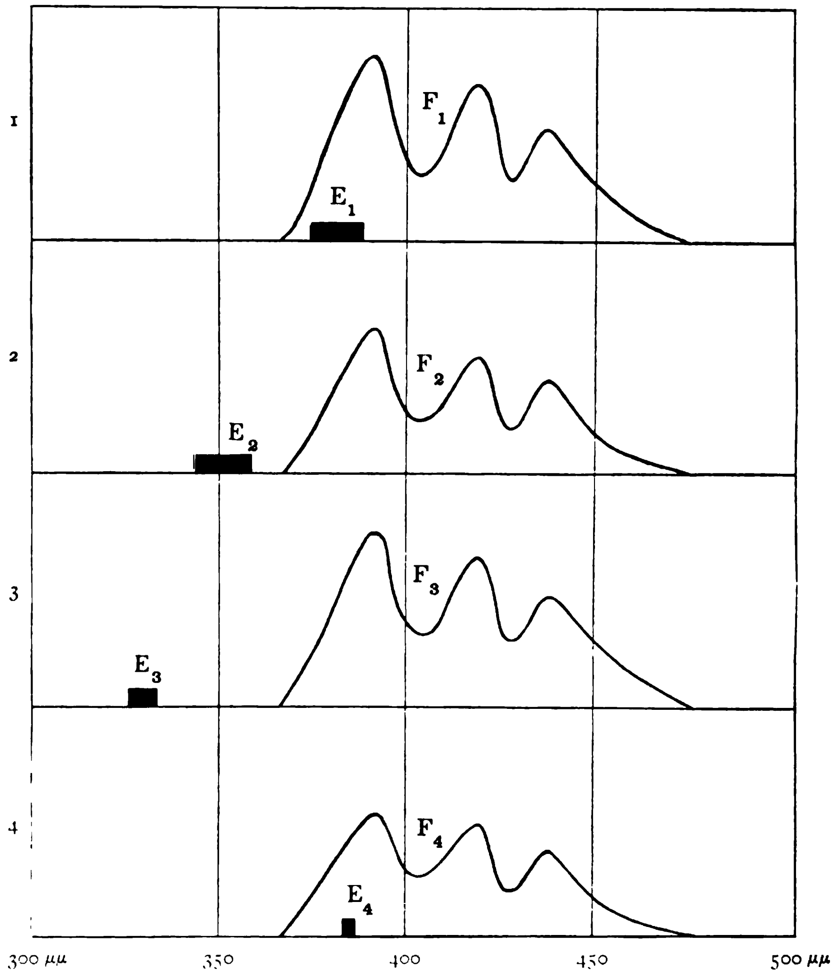


FIG. 3

gate intensity is negligible in comparison with that of the triplet, so that practically all of the exciting light from a pure magnesium spark is in the region 383–385 $\mu\mu$. It was found, however, that the elec-

trodes used contained zinc as an impurity. In order to screen off the lines due to the zinc, the solution C (see § VIII, 1) was used as an absorbing screen. It is opaque to ultra-violet light from 360 on out beyond 325 $\mu\mu$, and therefore to the group of zinc lines at 328–335 $\mu\mu$. When the magnesium spark, as thus screened, was used as the source and the fluorescence excited was photographed, it was found to give the same spectrum as the carbon and zinc arcs. The block E_4 and the curve F_4 in Fig. 3 represent respectively the exciting light and the fluorescence for this case.

In connection with the results given above, it should be remembered that the anthracene vapor used absorbs continuously from about 400 $\mu\mu$ to some point beyond 325 $\mu\mu$.

3. *Stokes's law.*—It is evident from the foregoing facts that Stokes's law, which states that the fluorescent light is of longer wave-length than the light which excites it, is not strictly true in the case of anthracene vapor; for in at least two cases (1 and 3 in Fig. 3) part of the fluorescent light is of shorter wave-length than the corresponding exciting light. In a general sense, however, the law may be considered as fairly representing the facts, since the fluorescent spectrum as a *whole* is of longer wave-length than the exciting light.

4. *Discussion of results.*—We are led to conclude from the foregoing facts (*a*) that the fluorescence of anthracene vapor is excited by light situated anywhere in the ultra-violet region of absorption of the vapor, and (*b*) that the character of the fluorescence is entirely independent of the source and wave-length of the light which excites it.

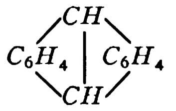
If we assume that the fluorescence is produced by a system of electrons within the molecule, then, in order to account for the fluorescent spectrum of anthracene vapor, which, as we found, was composed of three bands superposed upon a continuous spectrum, we may consider either that the electrons corresponding in period to the three bands are more numerous than those which give rise to the weaker, continuous parts of the fluorescence, or that the former are set in more violent vibration; also, that the system of electrons is so intimately connected in its parts that, when disturbed by the exciting light in any manner, be it direct or through an intermediary "luminophore," *all* the electrons in the system are set in vibration. If the disturbance of the system takes place through an intermediary

"luminophore,"[1] as seems more probable, then this luminophore undoubtedly consists of a connected system of electrons whose periods correspond to those of the absorption spectrum of the vapor.

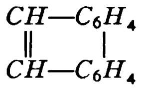
IX. THE FLUORESCENCE AND ABSORPTION OF PHENANTHRENE VAPOR, THE ISOMER OF ANTHRACENE VAPOR

Phenanthrene is a substance which has the same chemical composition, $C_{14}H_{10}$, as anthracene, but a slightly different structural composition as the following figure shows:

Anthracene



Phenanthrene



The only difference in structure, according to this, is in the manner in which the four groups or radicals are linked together.

When the fluorescent spectrum of pure phenanthrene vapor was photographed (Plate X, 6*a*), it was found to consist of the same bands as that of anthracene, but with an additional band at 360 $\mu\mu$.

Its absorption spectrum was photographed and found to be the same as that of the anthracene vapor.

It would seem from these facts that there is an intimate connection between the fluorescence of the vapors of the two substances, undoubtedly due to their common chemical composition and similar structural composition. Just what gives rise to the extra band in the fluorescent spectrum of phenanthrene vapor is not apparent.

X. SUMMARY

The following is a summary of the results of this investigation:

(1) The fluorescent spectrum of anthracene vapor consists of three bright bands at λ 390, 415, and 432 $\mu\mu$ superposed upon a continuous region extending from 365 to 470 $\mu\mu$. There is no evidence of lines. The absorption spectrum extends continuously from about 400 $\mu\mu$ to some point beyond 325 $\mu\mu$.

(2) The presence of inert gases in the vapor does not affect the fluorescence, so long as the pressure is below an atmosphere. But

[1] W. Kauffmann, *Ber. d. Phys. Gesell.*, 21, 375, 1905.

such gases as oxygen, chlorine, and sulphur dioxide, which at high temperatures react chemically with anthracene, almost completely extinguish the fluorescence at ordinary pressures.

(3) The intensity of the visible fluorescence decreases as the pressure of the gas inclosed with the anthracene is increased. It is not marked, however, at pressures lower than an atmosphere. The quality of the fluorescence is not affected by a change in pressure.

(4) Increasing the density of the fluorescing vapor has no effect upon the quality of the fluorescence, but diminishes its intensity slightly.

(5) The fluorescence of anthracene vapor may be excited by l'ght located anywhere within the ultra-violet region of absorption of the vapor.

(6) The same fluorescent spectrum is produced independent of what the source or wave-length of the exciting light may be, provided the condition noted in (5) is observed.

(7) There is an intimate connection between the fluorescence of anthracene vapor and its isomer, phenanthrene vapor.

This investigation was begun at the suggestion and has been carried out under the direction of Professor R. W. Wood, to whom I wish to extend my thanks for the interest which he has shown, and the encouragement and help which he has given me in the course of the work. I wish also to express my gratitude to Professor J. S. Ames for his unfailing courtesy and kindness.

JOHNS HOPKINS UNIVERSITY

ON THE RADIATION OF CANAL RAYS IN HYDROGEN

PART II. LINEAR SPEED AND INTENSITY OF RADIATION[1]

By J. STARK

§ 11. *Ionization behind the cathode; origin of the banded spectrum and of the stationary line-spectrum.*—Referring to Fig. 8, K is a cathode made of wire net having one mesh to the millimeter; in that portion of the tube which lies behind the cathode and which is traversed by the canal rays are introduced two pointed electrodes; with these are connected in series a four-volt battery and a galvanometer. The cathode is earthed. So long as no canal rays pass between the wire electrodes, the galvanometer shows no deflection; when they begin to pass, however, the galvanometer indicates a current and hence conductivity in the gas.

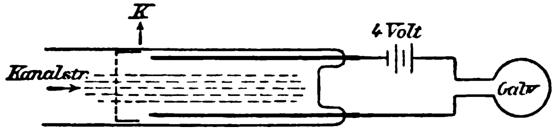


Fig. 8

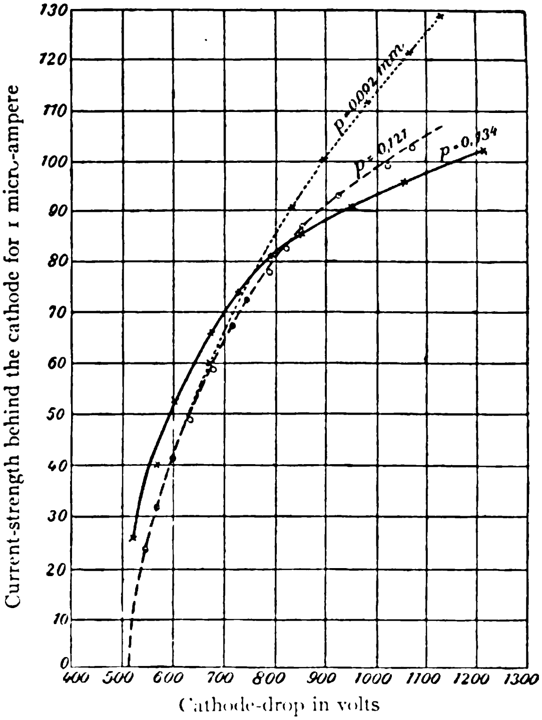


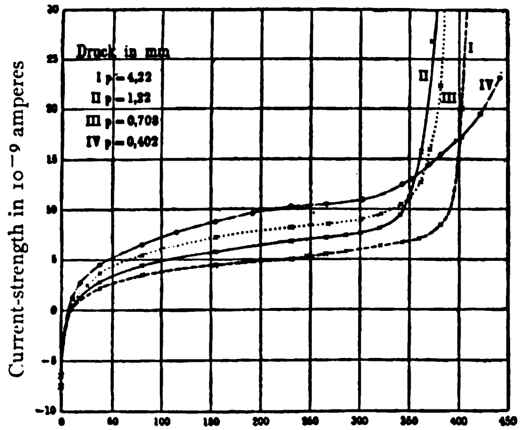
Fig. 9

In Fig. 9 are represented graphically three series of measurements, at three different pressures, in nitrogen. The cathode-

[1] Continued from page 44, January 1907.

drop in volts is plotted along the axis of abscissae; while the ordinates indicate the ratio of the current passing through the galvanometer to the current in the tube (*Glimmstrom*) which produces the canal rays.

It may be seen from the figure that, in the gas behind the cathode, the canal rays produce conductivity, and hence positive and negative ions; from 500 volts up the conductivity in the rear of the cathode rises at first rapidly, then more slowly, with the cathode-drop.



Difference of potential between cathode and negative end of thread

FIG. 10

Fig. 10 has already been published in another place.[1] It was obtained as follows. A copper wire was placed opposite to the incandescent filament of a very small electric lamp, at a distance of 6 mm; this copper wire served as cathode, the incandescent carbon filament as anode. On this there will be produced some positive ions in consequence of the high temperature; and these will be driven to the cathode by the potential difference, ΔV, between the electrodes; when the gas-pressure is low they will traverse this drop in potential freely—i. e., without collision—and will strike the surface of the cathode, with high speed, as "canal rays." It is evident that for small potential differences the current will be small and nearly "saturated." At the velocity corresponding to about 340 volts, however, the positive ions, by the suddenness of their impact upon the cathode, begin to create new ions; and hence the current suddenly rises to large values.

It would thus appear that canal rays when moving with a speed

[1] J. Stark, *Verhandl. d. deutsch. phys. Ges.*, 6, 104, 1904.

less than a certain limit (300 to 500 volts) are unable to ionize a gas by collision; with higher velocities, however, the gas is ionized. In the spectroscopic observations here described the cathode-drop always exceeded 1000 volts, so that the canal rays were capable of ionizing the gas behind the cathode, thus giving opportunity for the reunion of positive and negative ions, and hence, in terms of our hypothesis, for the emission of a banded spectrum. This radiation, then, has its origin in the potential energy which is set free by the reunion; it is an indirect consequence of the collision of the canal rays with the neutral atoms of the gas; the more frequent these collisions—i. e., the greater the gas-pressure—the more intense is the banded spectrum in the region traversed by the canal rays.

It has already been established, in §3, that the line-spectrum of this same region has both a displaced and a stationary intensity; the intensity of the line-spectrum in the negative glow is, at low temperatures, only stationary; no Doppler effect is here observable. In the region of the canal rays the stationary lines present the same appearance as the corresponding lines in the negative glow; and here the intensities of the series lines also diminish from red to violet in the same manner. In the negative glow the neutral atoms are ionized by collisions with cathode rays; behind the cathode they are ionized by collisions with canal rays. In the former case the newly formed positive ion always remains at rest because the mass of the incident cathode ray is so small; in the latter case either the ion just produced or the ion of the incident canal ray may remain at rest after the collision. Both cases have the feature in common that immediately after collision there is present a stationary positive ion. It is possible that this ion will be deformed by the collision, and will also have its internal energy increased—energy which it radiates immediately after the blow and while still at rest. We are thus led to infer that the stationary intensity of the line-spectrum has its origin in the deformation of a positive atom by collision. We may, therefore, infer that the stationary intensity of the line-spectrum is proportional to the number of collisions, and hence also proportional to the intensity of the banded spectrum.

Tables IV and V were obtained in the following manner. On a number of photographic plates the intensity of a strong band-line was

TABLE IV

Source of Light	Pressure: Current		Intensity of Band-Line 4928.8	Stationary Intensity of Series-Line $H\beta$
Negative glow........	0.5 mm;	0.006 amp.	1	1
	0.1	0.01	1	5
	1.0	0.03	1	5
Positive column.......	40	0.05	1	3
	10	0.03	1	1
	0.1	0.003	1	1.5

TABLE V

Pressure. Cathode-Drop (Canal Rays)		Intensity of Band-Line 4634.15	Stat.-Intens. Series-Line $H\gamma$
0.5 mm;	2000 volts	1	3
0.05	2500	1	3
0.80	2500	1	4
*0.03	3000	1	3
*0.03	3000	1	3
0.05	4000	1	5
0.02	4200	1	3
0.01	4200	1	4
0.01	5000	1	3

compared with the stationary intensity of a neighboring series-line; the intensity of the band-line was called unity. The measures indicated by an asterisk in Table V refer to canal rays traveling away from the observer.

The relation here indicated between the intensity of the banded spectrum and the stationary portion of the series-line holds only for low temperatures; by raising the temperature one may, as shown below, create a displaced intensity which will mask the stationary intensity.

§ 12. *Correspondence between speed of translation and temperature; comparison of stationary and displaced intensities.*—From the existence of the Doppler effect in canal rays one may infer that the "carrier" of the line-spectrum is in motion while in the act of radiating light; and since the carrier, the positive ion, undergoes at the same instant both translation and radiation, one may suspect a connection between these two processes. The source of light in canal rays is evidently a diminution of the kinetic energy of these rays;

as we have already seen, § 5, the maximum speed of the canal rays is less than that computed from the cathode-drop; not only so, but this velocity becomes smaller the farther the canal rays proceed from the cathode.

As one might expect, and as A. S. King[1] has shown by experiment, it is possible to obtain the line-spectrum of a metal by increase of temperature alone. According to the kinetic theory of gases, a rise of temperature means an increase in the speed of translation of the gas particles, and the temperature is proportional to the mean square of the speed.

A purely thermal emission, or temperature radiation, of the line-spectrum and its emission from canal rays have this feature in common, that the carrier of the line-spectrum, the positive ion, has a motion of translation through a material medium and is simultaneously emitting electromagnetic waves. Temperature in the first case appears as the analogue of the square of the velocity of the canal rays in the second case.

The emission of light by canal rays is not a thermal or "regular" radiation, but a case of luminescence.[2] The difference between these two special cases, of the general principle or feature just mentioned, lies in the distribution of the radiating particles among the various values and the directions of the velocities.

In the case of thermal radiation the motions of the carriers lie in all possible directions, while in canal rays they all lie in a single direction. In the former case the total number of carriers is distributed among all speeds, from zero to very high values, according to the Maxwell-Boltzmann law. In the latter case the carriers are distributed, according to another law, among speeds ranging from zero to the maximum given by the cathode-drop.

From this it would appear, therefore, that there is an analogy, but no identity, in the laws governing these two types of emission. The stationary radiation is also a sort of luminescence between which and the displaced radiation no parallelism is to be expected such as that which exists between this latter and thermal radiation.

This is evident from Tables VI, VII, and VIII.

[1] *Annalen der Physik*, **16**, 360, 1905.

[2] E. Wiedemann, *Annalen der Physik*, **34**, 446, 1888; **37**, 177, 1889.

TABLE VI

PRESSURE, CATHODE-DROP	Intensities: Band-Line 4205.2	Intensities: Series-Line 3970.2 ($H\epsilon$)
0.05 mm; 2500 volts	1	3
0.02 3300	1	5
0.01 4600	1	5
0.005 7500	1	10
0.005 7500	1	10

TABLE VII

PRESSURE, CATHODE-DROP	0.05 mm 2000 Volts		0.05 mm 2500 Volts		0.08 mm 2500 Volts		0.03 mm 3000 Volts		0.03 mm 3000 Volts		0.02 mm 4200 Volts		0.01 mm 4200 Volts		0.01 mm 5000 Volts	
Intensity	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.
$H\beta$	..	..	1	4	..	..	..	..	..	..	1	4	1	6	1	4
$H\gamma$	1	3	1	3	1	5	1	6	1	6	1	5	1	8	1	5
$H\delta$	1	2	1	2	1	3	1	4	1	5	1	4	1	6	1	3
$H\epsilon$	1	1	1	1.5	1	1.5	1	3	1	3	1	2	1	2	1	2

TABLE VIII

PRESSURE, CATHODE-DROP	2536.6 Intensity		3663.3 Intensity		4046.8 Intensity		4078.1 Intensity	
	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.	Stat.	Displ.
0.05–0.01 mm; 3700–11100 volts	..	..	10	1	50	1	great	0
0.05–0.005 mm; 6000–18000 volts	1	2	8	1	10	1	100	1
0.01–0.001 mm; 16000–48000 volts	1	3	3	1	6	1	20	1
0.005–0.0005 mm; 19000–57000 volts	..	..	2	1	5	1	4	1

The results of Table VI were obtained from grating spectrograms taken with the direction of observation perpendicular to that of the rays; the intensity here given for the series-lines is therefore the sum of the stationary and of the much greater displaced intensity. As will be seen, the displaced intensity of the series-lines is not proportional to the intensity of the banded spectrum, and hence not proportional to the stationary intensity of the series-lines.

Table VII was obtained from spectrograms on which the stationary and displaced intensities were separated by means of the Doppler effect, and could therefore be directly compared one with the other; for each separate line the stationary intensity is taken as unity. As will be seen, the distribution of intensity in the stationary spectrum follows a law different from that which governs the distribution in the displaced spectrum.

In Table VIII are the results from spectrograms of mercury vapor in which the stationary and displaced intensities were again separated by aid of the Doppler effect. Differing from hydrogen, the displaced intensity is here greater than the stationary only for the line $\lambda 2536$; in all other lines the reverse is true.

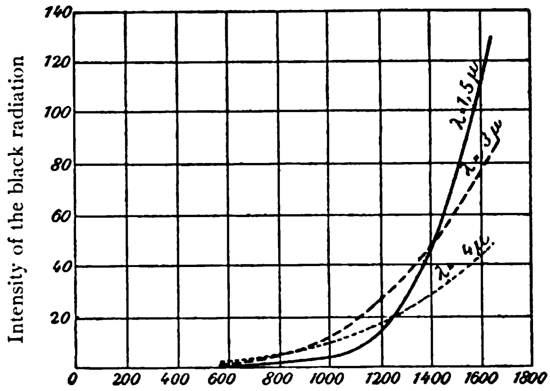


FIG. 11

A comparison of the three preceding tables shows that the displaced intensity does not behave like the stationary, becoming smaller as the number of collisions diminishes along with the gas-pressure. The displaced intensity appears rather to depend solely upon translational speed, with which it increases rapidly. This leads to the conjecture that the energy of the displaced intensity is transferred to the carrier of the radiation, not by collision, but from the kinetic energy of translation by means of radiation-pressure and during translation. The electron theory is confronted with the following question: Given a system of electrons in cyclic motion; suppose them so arranged that their total radiation is zero when the system is at rest with respect to the ether. Query: Will this neutral compensation between the radiations from the individual electrons still hold when the system has impressed upon it a translational velocity with respect to the ether?

§ 13. *Distribution of intensity, with Doppler effect.*—For the succeeding discussion let us make the following assumptions. The canal rays travel in parallel lines and experience no appreciable dispersion; the cathode-drop during exposure remains constant to within a few per cent. The distribution of the canal-ray ions among the different velocities is given by the broken curve in Figs. 12*a*–12*f*; squares of velocities are plotted as abscissae, and the number of canal-ray ions assigned to each velocity is given by the ordinates.

The intensity associated with any particular squared velocity (v^2) in the Doppler effect is proportional to the product of the number of particles (n) and the intensity of radiation (J) of the individual particle having the velocity in question, so that we have $E=n.J(v^2,\lambda)$. If J were independent of v^2—i. e., if $\delta J/\delta v^2=0$ then the distribution of intensity in the Doppler effect would be proportional to the velocity distribution ($E=kn$), since each of the n values would be multiplied by the same constant. We shall assume, therefore, that J is a function of v^2 in exactly the same manner that the black-body radiation (Fig. 11) is a function of the absolute temperature. The J,v^2-curve is that indicated by dotted lines in Figs. 12*a*–12*f*.

In Fig. 12*b* it is assumed that the intensity J increases with an increase of v^2 as indicated by the dotted curve. Since J is very small for small values of v^2, we find E also small, notwithstanding the fact that n is large. The displaced intensity falls off, therefore, with great rapidity in the region of small velocities; so that between the stationary and the appreciable, displaced intensity there lies a minimum of intensity.

In Figs. 12*c* and 12*d* it is assumed that, for two lines in the same series, the intensity J is a function of the wave-length λ; that, however, the intensities J_1 and J_2 vary in a constant ratio, giving us

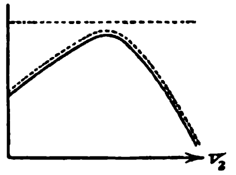
$$\frac{\delta J(\lambda_2)}{\delta v^2}:\frac{\delta J(\lambda_1)}{\delta v^2}=K\,.$$

In this case the total intensities in the Doppler effect, E_1 and E_2, would vary in the constant ratio K; the lines are similar; the widths of the intensity-minimum and the distances of the intensity-maximum from the stationary line are also equal.

In Figs 12*e* and 12*f* it is assumed that, for two lines of the same

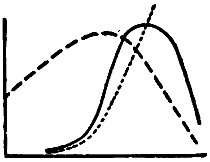
series, the intensity is a function of λ; that, however, the intensities do not vary in a constant ratio; accordingly

$$\frac{\delta J(\lambda_2)}{\delta v^2}=F(\lambda_1, \lambda_2, v^2)\frac{\delta J(\lambda_1)}{\delta v^2}.$$



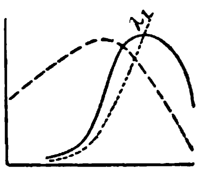
$$\frac{\partial J}{\partial v^2}=0$$

Fig. 12*a*



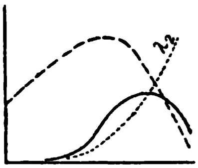
$$\frac{\partial J}{\partial v^2}>0$$

Fig 12*b*.



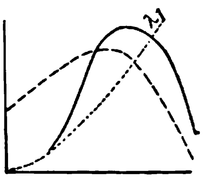
$$\frac{\partial J(\lambda_1)}{\partial v^2}=f(\lambda_1)$$

Fig. 12*c*



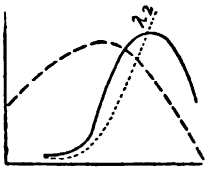
$$\frac{\partial J(\lambda_2)}{\partial v^2}=k\frac{\partial J(\lambda_1)}{\partial v^2}$$

Fig. 12*d*.



$$\frac{\partial J(\lambda_1)}{\partial v^2}=f(\lambda_1)$$

Fig. 12*e*



$$\frac{\partial J(\lambda_2)}{\partial v^2}=F(\lambda_1, \lambda_2, v^2)\frac{\partial J(\lambda_1)}{\partial v^2}$$

Fig. 12*f*

---- n = number of particles to v^2.
...... J = intensity for n = constant.
——— E = intensity in Doppler effect.

In this case the distribution of the total intensities, E_1 and E_2, are no longer similar. If we assume that $\lambda_1>\lambda_2$ and that

$$\frac{\delta J(\lambda_1)}{\delta v^2}<\frac{\delta J(\lambda_2)}{\delta v^2},$$

then the width of the intensity-minimum for λ_1 and the distance of the intensity-maximum from the stationary line are smaller than for λ_2. As the wave-length of the series-line diminishes, the width of the intensity-minimum, and likewise the distance of the intensity maximum, increase.

Before passing to experimental results concerning the distribution of intensity in the Doppler effect, the following sources of error should be distinctly recognized. It is not safe to compare with each other distributions of intensity in the Doppler effect for lines which do not have the same carrier. Nor is it admissible to compare with each other lines of the same series on different spectrograms; for it is impossible to produce two spectrograms under conditions which are exactly identical as regards cathode-drop, gas-pressure, and current-strength. The nv^2-curves of the two Doppler effects under comparison would not be identical in the two cases. The only reliable method is to compare the intensity-distribution of the Doppler effect for lines of the same carrier and upon the same spectrogram.

After this condition is satisfied, the problem is to compare distributions for lines of different intensities. Here errors are possible, owing to imperfections of the photographic method: the intensity, as measured by the blackening of the plate, may be underestimated in consequence of either over- or under-exposure. The results of § 15 and 16 are certainly subject to such errors. In order to secure some accuracy for the results of § 16, I have employed as many spectrograms as possible both of long and short exposure.

§ 14. *The intensity-minimum of the Doppler effect; condition necessary for its occurrence.*—We define the width of the intensity-minimum of the Doppler effect as the distance between the red and blue edges of the displaced band and the red and blue edges, respectively, of the stationary line, corresponding to positions b and c, respectively, of Fig. 4. If the intensity of the stationary line is great or the intensity of the displaced band small, then the photographic method yields too large a value for the width of the intensity-minimum. Indeed, an exact measurement of it is impossible; the numerical data concerning it have only a qualitative value.

This intensity-minimum is found in all series-lines of hydrogen, and in all lines of mercury and nitrogen. Among the spectrograms

now before me I fail to find it only in the second doublet of the principal series of potassium; this appears to be due partly to the fact that its width in this case is small, and partly to the fact that the dispersion of the spectrograph was too small.

The appearance of the intensity-minimum is to be explained not by the absence of the small velocities, but rather along the lines laid down in § 13, namely, that with small velocities the intensity of radiation from canal rays is small, just as with a low mean temperature the intensity of the purely thermal radiation is small. If, therefore, one wishes to demonstrate the existence of the Doppler effect with appreciable intensity for the lines of an element, he must work with canal rays of a maximum speed which is considerably greater than that corresponding to the width of the intensity-minimum. In Table IX are collected the widths of the intensity-minimum for some lines of the elements *K*, *H*, and *Hg*.

TABLE IX

Element	Wave-length	Width $\left(\frac{\Delta\lambda}{\lambda}\right)$ of Minimum	Necessary Cathode-Drop in Volts	Dependence of Intensity upon Temperature
K.........	4047.4 4044.3	very small, not measured	small	Found in Bunsen flame
H...........	4861.5	$8.4.10^{-5}$	220—687	May be produced in arc; not in Bunsen flame
Hg........	2536.7	$2.3.10^{-5}$	2743—8229	Found in Bunsen flame
	4046.8	$3.7.10^{-5}$	2589--6767	Not in Bunsen flame; increases slowly with temperature
	4078.1	$6.1.10^{-5}$	5160—15480	Not in Bunsen flame; increases very rapidly with temperature

In the fourth column two values of the cathode-drop are given for each line; the smaller is computed from $\Delta\lambda$ and gives the actual velocity measured in volts; the second and larger value gives the drop which must be effective at the cathode in order to produce the observed velocity in the canal rays; according to § 5, the first value is from 1.5 to 3 times smaller than the second. If, for instance, one is working with mercury with a cathode-drop of less than 3000 volts, he will not find a trace of the Doppler effect at λ 4046; and at $\lambda\lambda$ 4078

and 5790, he will find, even with 10,000 volts, scarcely a trace of the Doppler effect and only the stationary line.

It appears from Table IX that the Doppler effect for any one of these lines appears for velocities which are smaller in proportion as temperature at which the line appears in purely thermal radiation is smaller. This is a result which might have been anticipated from what has been said in § 12 concerning the analogy between temperature and square of velocity in canal rays.

It is well known that observers have sought in vain for the Doppler effect in the positive column of the glow-discharge and in the electric arc. This negative result is a confirmation of the inference drawn above, namely, that there is associated with a small speed of translation only a low intensity of radiation. The potential-drop in the positive column is much smaller than at the cathode, and hence the ions there located acquire a relatively small speed in the direction of the current.

§ 15. *The distribution of intensity in a series-term.*—The three components (I, II, III) of the first subordinate series of mercury triplets are themselves compound, as indicated in Table X. For these I have tabulated the widths of the intensity-minimum in the grating spectrum of the first and second order. As will be seen, these widths are equal for all components of the series-term; and I have found the same to be true for the distance of the maximum intensity from the stationary line. Within a series-term, therefore, the distribution of intensity of the Doppler effect is the same for all components; according to § 13 the intensities of the components of a series-term vary in a constant ratio; this is independent of the square of the speed.

A comparison of the last two columns of Table X shows that within the series-term the ratio of the displaced to the stationary intensity is constant; the intensity of the line 2967 with which the others are compared is taken as unity.

It has been found by Küch and Retschinsky[1] that on varying the load (temperature, vapor-pressure) of a mercury lamp the lines 5461, 4359, and 4047 increase and decrease in a constant ratio. These three lines are the components of the first term of the second subordinate series of mercury. They find the same phenomenon in the

[1] *Annalen der Physik*, 20, 563, 1906.

TABLE X

Wave-Length	Width First Order	Width Second Order	Displaced Intensity	Stationary Intensity
I { 3663.46, 3663.05 }	458	969	2	2
I 3655.00	429	921	3	3
I 3650.31	...	949	10	10
II { 3131.95, 3131.66 }	436	998	4	4
II 3125.78	432	995	3	3
III { 2967.64, 2967.37 }	438	977	1	1

case of the lines at $\lambda\lambda$ 6908, 6234, 5790, 4348, and 4078. We may therefore conclude that these lines represent the components of one and the same series-term (§ 6).

The preceding results suggest the following generalization and hypothesis. In any one term of a spectral series the distribution of intensity among the components is independent of the mode of excitation; but the absolute intensity of any component (and hence of all the other components), or the absolute intensity of the entire term, is a function of the mode of excitation (speed of canal rays, temperature). When viewed from an energy standpoint, or with reference to the amplitude of acceleration in the source of emission, the components of a series-term appear to be connected in some way in the ion.

It may be noted, in passing, that this relation between the components of a series-line may serve as a means for finding which lines *are* the components of any term.

§ 16. *Distribution of intensity in a series.*—In the second column of Table XI are given the reduced widths of the intensity-minimum of the Doppler effect for the series-lines $H\beta$, $H\gamma$, $H\delta$, $H\epsilon$, as observed on a grating spectrogram; in the third and fourth columns are collected the displaced and stationary intensities. We see that the width of the intensity-minimum increases as the wave-length diminishes through the series.

In Table XII are given the values of the squared velocities, as derived from a number of spectrograms, for which the intensity in the Doppler effect is a maximum for the series-lines $H\beta$, $H\gamma$, $H\delta$,

TABLE XI

Wave-Length	Breadth of Minimum $\frac{\Delta\lambda}{\lambda}$	Displaced Intensity	Stationary Intensity
$H\beta$ 4861.5...........	426	7	5
$H\gamma$ 4340.7...........	470	6	6
$H\delta$ 4101.8...........	492	3	3
$H\epsilon$ 3970.2...........	516	1	1

and $H\epsilon$. As will be observed, the intensity-maximum is associated with larger velocities as the wave-lengths diminishes. The three bracketed values for $H\epsilon$ appear to form an exception to this rule, probably because the measurement of $H\epsilon$ is rendered uncertain by a neighboring line not belonging to the series.

TABLE XII

SQUARE OF VELOCITY CORRESPONDING TO MAXIMUM INTENSITY

(Unit $= 10^{14}$ cm^2/sec^2)

Wave-Length	1	2	3	4	5	6	7	8	9	10
$N\beta$ 4861.5....	4.48	....	5.53	3.65	4.86	....	....	.....	.....	7.47
$H\gamma$ 4340.7....	5.02	....	6.04	7.03	6.90	7.95	....	8.22	9.37	12.89
$H\delta$ 4101.8....	....	5.58	....	7.24	7.35	9.24	8.15	8.46	12.00	12.92
$H\epsilon$ 39.702....	....	5.59	....	....	....	[8.84]	9.13	[7.10]	12.33	[12.74]

When the direction of the canal rays and the direction of observation are at right angles, one obtains, as already indicated, the sum of the stationary and the displaced intensities. However, the latter is, in the case of hydrogen, considerably greater than the former; hence the error will not be large if one places the observed total intensity equal to the stationary (displaced ?).

TABLE XIII

Wave-Length	Observed Cathode-Drop in Volts				
	2500	3300	4600	7500	7500
$H\beta$ 4861.5................	24	24	18	24	24
$H\gamma$ 4340.7................	12	12	12	32	36
$H\delta$ 4101.8................	4	4	4	8	9
$H\epsilon$ 3970.2................	2	2	2	2	3
$H\zeta$ 3889.1................	1	1	1	1	1

TABLE XIV

Wave-Lengths	Observed Cathode-Drop in Volts					
	2000	3300	3500	5000	7500	7500
$H\beta$ 4861.5	20	24	26	24	27	27
$H\gamma$ 4340.7	16	30	36	36	54	54
$H\delta$ 4101.8	8	12	12	18	27	27
$H\epsilon$ 3970.2	4	4	4	6	9	9
$H\zeta$ 3889.1	2	2	2	3	3	3
$H\eta$ 3835.6	1	1	1	1	1	1

This has been done in Tables XIII and XIV, the former having been obtained from a grating spectrogram, the latter from a prism. In each of these tables the values of the displaced intensities are given for different values of the cathode-drop, the unit being the weakest line of the series, $H\zeta$ and $H\eta$ respectively; it is to be remembered that the sensitiveness of the photographic film in the $H\gamma$–$H\zeta$ region is somewhat variable, while at $H\beta$ it is considerably smaller than at $H\gamma$.

If one compares the intensity of distribution for various values of the cathode-drop, as observed on the electrometer, he is struck by two features which repeat themselves with regularity. First, in the hydrogen series—the intensity-maximum shifts toward the smaller wave-lengths as the velocity increases; and, secondly, the decrease of intensity, in the ultra-violet terms, just above the maximum, is steeper in proportion as the maximum moves toward the shorter wave-lengths.

The analogy between these two phenomena and the dependence of the intensity of black-body radiation upon temperature is at once evident. As is well known, and as may be seen at a glance from Fig. 13, the maximum of intensity of black-body radiation shifts in the direction from red to blue with rising temperature; and further the decrease of intensity on the side of the short wave-lengths, just above the maximum, becomes steeper as the maximum approaches the blue—i. e., as the temperature increases.

We may suppose that the line-spectrum emitted by the arc or by the condensed spark at atmospheric pressure is principally a thermal radiation. After finding the two characteristic features of intensity-distribution in a series which have been mentioned above, the follow-

ing question occurred to me. In the case of thermal radiation of a series of lines, does an analogous law hold for the distribution of intensity throughout the series? If so, and if the temperature of the condensed electric spark is higher than that of the arc, then we should expect the maximum of intensity for the series to lie nearer the blue in the case of the spark than in the case of the arc; not only so, but we should expect the blue side of the maximum to be steeper in the case of the spark than with the arc. So far as I have been able to test these conclusions, from the data concerning intensity which have been given in an incidental way by those who have measured wave-lengths, I find them verified. I will give merely the following example.

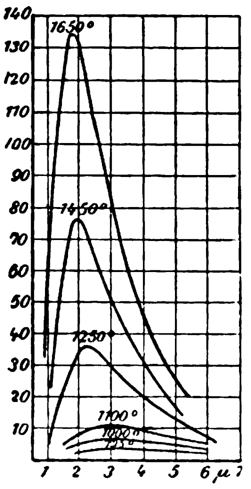


Fig. 13

The values for the intensity of arc-lines given in Tables XV–XIX are from Kayser and Runge; those for the intensity of the spark-lines in Tables XV, XVIII, and XIX from Eder and Valenta; and those in Tables XVI and XVII from Exner and Haschek. Tables XV and XVI contain doublets; Tables XVII, XVIII, and XIX, triplets. In Table XVIII the principal line of each series-term is indicated by an asterisk.

On comparison of the five following tables, one observes the following regularities. First, if the intensity-maximum lies within the series (Tables XV and XVII), it falls upon a shorter wave-length in the case of the spark than in the case of the arc. Secondly, the intensity of the series-terms just above the maximum falls off much more rapidly in the spark than in the arc.

Tables XVIII and XIX show also that the intensity-distribution in a series-term (§ 15) does not follow the above law for the intensity-distribution in the series. For instance, the second or third component of the *m*th term of a series may have a shorter wave-length and yet a greater intensity than the first component of $m+1$th term.

§ 17. *Widening of lines in a series by means of Doppler effect.*—In a gas which is made luminous by purely thermal means, and which is emitting a line-spectrum, the "carriers" of the radiation have their

velocities in all possible directions; in consequence of Doppler's principle the lines will therefore be widened on both sides. If the density of the luminous particles remains constant, this widening must increase with the temperature. These are inferences which have already been drawn by other observers. Our purpose here is merely to call attention to the bearing which the above results concerning the dependence of the intensity of radiation upon translational velocity has upon this type of widening.

TABLE XV

POTASSIUM, PRINCIPAL SERIES

Wave-Length	Intensity: Arc	Intensity: Spark
7699	10	8
7666	10	8
4047.04	8	10
4044.3	8	10
3447.5	6	10
3446.5	8	
3217.8	4	2
3217.3	6	
3102.4	2	1
3102.2	4	
3034.9	4	..
2992.3	2	..
2963.4	1	..
2942.8	1	..

TABLE XVI

ALUMINUM, SECOND SUBORDINATE SERIES

Wave-Length	Intensity: Arc	Intensity: Spark
3961.68	10	100
3944.16	10	50
2660.49	10	3
2652.56	10	2
2378.52	6	?
2373.45	4	2
2263.52	8	1
2258.27	2	..
2204.73	4	..
2199.71	1	..

TABLE XVII

ZINC, SECOND SUBORDINATE SERIES

Wave-Length	Intensity: Arc	Intensity: Spark	Wave-Length	Intensity: Arc	Intensity: Spark
4810.71	10	?	2567.99	5	1
4722.26	10	500	2542.53	5	..
4680.38	10	300	2530.34	4	..
3072.19	10	10	2493.67	4	..
3035.93	10	5	2499.72	3	..
3018.50	8	3	2457.72	2	..
2712.60	7	2	2449.76	1	..
2684.29	7	2	2427.05	1	..
2670.67	6	..	2415.54	1	..

TABLE XVIII

MERCURY, FIRST SUBORDINATE SERIES

Wave-Length	Intensity	
	Arc	Spark
3663.46	5	9
3663.05		
3655.00	5	8
3650.31	10*	10*
3131.95	8	10
3131.66		
3125.78	10*	10*
2967.64	10*	10*
2967.37		
3027.66	2	..
3023.64	2	..
3021.68	4*	8*
2655.29	5	1
2653.89	5	1
2652.22	8*	4*
2536.12	10	10
2534.89	7*	7*
2803.69	4*	3*
2482.14	4*	1*
2378.40	5*	2*
2699.74	2*	..
2399.64	4*	..
2301.57	1*	..

TABLE XIX

MERCURY, SECOND SUBORDINATE SERIES

Wave-Length	Intensity	
	Arc	Spark
5460.97	10	10
4358.56	10	10
4046.78	5	10
3341.70	5	8
2893.67	5	10
2752.91	9	8
2925.51	8	3
2576.31	8	6
2464.15	5	4
2759.83	5	2
2446.96	5	1
2345.41	4	2
2675.20	1	..

At a distance $\Delta\lambda$ from a stationary line produced by thermal radiation one may observe an intensity which is due to a large number of carriers; all velocities greater than $c\times\Delta\lambda/\lambda$ may indeed contribute to this intensity. If the angle between the direction of the velocity and the direction of observation be denoted by α, then all particles for which

$$v \cos \alpha = c \cdot \frac{\Delta\lambda}{\lambda}$$

will contribute to the intensity E at $\Delta\lambda$. Let n_i be the number of particles which have the speed v_i and have also the direction (α_i) prescribed by the preceding equation. Then, according to § 13, the total intensity becomes

$$E = n_1 \cdot J(\lambda, v_1^2) + n_2 \cdot J(\lambda, v_2^2) +, \text{etc.}$$

It is at once clear that the widening, due to the Doppler effect in a line produced by thermal radiation, is a complicated function of the

temperature; and so long as the function $J\ (\lambda, v^2)$ is unknown this distribution of intensity cannot be computed.

According to § 15 we have for the components of any one term in a series

$$\frac{\delta J(\lambda_2)}{\delta v^2} : \frac{\delta J(\lambda_1)}{\delta v_2} = k\ .$$

This shows that these lines, widened by means of the Doppler effect, have the same distribution of intensity when the reduced measure, $\Delta\lambda/\lambda$ is employed.

But for the terms of a series this theorem does not hold. For according to § 16, we have for this case

$$\frac{\delta J(\lambda_2)}{\delta v^2} = F(\lambda_1;\ \lambda_2,\ v^2) \cdot \frac{\delta J(\lambda_1)}{\delta v^2}\ .$$

Hence the intensity-distribution in the reduced widening of the lines of two different terms in a series is not the same. The larger $\Delta\lambda/\lambda$ becomes, the greater is the intensity in the widened parts of the short wave-lengths as compared with the long wave-lengths. As the temperature rises, the broadening which is visible increases more rapidly for the blue than for the red terms of the series.

PART III. TRANSLATIONAL VELOCITY AND WAVE-LENGTH

§ 18. *Introduction, sources of error.*—As indicated above, the ions in the canal rays have a high speed and emit at the same time a line-spectrum whose intensity increases rapidly with the square of the velocity. Those amplitudes in the centers of emission (negative electrons) in the ion which are sufficient to emit light must therefore increase rapidly with the square of the translational velocity, and they must simultaneously experience in a direction opposite to that of their translation, a light-pressure whose value is proportional to the rapidly increasing intensity of radiation and to the velocity of translation. Both the increase in the amplitude of the centers of emission and the light-pressure will combine to produce a slight deformation of the "carrier" of the radiation, the positive ion. The question now arises as to whether this deformation will produce a perceptible change of wave-length.

This question may be replaced by another consideration which would appear to be valid when the system of electrons (positive ion) in translation emits no electromagnetic waves during the translation. Between the electrons of an ion (*Atomion*) there are doubtless electromagnetic forces at work. The variations of these quantities are propagated through the ether with the speed of light c; and if the sources of the electric lines of force, the electrons, have themselves a velocity v, there will result from the composition of these two velocities a modification of the electromagnetic forces between the electrons, which will be measured by the even powers of the ratio v/c. The question now is whether this modification of the electromagnetic forces in the ion will bring about an appreciable change in the wavelengths emitted.

Since the variation of wave-length in question is a function of v^2/c^2, it ought to be possible to observe it by eliminating the Doppler effect—i. e., by placing the direction of observation at right angles to the direction of translation, hence normal to the direction of the canal rays. In a preliminary paper I have already described some observations of this kind made with a prism spectrograph; as the speed of the canal-rays increased, the wave-lengths emitted seemed to shift toward the red: in two instances $H\epsilon$ appeared to break up into two components; in every case the total radiation was polarized in such a way that the vibrations parallel to the direction of translation were a trifle stronger than those normal to this direction.

The experiment dealing with the polarization of light from canal rays I mean to take up again. The question concerning the displacement of the lines I have already investigated with the concave grating. The results are described below. But first let me state that, in my opinion, these results do not conclusively prove that the translation of a radiating particle produces a change of wave-length proportional to the square of the velocity. I am unable to say to just what extent the observations are affected by error.

It is possible, of course, that the observed displacement is partly due to the Doppler effect. In order to separate these two, the direction of observation—i. e., the line joining the slit and the center of the grating—must be exactly perpendicular to the beam of parallel canal rays. By the aid of a large iron square I have always taken

particular pains to make these two directions mutually perpendicular; in several cases I left this part of the adjustment to Messrs. Kinoshita and Siegel. In spite of this care, and notwithstanding the fact that in every case the displacement was toward the red end, it is possible we have systematically placed these two directions out of plumb; in any event, it is not possible to make any definite statement concerning the amount of the error. In one case where the cathode-drop was 7500 volts the displacement of the middle of $H\gamma$ was 0.42 Å. U. toward the red. To produce such a shift by means of the Doppler effect with a maximum velocity of 0.5×7500 volts (or 8.38×10^7 cm sec.) would require that the canal rays move away from the slit at an angle of 91° 59′, the error in adjustment thus amounting to 1° 59′.

It is possible, too, that my observations and results are affected by still other errors. Since the experiments are new, and since definite experimental evidence of shift of the lines toward the red would be a matter of great import, my observations may be fairly received with a certain amount of mistrust. However, I here state them in order that the reader may form his own judgment as to what has been obtained by the means at my disposal, and in order that this may possibly lead to more reliable investigations.

§ 19. *Widening of lines when directions of observation and translation are at right angles.*—In Table XX are given the widths of the lines $H\beta$, $H\gamma$, and $H\delta$ as they appear in the negative glow and in canal rays for different values of cathode-drop. In the first half of each double column the value of the line is given in terms of $\frac{1}{210}$ mm; in the second half the value is given in terms of $H\delta$ as a unit. It will be observed that in general the width of each line increases with the speed of translation, $H\delta$ widening more rapidly than $H\gamma$, and $H\gamma$ more rapidly than $H\beta$.

TABLE XX

WIDTH OF LINES AT DIFFERENT CATHODE-DROPS (VOLTS)

(Unit = $\frac{1}{210}$ mm)

Line	Neg. Glow		2500 Volts		3400 Volts		4200 Volts		7500 Volts		8000 Volts	
$H\beta$...	13.53	2.76	19.33	2.21	20.20	2.12	18.03	1.94	21.23	1.93	28.80	1.76
$H\gamma$...	6.75	1.38	15.23	1.74	15.27	1.60	13.70	1.41	19.33	1.40	25.23	1.54
$H\delta$...	4.90	1	8.70	1	9.53	1	9.76	1	11.00	1	16.33	1

The aperture of the grating used was 8.6 cm, the distance of the slit from the center of the grating being 100 cm. Those canal rays which sent light just to the edge of the grating would furnish rays of light to the observer under an angle of $2^\circ\ 27'$ with the axis of the grating. From this would result a widening of the line by Doppler's principle. In the second column of Table XXI are given the values of these widths computed on the assumption that the actual speed is 50 per cent. of that which one would expect from the cathode-drop. The third column contains the observed values computed in Ångström units from Table XX.

TABLE XXI

Cathode-Drop in Volts	Width of $H\gamma$ in Å. U.	
	Computed	Observed
2500	0.60	1.24
3400	0.70	1.25
4200	0.77	1.12
7500	1.04	1.58
8000	1.07	2.06

§ 20. *Displacement of middle of line toward red.*—By the aid of a Zeiss comparator, I measured the distance of the following lines from $H\beta$, several times on each grating spectrogram, namely, several band-lines [2), 3), 4), 7)], two mercury lines [5) =4358, 8) =4047], and the line $H\gamma$. Table XXII gives three examples of such measurements. In the first part of each principal column is given the mean of three series of measures; besides this the mean error is also stated. Those columns which are headed "reduced" contain values computed as follows. Under the conditions employed it seems probable that the band-lines and the mercury lines in the canal rays suffer no displacement; accordingly the spectrograms of the canal rays were reduced to those of the negative glow as appears from Table XXII. For each spectrogram the following differences were computed:

8)−7), 8)−5), 8)−4), 8)−3), 8)−2), 7)−5), 7)−4), 7)−3), 7)−2), 5)−4), 5)−3), 5)−2), 4)−3), 4)−2), 3)−2)

Then the quotient of each difference for the negative glow by the homologous difference for the canal rays was taken; the mean of all

TABLE XXII

Line	Neg. Glow		Canal Rays 4200 Volts			Canal Rays 7500 Volts		
		Mean Error	Dist. of $H\beta$ Obs.	Dist. of $H\beta$ Reduced	Mean Error	Dist. of $H\beta$ Obs.	Dist. of $H\beta$ Reduced	Mean Error
1) $H\beta$	....	....	....	....	....	....	....	....
2)	13958.7	0.66	14013.3	14015.1	3.32	13994.3	14001.3	1.60
3)	22327.3	0.88	22371.7	22374.6	3.27	22351.0	22362.1	2.18
4)	27585.0	3.70	27651.3	27654.9	4.03	... ...	....	....
5) Hg ..	30910.3	2.85	30967.3	30971.3	3.27	30957.7	30973.2	2.05
6) $H\gamma$ ..	32005.3	0.66	32048.3	32073.5	2.44	32062.0	32078.0	0.13
7)	40312.0	3.21	40355.0	40360.2	3.87	40325.7	40345.8	2.72
8) Hg ..	50024.3	2.96	50079.7	50086.2	3.91	... ...	....	....

these quotients gave the reduction-factor for the spectrogram of the canal rays; the "reduced" distances were obtained by multiplying all the observed distances on any one spectrogram by its reduction-factor. In those plates taken with 4200 volts, the reduction-factor was 1.00013; with 7500 volts it was 1.00050. After the reduction of the spectrograms, the following differences were computed:

$$2)_k-2)_g,\ 3)_k-3)_g,\ 4)_k-4)_g,\ 5)_k-5)_g,\ 7)_k-7)_g,\ 8)_k-8)_g$$

where the subscripts $_k$ and $_g$ refer to spectrograms of canal rays and glow discharge, respectively. The mean of these differences is the displacement of the middle of the line $H\beta$ in the canal rays with respect to the middle of the same line in the negative glow. Besides these the following differences were computed.

$$[6)-2)]_g-[6)-2)]_k,\ \ [6)-3)]_g-[6)-3)]_k,\ \ [6)-4)]_g-[6)-4)]_k,$$
$$[6)-5)]_g-[6)-5)]_k,\ \ [7)-6)]_k-[7)-6)]_g,\ \ [8)-6)]_k-[8)-6)]_g.$$

The mean of these differences is the displacement of the middle of the line $H\gamma$ in the canal rays with respect to the same line in the negative glow.

TABLE XXIII

Cathode-Drop in Volts	Displacement toward Red in Ångström Units $H\beta$	Displacement toward Red in Ångström Units $H\gamma$
4200	0.03	0.17
7500	0.71	0.42

Table XXIII contains the displacement of $H\beta$ and $H\gamma$ computed in this manner for cathode-drops of 4200 and 7500 volts.

§ 21. *Displacement of series-lines by rise of temperature.*—In the second part of this paper we have discussed the far-reaching analogy between speed of translation and intensity of radiation for canal rays, on the one hand, and temperature and intensity of thermal radiation, on the other hand. If the displacements observed above have not been produced by the Doppler effect, but are really a function of v^2/c^2, we may draw the following conclusion from the analogy just stated: Lines emitted by a gas made luminous by thermal means, when compared with stationary lines, such as those of the negative glow, should be displaced toward the red in proportion as the temperature of the luminous gas is increased.

It has been certainly proved[1] by Jewell, Humphreys, Mohler, and Ames that the lines of the arc-spectrum are slightly shifted toward the red as the pressure about the arc is increased. My own experience, and the results of Küch and Retschinsky[2] with the mercury arc, leave no doubt that along the axis of the arc the temperature rises with the potential-drop and with the energy used per cubic centimeter; on the other hand, when the pressure is constant, the temperature varies only slowly with the current-strength; for, in this case, it is only the cross-section of the arc and not the energy per unit volume that increases.

Ames and Humphreys[3] report that the difference of wave-length, for the D lines, between the Bunsen flame and the arc does not exceed 0.002 Å. U. Now, the temperature at the center of the arc is very considerably higher than that of the Bunsen flame; however, its value falls off rapidly from the axis to the outer layers of the arc. For any one line, therefore, the observed intensity is made up of components from different layers of the arc, each at a high temperature. If the line is found in the layers of low temperature—i. e., the outside layers—then the radiation from the inner layers will be absorbed by the outer layers, and the observed intensity will be principally that of the outer, not that of the inner and more highly heated, layer.

Thus the lines actually observed in the principal and two subordinate series may have their origin in one and the same arc, at the same

[1] Kayser, *Handbuch der Specktroskopie*, **2**, 322, 1902.

[2] *Annalen der Physik*, **20**, 563, 1906.

[3] *Phil. Mag.*, **44**, 119, 1897.

pressure, but at different temperatures. The above-mentioned investigators found that in the three series the pressure-shift was in the ratio 1:2:4.

Still another inference is to be drawn. According to § 16 the radiation-temperature for a series of lines is higher in the spark than in the arc. If, therefore, the influence of temperature upon wave-length is that which is here imagined, the spark-lines should be slightly displaced toward the red when compared with the arc lines. As to whether this is the case, there appears as yet no agreement[1] between observers who have studied the question, namely, Exner and Haschek, Kayser, Eder and Valenta, and Kent.

My thanks are due to Professor Riecke, who has throughout this work been cordial in his support and has placed at my disposal the necessary apparatus. I wish also to thank Professor Runge for kindly loaning me his short-focus concave grating.

[1] Kayser, *Handbuch der Spectroskopie*, **2**, 308; *Zeitschrift für wissenschaftliche Photographie*, **3**, 308, 1905.

THE CHARACTER OF THE STAR IMAGE IN SPECTROGRAPHIC WORK

By J. S. PLASKETT

The object of this paper is to describe some experiments on the size and form of the star image given by the combination of objective and correcting-lens, with an investigation into the causes of the observed effects and suggestions for the improvement of existing conditions.

The equipment of the Dominion Observatory, Ottawa, for radial-velocity work consists of a 15-inch telescope with a Brashear visual objective and photographic correcting-lens, and a spectroscope of the universal type, also by Brashear. The objective for visual purposes is excellent, and the spectroscope is admirably adapted for general spectroscopic work, but, as the experience of others as well as myself has shown, is not suitable for the accurate determination of radial velocities. Its design as a universal spectroscope does not give sufficient stability, and, in exposures of any length, flexure will not only ruin the definition, but is liable to introduce systematic errors in the velocities obtained. Pending the construction of a spectrograph specially designed for the required purpose, an attempt was made to render the present instrument capable of giving accurate velocity values. The investigation and removal of the known sources of error led to the discovery of the aberrations to be presently described. A brief description of the steps leading thereto may be of interest.

Trusses connecting the various parts of the instrument, where flexure could occur, with the supporting tubes were applied to such effect that an initial displacement of the spectral lines, equivalent to a velocity of 30 km per second, occasioned by a movement of telescope and spectroscope through two hours in right ascension, was reduced to 1½ km. The prisms were firmly clamped in place, without inducing strains in the glass, by screws passing through the base of the prism-box and the minimum-deviation linkwork into the prism-cells. The slit-jaws, originally too thick on the edge, were reground,

and the occulting diaphragms for star and spark light were removed from the slit-head and placed on an independent frame attached to the supporting tubes. The comparison apparatus was remodeled, the direction of the spark being made transverse to, instead of parallel with, the slit-jaws, and many other smaller details were carefully attended to.

After all known sources of error in the spectroscope itself had been overcome, and after it had been placed in thorough adjustment, it was found that test spectra of the standard-velocity stars occasionally gave values differing by as much as 3 km per second from those obtained by other observers. As the probable error of the mean of the measured lines did not exceed four-tenths of a kilometer, and as all the other known causes of systematic error had been overcome, it seemed probable that this might be due to unsymmetrical distribution of the star light over the collimator and camera lenses. Evidently such unsymmetrical distribution can cause a displacement of the lines only when the camera is not in exact focus. The camera was always carefully focused by a modification of Newall's method, which readily detected displacements of the sensitive surface from the focal plane of less than 0.05 mm in a focal length of 375 mm. But as the plates are supported only at the ends of the plate-holders, differences in the curvature of the glass may easily cause differences of 0.1 mm or more in the position of the center of the sensitive surface, where all measurements are made. In the case of a displacement of 0.1 mm from the focus, a distribution of the star light on the collimator objective so that its center of intensity is 5 mm to one side of the axis, is sufficient to cause a displacement of the spectral line $\frac{5}{375} \times \frac{1}{10} = \frac{1}{750}$ mm equivalent to a velocity of 1.8 km per second.

An examination of the illumination pattern on the collimator lens, both visual and photographic showed how easily such or even greater displacements of the center of intensity could occur even with the utmost care in guiding. The illumination could never be made uniform, no matter how the relative positions of slit and correcting-lens were altered. The pattern was either a diametrical bar parallel to the slit of a width about one-third or one-fourth the aperture, or else such a bar with the addition of a peripheral ring; while a very slight movement of the slit-jaws to one side or other was sufficient to cause

one side only of the lens to be illuminated, without causing any appreciable change in the appearance of the image in the guiding telescope, guiding being done by means of light coming through the slit. It is easy to see how the center of intensity of the star light could be displaced without the observer being aware of the fact, thus causing a displacement of the star lines unless the plate were in exact focus.

The appearance of this pattern and its behavior for change of slit position indicated spherical aberration of the condensing system. That aberrations of some nature were present was indicated not only by the long exposures required—upward of two hours for a star of the fourth photographic magnitude—but also by the large effective diameter of the image as shown by the wide opening, 0.25 mm, of the slit required to obtain uniform illumination.

An examination of the correcting-lens showed that part of the difficulty might arise from the accidental inversion of the diverging element, which had been so placed in the cell that surfaces of unlike curvature were adjacent to each other. On inverting this concave element so that surfaces of like radius of curvature were in contact, the illumination pattern became more uniform, the required exposure time was diminished by 50 per cent. and no errors of a greater magnitude than should be expected with the dispersion employed, appeared in velocity determinations of standard stars. If the diameter of the object-glass, 15 inches, and the linear dispersion of the spectrograph, 18.6 tenth-meters per millimeter at $H\gamma$, be taken into account, the exposures required—less than an hour for stars of the fourth photographic magnitude—compare very favorably with those of other equipments.

Notwithstanding the great improvement shown, photographic tests of the star focus for different temperatures indicated that the star spectrum was much wider than could reasonably be accounted for by atmospheric disturbance, and I was led to make thorough tests of the character and diameter of the image.

To determine whether a narrower spectrum could be obtained by a change in adjustment, a plate was made for each of six settings of the correcting-lens, above and below its computed position, over a range of four inches. A simple device applied to one of the plate-

holders enabled ten successive star spectra to be made side by side on each of these plates, at different settings of the slit position in the neighborhood of the star focus; the sixty spectra forming a record of the diameter of the star image under varying conditions. To insure that the spectrum had not been widened by a drift of the star image along the slit, the spectroscope was turned in position angle until the slit-jaws were parallel to an hour circle. By opening the slit 0.2 mm, and by using a bright star, *Vega*, a fully exposed linear spectrum was obtained in eight or ten seconds, evidently with no chance of widening due to drift. The width of the narrowest part of the narrowest spectrum on each plate, presumably where the star was in focus on the slit, was measured, and these widths ranged from 0.085 to 0.115 mm. As the camera and collimator objectives are of the same focal length, and as one second of arc in the focus of the refractor is equivalent to 0.0275 mm, the diameter of the star image according to this test must be between $3''$ and $4''.5$. The diameter of the central diffraction disk as given by the formula $d=\frac{1.2197\lambda}{r}$ is, for a 15-inch objective and $H\gamma$ light, about $0''.57$, while the actual effective diameter as obtained from the width of star spectra is five to eight times as great.

This enlargement of the diffraction image may be due to three causes: (1) aberrations in the spectroscope; (2) atmospheric disturbances; (3) aberrations in the system of objective and correcting-lens.

1. *Aberrations in the spectroscope.*—It is a simple matter to determine whether the wide star spectra obtained are due to this cause, for by direct photography of the star image no aberrations in the spectroscope can affect the result. A series of star trails was therefore made on ordinary plates by the system of objective and correcting-lens. A small plate, held in guides in the slit-cap of the spectroscope, could be moved in these guides between exposures so as to make a number of trails on each plate. The collimator tube, carrying the plate with it, was moved by the rack and pinion about a quarter of a millimeter between each exposure, to insure having one of the trails within an eighth millimeter of the focus. A plate each was made of six stars ranging from the third to the sixth magnitude, and the width of the narrowest trail on each plate, corresponding to the

position where the star was most nearly in focus, was measured. Although the conditions of seeing both for trails and spectra were above the average, about 3 in a scale of 5, the trails were not continuous but broken and jagged, owing to atmospheric disturbances, and the measurements were made in two ways: first, of the width of narrow short parts of the trails where the seeing had been momentarily steady; and, second, of the average width of a longer strip of trail. In the first series of measurements the widths varied from 0.070 mm in the fainter stars to 0.110 mm in the brighter stars, while the average widths of longer strips were about 20 per cent. greater. Since the widths of spectra were practically the same, it is evident that the cause must be sought in the star image itself, and is not due to aberrations in the spectroscope.

2. *Atmospheric disturbances.*—Newall in his paper on the design of spectrographs[1] has introduced a very useful conception, that of tremor-disks, and he states that atmospheric disturbances enlarge the effective diameter of the star image. Such enlargement may be due either to bodily displacements of the image from its mean position or to the spreading-out of the central image into a more or less expanded disk. He considers that the actual effect, so far as getting light through the slit of a spectrograph is concerned, is the same as if the image consisted of a central core from 1″ to 2″ in diameter surrounded by a more or less diffuse and gradually diminishing portion, the whole diameter being in the neighborhood of 4″ or 5″. If we accept Newall's estimates as correct, and if we remember that in no case was a sufficiently long exposure given to allow the outlying parts of the tremor-disk to increase the width of spectrum or trail, then the diameter of the image given by the Ottawa objective and correcting-lens, even allowing the extreme limit assigned by Newall for atmospheric disturbances, is nearly twice as great as it should be.

It is also a simple matter to test this conclusion experimentally. As the objective gives excellent visual definition, it may be safely assumed that the visual star image is of normal diameter. A measurement of the width of spectra and trails produced by the visual image, and a comparison with the widths given by objective and correcting-lens in photographic light, should at once decide whether the observed

[1] *Monthly Notices*, 65, 808, 1905.

effect is due to atmospheric tremor. The correcting-lens was therefore removed, the spectroscope was adjusted for yellow light, and spectra were made similarly to the previous ones, though on Cramer Isochromatic plates, which have a pronounced band of sensitiveness almost identical in wave-length with the turning-point of the color-curve of the objective. The widths of the spectra produced varied between 0.050 and 0.065 mm, about 2″, but as the seeing was very unsteady (about 1½ in scale of 5), these widths are doubtless about 25 per cent. greater than would be the case with good seeing. For the star trails the same make of plate was used, light of shorter wave-length than λ 5000 being absorbed by a yellow screen of plane glass placed in contact with the plate. Owing to the insensitiveness of the plate to light of wave-lengths between λ 5000 and λ 5400, and to longer waves than λ 5800, only the light which is effective in forming the visual image can act in producing the trails. As before, the width of the trails varied with the brightness of the stars, ranging from 0.025 mm in faint trails to 0.055 mm in stronger trails, or from 1″ to 2″, while the average width over a longer strip of trail was about 20 per cent. greater. Notwithstanding the bad seeing, both trails and spectra were much more sharply defined than those made with the correcting-lens in photographic light and of only half the width.

These experiments conclusively prove that the abnormal width of spectra and trails in photographic light is not due to aberrations in the spectroscope nor to atmospheric disturbances, and clearly point to aberrations in the condensing system as the cause of the observed effects. A short summary of the experimental data will render this more evident. The theoretical diameter of the central disk, or rather of the first dark ring, for visual light λ 5600, is 0″.74, for photographic light, λ 4340, is 0″.57. The actual width of visual spectra and trails is from 1″ to 2″, or one and one-half to three times the theoretical diameter. The actual width of photographic spectra and trails is from 3″ to 4″.5, or five to eight times the theoretical diameter.

Some further information regarding the size and character of the photographic image may be gained by considering its effective diameter under another aspect, that of the loss of light at the slit. Referring again to Newall's paper, and taking, as he does for an example, a tremor-disk of 5″ diameter with a core of 2″, we find that

a slit 0.025 mm wide will transmit 31 per cent. of the incident star light; a slit 0.037 mm, 44 per cent.; a slit 0.05 mm, 58 per cent.; and so on. I am indebted to a suggestion by Professor Campbell for a method of testing this theoretical result experimentally. A series of star spectra were made at different slit-widths, and the resulting intensities were compared. As it is practically impossible to make a number of wide spectra of uniform intensity throughout their width, photometric measurements cannot be relied upon and recourse must be had to visual estimates. Such estimates can be made more accurately if the exposures are so regulated as to give spectra of equal intensity, and, moreover, within the limits of exposure time and intensity used here, errors due to the characteristics of the plate employed are to a great extent avoided. The spectrum of *a Lyrae*, the star used, is practically continuous except for the *H* series, and is therefore well suited for the estimation of intensities, while its brightness is such that only short exposures are required. Ten different slit-widths between 0.012 and 0.25 mm were used, and ten spectra, one through each slit-opening, were made side by side on the same plate. The exposures were so regulated as to render the resulting spectra as nearly equally intense as possible, and the final estimate is the mean from a number of plates and from spectra of different widths. To render the comparisons more direct, slit-widths will be represented by divisions, a single division corresponding to 0.025 mm, and the relative exposure times will be reduced to a unit of 100 with a slit-width of one division, 0.025 mm, or $0\overset{''}{.}91$, the normal width with the dispersion employed here.

The following table shows that the exposure required is inversely proportional to the slit-width until this reaches 0.1 mm, leaving out of account widths less than a single division, where diffractional loss within the collimator plays an important part. It also shows that with normal slit-width less than 17 per cent. of the light incident on the slit is transmitted. In Newall's hypothetical case 31 per cent. would be transmitted. The experimental data given above, using Newall's method of calculation, indicate a tremor-disk 8″ or 10″ in diameter with a core of about $3\overset{''}{.}5$, and, as the previous experiments have shown, this is much larger than can be accounted for by atmospheric disturbances.

TABLE I

LOSS OF LIGHT AT SLIT

SLIT-WIDTHS			COMPARATIVE TIMES FOR EQUAL INTENSITY	
Divs.	Mm	Secs.	Experimental	Computed: $\tau=5''$ $\gamma=2'$
½	0.012	0.45	300	
1	.025	0.91	100	100
1½	.037	1.35	67	70
2	.050	1.82	50	54
3	.075	2.73	33	39
4	.100	3.64	28	34
5	.125	4.55	25	31
6	.150	5.45	21.7	31
8	.200	7.27	18.3	31
10	.250	9.07	16.7	31

The above experiments point conclusively to aberrations in the system of objective and correcting-lens, when used with photographic light, as the cause of the observed effects, but they give no information concerning the nature of these aberrations beyond indicating in a general way, from the appearance of out-of-focus photographs of spectra and trails, that spherical aberration is present. It was decided therefore, to make quantitative tests to ascertain if possible the nature and magnitude of the aberrations and the best means of removing them.

The most simple and accurate method of determining the zonal errors and axial astigmatism of a telescope objective is Hartmann's method[1] of extra-focal measurements. The principle of the method and the measurements and reductions necessary are extremely simple, while it gives accurate values with the expenditure of comparatively little time and without the use of any appliances except such as can be readily made by anyone. For the benefit of those who have not the above paper at hand, and in order to render the present article complete, the essential principles of the method will be briefly described.

It depends upon the determination of the intersecting point of pencils of light coming from different parts of the objective. Suppose a diaphragm containing two small openings, equidistant from the

[1] *Zeitschrift für Instrumentenkunde*, **24**, 1, 33, 97, 1904.

center and along a diameter, be placed over the objective. If the distance between the pencils of light coming from these openings be measured at two points, one within and one without the focus, the point of intersection of the pencils, and consequently the focus for the particular zone in question, can be at once obtained from similar

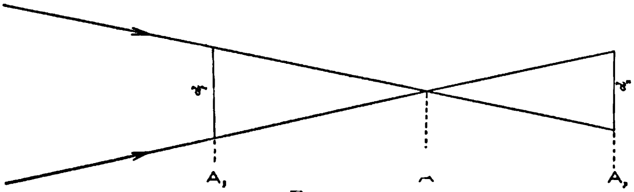


FIG. 1

triangles. For let d_1, Fig. 1, be the distance between the pencils at the scale-reading A_1 within the focus, d_2 the distance at the scale-reading A_2 beyond the focus. Evidently then the scale-reading for the focus A is $A_1+\left(\frac{d_1}{d_1+d_2}\right)(A_2-A_1)$. The distances d_1 and d_2 may be determined directly by micrometer measurements on the pencils from a star or distant artificial point-source, or by making exposures on photographic plates in the two positions and measuring the distances between the resulting images by a measuring microscope. The latter method is preferable and was used exclusively, except that the photographic determinations were checked by micrometer measures.

A zone plate A, Fig. 2, similar to that described by Hartmann, was employed. The apertures, except the four inner ones, were

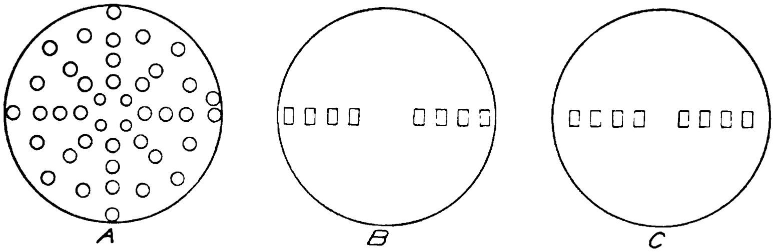


FIG. 2.—Zone Plates

each about 25 mm in diameter, and the radii of the nine zones were respectively 28, 47, 66, 85, 104, 123, 142, 160, and 178 mm. In order to determine the astigmatism along the axis, each pair of open-

ings is duplicated by a second similar pair at right angles, so that the focus of each zone of the objective is determined for two elements perpendicular to each other. In the case of the zone of 142 mm radius the focus can be obtained for four elements 45° apart. Thus an exposure within the focus, and a second one without the focus, give data sufficient to determine the focus of each of nine zones of the objective in two directions perpendicular to each other. These two directions are distinguished from one another in the measurement by making an extra aperture in the zone plate, which, on being reproduced in the negatives, serves to identify the origin and direction of the angle ϕ.

To determine the zonal errors of objective and correcting-lens, the zone plate was placed in position in front of the objective and a small photographic plate was placed in the guides in the slit-cap of the spectroscope. The spectroscope is supported on two parallel tubes carried by an adapter on the eye-end of the telescope, and can be readily moved up and down through a range of about 20 cm. Experience showed that the images were most sharply defined, and the best measurements could be obtained when the plates were between 6 and 10 cm from the focus. As the photographic focus was to be tested, an ordinary Seed 27 plate was first tried; but it was not found possible to make very accurate settings, as the pencils from the zone plate were spread out into radial spectra owing to the long range of wave-length (λ 5000 to the limit passed by the object-glass, say λ 3600) to which such a plate is sensitive. Several means of overcoming this difficulty were tried. As a yellow screen in front of an ordinary plate did not improve matters, the dispersion of the pencils must evidently be chiefly due to the light around $H\beta$. An ordinary lantern plate, which is sensitive from about λ 4600 down, was therefore next tried, and gave good images capable of accurate measurement; while if a yellow screen were used with such a plate the resultant images were again elongated, showing that the prolonged exposure entailed thereby had extended the action on the plate toward the red and reintroduced the first difficulty. A yellow or red star was used in preference to a white or blue, as limiting the action in the violet, shortening the effective range of spectrum, and thus giving images with less spectral dispersion and with no apparent elongation.

Four sets of extra-focal plates were made which, on being measured, reduced, and averaged, gave the focal positions of the nine zones as tabulated below (Table II). All four measures are in substantial agreement, which of course is closer for the outer zones where the convergency of the pencils is greater. There the probable error of a single determination of the focus does not exceed 0.1 mm, while near the center it may be as great as 0.5 mm. It will be noticed that the focus for the edge of the objective and correcting-lens is upward of 2 mm longer than the focus near the center, and if astigmatism be taken into account also, the difference is greater than 2.5 mm. The values are plotted graphically in the curve (*A*) of Fig. 3, the vertical distances being magnified some six or seven times, the appended scale representing millimeters. The horizontal line is drawn in the position of focus 75.34 that gives the smallest circles of confusion, in this case 0.04 mm in diameter. The astigmatism will increase this to some extent, so that probably the diameter will be nearly 2″. Unless the slit is set exactly at this mean position, which is not likely, the diameter of the confusion disks will be still further increased, so that we may consider 2″ as a moderate estimate. It must be remembered, however, that in speaking of circles of confusion the conceptions of geometrical optics alone are being considered, and no account is taken of diffraction phenomena, which may have some effect on the geometrically calculated dimensions of the star disk resulting from aberrations of the magnitude here present. However, the experiments on the width of spectra and trails showed conclusively that the photographic image was about 2″ greater in diameter than the visual image, presumably unaffected by aberrations, and this agrees with the geometrical theory.

To determine where the aberrations arise it is necessary to accurately compare the performance of the objective used visually with the performance of the objective and correcting-lens in the photographic part of the spectrum. Zonal tests were therefore made of the objective alone. For this purpose the wave-length of the light used must be limited to λ 5400–λ 5800, the range to which the eye is most sensitive, which is the most luminous in the spectrum, and which coincides with the turning-point of the color-curve of the objective. Fortunately, as the band of color-sensitiveness of Cramer Isochro-

TABLE II

ZONAL FOCI OF 15-INCH OBJECTIVE

Radius of Zone	ϕ	Objective and Correcting-Lens Photographic			Objective Alone Visual		
		Focus	Mean	Astigmatism	Focus	Mean	Astigmatism
28	45°	73.54		−0.20	106.43		−0.05
	135	73.94	73.74	+ .20	106.54	106.48	+ .06
47	0	74.19		+ .08	108.35		+ .42
	90	74.03	74.11	− .08	107.51	107.93	− .42
66	45	73.54		− .30	106.67		− .13
	135	74.14	73.84	+ .30	106.93	106.80	+ .13
85	0	74.15		+ .11	106.42		+ .26
	90	73.94	74.04	− .10	105.91	106.16	− .25
104	45	74.65		− .23	106.15		− .08
	135	75.11	74.88	+ .23	106.31	106.23	+ .08
123	0	75.68		+ .22	106.20		+ .09
	90	75.25	75.46	− .21	106.02	106.11	− .09
142	22.5	75.93		+ .24	106.08		+ .20
	67.5	75.32		− .37	105.77		− .11
	112.5	75.67		− .02	105.82		− .06
	157.5	75.83	75.69	+ .14	105.83	105.88	− .05
160	45	75.58		− .15	105.91		+ .04
	135	75.88	75.73	+ .15	105.83	105.87	− .04
178	0	76.11		+ .21	105.93		− .01
	180°	75.69	75.90	− .21	105.95	105.94	+ .01
Mean focus		.....75.34			106.01		

matic plates almost exactly coincides with the same region, all that is necessary in order to obtain photographic test plates is to absorb the blue and violet light by a suitable screen, and thus confine the action to the visual part of the spectrum. A deep yellow screen with plane parallel surfaces was used in contact with the plate. Although the pencils from the zone plate are displaced slightly on passing through this screen, these displacements are proportional, and the only effect will be to lengthen the focus for all the zones by the same amount, about one-third the thickness of the screen, without in the least altering the relative positions of the pencils. An exposure of about a minute on *Capella*, through the screen, with the plate from 60 to 100 mm from the focus, gives a negative of good intensity in which the images of the pencils are quite round and free from any noticeable spectral elongation, thus allowing accurate measurement.

Five sets of extra-focal exposures were made in the visual part of the spectrum, and the mean values resulting from the measurement and reduction of these plates are given in Table II and plotted graphi-

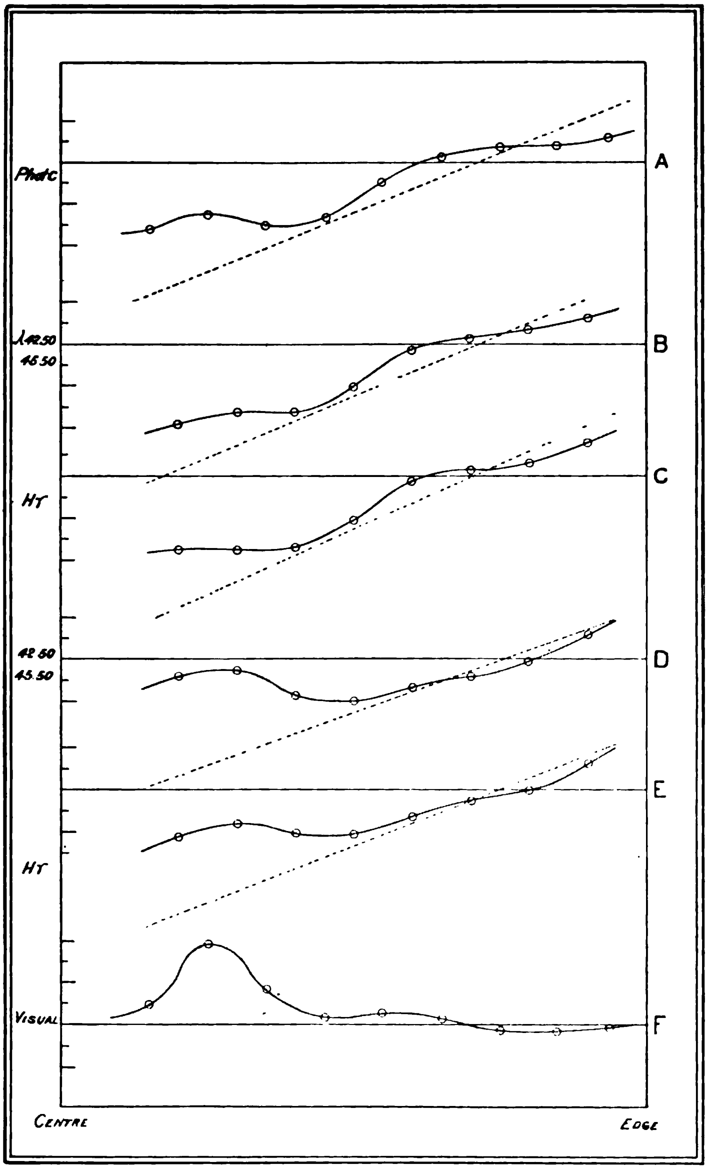


FIG. 3.—Zonal Differences of Focus

cally in curve F of Fig. 3. An examination of this curve shows that no point or focus is at a greater distance than 0.2 mm from the position

of mean focus, shown by the horizontal line, except a small region near the center of the objective, which has a longer focus. The effect of this region on the performance of the objective must, however, be exceedingly small, owing to its small area, less than one-tenth of the objective, and to the weak convergency of the pencils proceeding from it. In fact if Hartmann's criterion T[1] as to the quality of an objective be computed from the above mean values, it is found to be 0.141. According to this classification an objective is moderately ("mässig") good when T is greater than 1.5, good when T is between 0.5 and 1.5, and exceedingly ("hervorragend") good when T is less than 0.5. In the ideal, absolutely zoneless objective T is 0.

Evidently the objective when used visually is of the very first quality, and the aberrations appear only when it is used in conjunction with an auxiliary corrector for spectrographic work. Whether the aberrations there present are due to the correcting-lens, or to the objective when used in the photographic part of the spectrum, remains to be determined. For this purpose a further application of Hartmann's method was necessary to find the color-curves of the objective alone, and of the system of objective and correcting-lens for a number of zones. It was hoped that such observations would throw light on the cause of the aberrations and suggest a possible remedy. They would also serve as a check upon the zone-plate determinations, as, in this case, no spectral dispersion of the pencils could affect the accuracy of setting. To find such color-curves, the pencils of light coming from a zone plate fall on the spectroscope slit, and the distance between the resulting spectra taken with the slit within and beyond the focus gives a measure, calculated in the same way as before, of the focal position of any desired wave-length for any particular zone.

It was decided to determine the color-curves of eight zones of 38, 57, 76, 95, 114, 133, 152, 171 mm radius; and, to prevent the spectra from merging into one another, two zone plates were required, one (B), Fig. 2, of the four zones of 57, 95, 133, and 171 mm radius, and the other (C), Fig. 2, of the remaining four. The central openings were each 20 mm square, and the outer 20 by 25 mm. The zone plates were so placed on the objective that the row of openings was parallel to an hour circle, and the spectroscope was turned in

[1] *Zeitschrift fur Instrumentenkunde*, **24**, 46, 1904.

position angle until the slit was parallel to the openings, in order that irregularities in driving would not widen the spectra. To diminish the exposures as much as possible, bright stars, *Vega* and *Sirius*, were used and the slit was widely opened, as no inaccuracy would be thereby introduced in the distance between the spectra. The exposures were made on a night when the temperature was nearly stationary, and were arranged in the following order:

Plate 1;	Zone Plate (*B*) Fig. 2;	slit	about	50	mm	within	the	focus
2;	(*C*)	"	"	50	"	"	"	"
3;	(*C*)	"	"	40	"	beyond	"	"
4;	(*B*)	"	"	40	"	"	"	"

This procedure was followed to avoid as far as possible any relative displacement of the focal determinations of the two sets, due to slight changes of temperature of the objective. That no measurable displacement has occurred is shown by the continuity of the zonal curves of Fig. 3 drawn from the combination of the two separate determinations, and by their agreement with those made by the regular zone-plate method.

Each of these plates contains eight spectra side by side, one from each light pencil transmitted by the zone plate, and the position of the focus for each zone and for any desired wave-length in the range on the plate can be determined in exactly the same way as before. The hydrogen lines, in the first type stars used, serve as datum marks for the identification of wave-lengths, and measurements were made at eleven positions between λ 3970 and λ 5030. The corresponding focal points, as calculated from these measurements, are given in Table III for eight zones of the objective alone, and in Table IV for the same eight zones of the objective with correcting-lens, the latter being about 40 mm nearer the focus than its computed position.

The reason for using the correcting-lens below its computed position at once appears on inspection of Fig. 4, which represents, in their correct relative positions, the color-curves of a median zone of 108 mm radius, determined in exactly the same way as above. Curve *A* (Fig. 4) is the color-curve of the visual objective between the limits λ 6250 and λ 3970, which shows that the minimum focus is at about λ 5600, exactly in its computed position. Curve *B* is the color-curve of the system of objective and correcting-lens between λ 6250 and

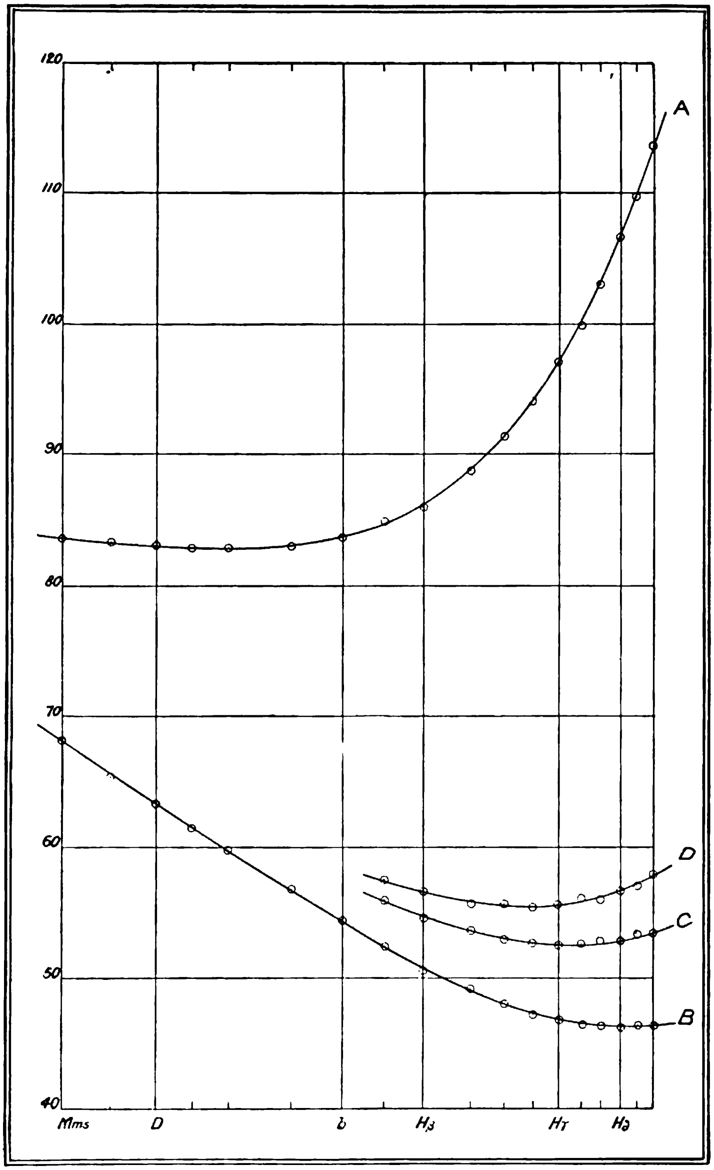


FIG. 4.—Color-Curves for a Median Zone

λ 3970, which shows that the minimum focus is at about $H\delta$, instead of $H\gamma$, its computed position. When the correcting-lens is moved

TABLE III

COLOR-CURVES OF OBJECTIVE ALONE

Radius of Zone	Wave-Lengths										
	5030	$H\beta$ 4861	4680	4550	4440	$H\gamma$ 4340	4250	4175	$H\delta$ 4102	4035	$H\epsilon$ 3970
38	85.57	86.87	89.64	92.02	94.28	96.30	99.78	102.48	105.82	109.59	110.96
57	85.30	86.30	88.95	92.00	94.28	96.60	100.25	102.74	105.95	108.75	111.61
76	83.84	85.78	88.76	91.09	93.67	96.39	99.50	102.34	105.31	108.69	112.31
95	84.67	85.42	88.41	90.82	93.56	96.34	99.37	102.61	105.68	109.11	112.12
114	84.38	85.78	88.68	91.16	93.87	96.77	99.58	103.06	106.19	109.63	112.65
133	84.71	85.93	88.68	91.08	93.91	97.16	100.21	103.16	106.72	110.11	113.08
152	85.06	86.29	89.18	91.49	94.41	97.42	100.53	103.71	106.79	110.10	113.38
171	85.41	86.87	89.65	92.03	95.02	98.04	101.29	104.62	107.81	111.10	114.53

TABLE IV

COLOR-CURVES OF OBJECTIVE AND CORRECTING-LENS

Radius of Zone	Wave-Lengths										
	5030	$H\beta$ 4861	4680	4550	4440	$H\gamma$ 4340	4250	4175	$H\delta$ 4102	4035	$H\epsilon$ 3970
38	55.12	54.75	53.11	51.18	50.65	50.92	50.92	51.17	51.36	51.68	51.91
57	53.38	53.89	52.55	51.98	51.24	51.04	50.90	50.91	50.95	51.30	51.82
76	54.51	53.67	52.54	51.60	51.19	51.14	51.16	51.11	51.20	51.46	51.26
95	55.57	54.37	53.16	52.46	51.95	51.70	51.60	51.74	51.96	52.30	52.66
114	55.45	54.82	53.62	53.12	52.79	52.59	52.65	52.79	53.03	53.24	53.31
133	55.94	55.10	53.88	53.33	53.06	52.89	52.93	53.05	53.31	53.60	53.73
152	55.84	55.13	54.05	53.54	53.26	53.07	53.25	53.38	53.49	53.62	53.64
171	56.05	55.39	54.38	53.90	53.60	53.53	53.56	53.97	54.15	54.27	54.34

down, away from the objective, some 40 mm we get curve *C*, and at 70 mm, curve *D*. In curve *C* the minimum focus is nearly at $H\gamma$, and in *D* at λ 4460. Evidently the lowering of the correcting-lens some 40 mm effects considerable improvement in the color-correction without, as the earlier experiments showed, appreciably enlarging the image, and the lens has been used in this position almost from the first.

Although all the data in regard to the complete color-curves are given in Tables III and IV, still the actual curves drawn from these figures show all the conditions at a glance, and are hence worth giving. To prevent too great a confusion of lines, the curves for four zones only (zone plate (*B*), Fig. 1), of 57, 95, 133, 171 mm radius, are shown here in Fig. 5, the upper curves being of objective

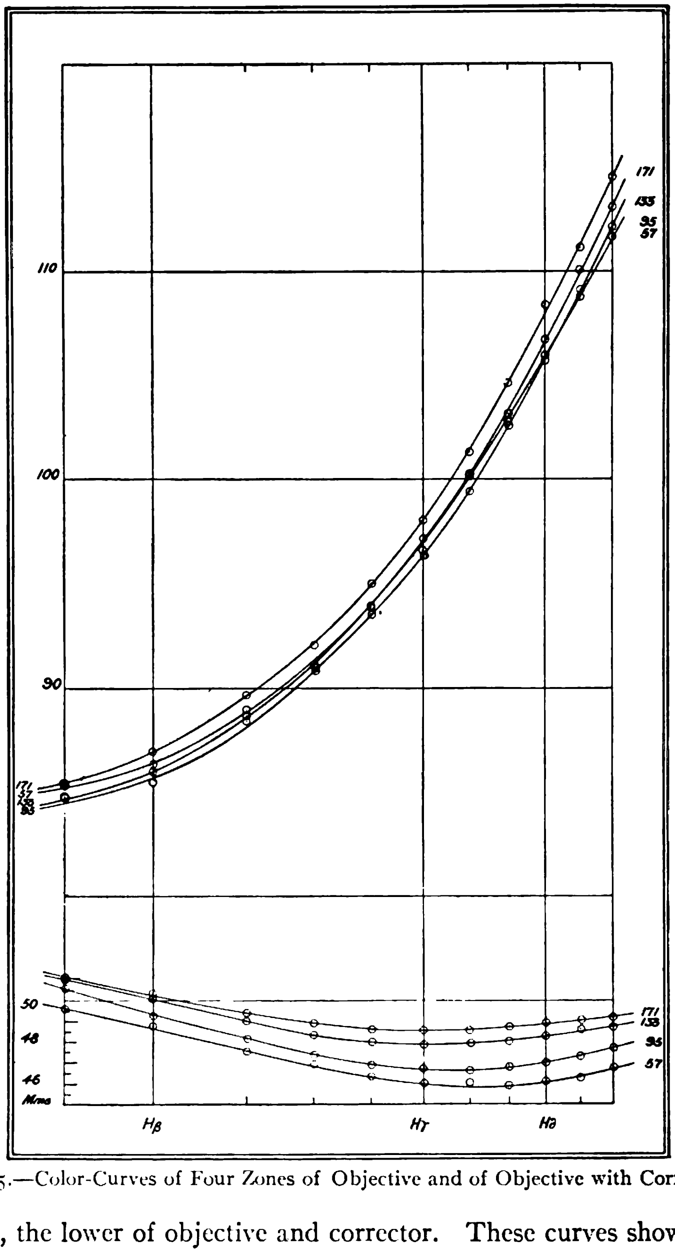


Fig. 5.—Color-Curves of Four Zones of Objective and of Objective with Corrector

alone, the lower of objective and corrector. These curves show at a glance that, in the photographic part of the spectrum, the focus for

the edge of the objective is longer than the focus for the center, that it has negative spherical aberration. This chromatic difference of spherical aberration is inherent in two-part objectives of the ordinary glasses, and the only remedy is to compensate for it by introducing the correct amount of positive aberration by the correcting-lens. However, the lower curves show that, instead of compensating for this chromatic difference, the correcting-lens has, on the contrary, increased it somewhat, and the focus for marginal rays is upward of 2 mm longer than the focus for central rays. This agrees almost exactly with the previous determination of the zonal foci of objective and corrector, and is good evidence of the substantial accuracy of the determinations. Before leaving these curves it may be pointed out that the crossing of the curve from the 57 mm zone over the others in passing from short to long waves is due to the longer focus of the central zones in the visual part and is further evidence in favor of the accuracy of the determinations.

To obtain a still more striking comparison of the cause and magnitude of the aberrations present in the system, the color-curves can be presented in another form, that of zonal foci curves like *A* and *F*, Fig. 3, previously determined. We have the color-curves, or the positions of focus, of the whole photographic region for eight zones of the objective in Tables III and IV, and these can be readily plotted in the same way and on the same scale as *A* and *F*, Fig. 3. If such curves were plotted for every wave-length in these tables, they would show a striking agreement in form, but I have satisfied myself with representing the positions of the focus of eight zones for $H\gamma$, the wave-length for which the system was computed, and for the mean of λ 4250, 4340, 4440, and 4550, the range of spectrum used here in velocity determinations. *E*, Fig. 3, is the curve for $H\gamma$ of the objective alone; *C* is the curve for $H\gamma$ of objective and corrector. *D* is the curve for λ 4250 to λ 4550 of the objective alone; *B* is the curve for λ 4250 to λ 4550 of the objective and corrector.

A comparison of curves *D* and *E* with *F* shows in a striking manner the chromatic differences of spherical aberration in the objective when used with photographic light. If we leave out of account or allow for the deviations in the central zones, we see that the focus of the outer is about 1.8 mm longer than the focus for the central

zones, a figure that agrees almost exactly with the computed difference as furnished me by Professor Hastings. A comparison of curves A, B, and C with D and E shows that this difference, instead of being removed or diminished by the introduction of the correcting-lens, has on the contrary been increased by about 0.6 mm, so that the difference in focus between outer and central zones is now about 2.5 mm, which, as before stated, will give a confusion disk nearly 2″ in diameter. I wish to point out, before leaving these curves, how the form of the curve is maintained throughout from F up to A except that the axis of the curve is inclined downward by the chromatic differences in the photographic region, and further tilted by the introduction of the correcting-lens. To show this I have dotted in approximate positions of such axes in the curves E to A to correspond with the horizontal axis in F. It will be noticed that the irregularities in the visual curve are continued throughout, but in an intensified form, as is to be expected when it is considered that the objective was computed and figured for visual work, and its use in the photographic region with an auxiliary corrector was only a secondary consideration.

I see no reason to doubt, however, if sufficient positive aberration were left in the correcting-lens to compensate for the negative aberration introduced by the chromatic differences, that the performance of the system could be much improved, although it is not likely, from the magnifying of the unavoidable zonal aberrations, that it would equal its visual quality. If the curve A, Fig. 3, representing the present condition of the system, could be tilted through the angle between the horizontal and dotted lines, by such a change in the correcting-lens, the resulting confusion disk would certainly have a diameter less than half its present magnitude, while the percentage of the incident star light transmitted by the slit would be considerably increased, probably doubled, with a proportionate diminution of the required exposure times for stellar spectra.

Such an improvement would be well worth considerable effort, and I have been in communication with the Brashear Company and with Professor Hastings to that end. With their well-known willingness, I may even say anxiety, to produce the highest quality of optical work and to make any improvements that may be suggested

to them, the Brashear Company are undertaking to make a new correcting-lens to computations by Professor Hastings, to whom I I am very much indebted for criticisms and suggestions on the present paper. I may say that Professor Hastings finds a very marked agreement between his computed data of the objective, color-curves, and chromatic differences, and my observations. He explains the failure of the correcting-lens to compensate for the chromatic differences of focus, which it was computed to do, by the fact that this lens has to correct the errors of an objective of nearly fifty times the area, that the small departures of the wave-surfaces from a true sphere have grown enormously when these surfaces have contracted to one-fiftieth their original area, and that a very perfect correction by spherical surfaces can hardly be hoped for. He thinks, however, that considerable improvement can be effected, and I have no doubt myself that he and the Brashear Company can do much better than he says when they have quantitative values of the existing aberrations.

The reason for publishing this paper in its present incomplete form, before the new correcting-lens is ready, is to bring before stellar spectroscopists the important matter of the size and character of the star image given by their telescopes. I have gone fully into the details of the investigation and explained the difficulties that arose with the means of overcoming them, in order to smooth the way for similar investigations into the character of the star image given by other systems of objective and correcting-lens. It seems to me extremely probable that, in the major part if not all of the telescopes employed in spectrographic work, aberrations of the same or a similar nature are present. If a correcting-lens computed to compensate for the chromatic difference fails in one case, it is possible, even probable, that it may fail in others. Another basis for this belief is a comparison of the relative exposure times required for different installations taking into account size of object-glass, slit-width, and dispersion of the spectrograph. I am well aware that such a comparison must necessarily be incomplete, and the results reached subject to an uncertainty, say, of 25 per cent., owing to the difficulty of comparing different installations under different conditions of seeing, etc. We have already seen how important a part is played by atmospheric disturbances in enlarging the star image so that the linear

diameter of the image increases nearly in proportion with the focal length, and therefore approximately, as the ratio of aperture to focal length does not vary much in large instruments, with the diameter of the object-glass. Consequently, the effective value of increase of aperture is not proportional to the increase of area, but more nearly to the increase of diameter, which was accordingly used in the comparison. So far as regards the relative dispersion of different instruments, the exposure time was taken as directly proportional to the linear dispersion, presuming the same height of spectrum in each case. No account was taken of the difference in the loss due to absorption and reflection in the prism-train, although this may be quite important in some cases. The exposure time required was taken as inversely proportional to the slit-width, and this, as one of the experiments detailed above shows, is probably nearly in accordance with the facts. In the following Table V, data of the various equipments which are and have been used in radial velocity work, so far as they were available to the writer, appear, but these data are incomplete and may in some cases be in error, although probably not to a marked degree.

TABLE V

COMPARISON OF EFFICIENCIES OF INSTALLATIONS

Equipment	Diameter of Objective, inches	Ratio of Diameters	Ratio of Areas	Linear Dispersion, mm per Tenth Meter	Slit-Width, mm.	Theoretical Exposure	Actual Exposure Required		
							β Ophiuchi	*γ Aquilae*	*α Boötis*
Ottawa	15	1	1	18.6	0.025	1	50m	60m	6m
Yerkes	40	2.67	7.1	10.8	.038	0.42	75	115	15
Lick	36	2.4	5.76	12.5	.025	0.62	25 ?	25 ?	4 ?
Lowell	24	1.6	2.56	11.4	.025	1.02	120	120	20 ?
Newall	25	1.67	2.78	14.6	.025	0.76	70	75	15
Bonn	12	0.8	0.64	15.2	.020	1.91	75	75	15
Pulkowa	30	2.0	4.0	13.0	.020	0.89	65 ?	65	15
Lord	12½	0.83	0.69	18.6	.025	1.20	60 ?	60 ?	4

The above comparison shows that the Lick, Bonn, and Lord equipments *in practice* approach more nearly the theoretical efficiency than the Ottawa, but the Yerkes, Lowell, Newall, and Pulkowa depart farther from it.

There seems therefore reasonable ground for believing that considerable improvement in the efficiency, and considerable increase in the range of the majority of spectrographic equipments can be attained by looking into the character of the star image given by the condensing system. Although the exact effect of atmospheric disturbances on the effective diameter of the star image is difficult of determination, I feel satisfied, if I can obtain a correcting-lens that will give a star image reasonably free from aberration, that the exposure times required here can be very materially reduced, I hope by 50 per cent., and I see no reason why a similar or even greater improvement could not be effected in some of the other equipments.

I acknowledge with pleasure my indebtedness to Dr. W. F. King, the Director of the Observatory, for help and encouragement in the prosecution of the work, and to Mr. W. E. Harper for making duplicate measures for comparison purposes on some of the test plates.

Dominion Observatory, Ottawa
January, 1907

ON A NEBULOUS GROUNDWORK IN THE CONSTELLATION *TAURUS*

By E. E. BARNARD

I have elsewhere at various times called attention to the connection of nebulosities with some of the vacant regions of the sky. The finest example of this remarkable and suggestive peculiarity is shown in the great nebula of ρ *Ophiuchi*. In connection with some of these vacant regions I have remarked on the singular fact that in some cases—especially in the regions of θ and ρ *Ophiuchi*—these vacancies are vacancies only in the apparent absence of stars, for they are really often filled with a luminous veiling in which darker perforations occur.

The extraordinary vacant lanes among the Milky Way stars, in *Ophiuchus* and elsewhere, have often suggested that they are not only devoid of stars, but that they are darker than the immediate sky. In some cases there has been a suspicion that this was a matter of contrast, and that, if the remaining stars were removed, the lanes would also disappear. While this might be true in some cases, there are others where the appearance is strictly conclusive that the vacancies are not only due to the absence of stars, but that the channels are in a bed-work or nebulous substratum, and that, if the stars were removed, the lanes would still exist.

It will be seen that much importance depends upon whether these lanes are subjective—due to the scarcity of stars alone—or whether they reveal to us a nebulous substratum in certain parts of the sky,

In some of my early photographs north and east of the *Pleiades* the plates showed the existence of peculiar lanes far to the east of the cluster. Opportunity did not offer itself until the past winter to investigate their peculiarities by photography.

In the first part of January of this year I made several long exposures which covered the region in question. The result is very striking, and I believe of great importance; for the plates show that these lanes are undoubtedly in a substratum of some kind, as well as among the stars themselves.

The dying-out of nebulae—since it does not seem any longer necessary to use these vast bodies of gaseous matter for the making of suns—is a probability fully as warranted as the belief and certainty that the stars must die out. What would be the condition of a nebula that no longer emitted light, is a question; but as this light in all probability is not the product of heat or combustion in the ordinary sense, it is likely that we should simply have a dark nebula which would not be visible in the blackness of space unless its presence were made known by its absorption of the light of the stars beyond it—if this absorption were sufficient to be effective.

We have rather looked upon the nebulae as transparent bodies, like the comets; but there are no observations to warrant this idea, since in no case do we know that a nebula is on this side of the stars or beyond. True, there are cases where a star is palpably involved and seen through at least a portion of the nebula. There is nothing, however, to show whether the light of such a star has not been very greatly reduced by the interposition of the nebulous matter, and whether a much smaller star would not have been entirely invisible through the veiling of nebulosity.

This idea of the absorption of the light of the stars by a dead nebula or other absorbing matter has been used by some astronomers as an explanation of the dark or starless regions of the sky. Though this has not in general appealed to me as the true explanation—an apparently simpler one being that there are perhaps no stars at these places—there is yet considerable to commend it in some of the photographs.

The immediate neighborhood of the great nebula of ρ *Ophiuchi*—as if the outer boundaries of it were devoid of light—is a good example of an apparently absorbing medium by the dying-out of the outer portions of the nebula.

From their appearance in the sky, I believe the nebulae in general are transparent, yet there are some cases, especially among the planetary nebulae, where the appearance is quite otherwise.

The beautiful veil of nebulosity extending from the star ν *Scorpii* strongly gives the impression of a dulling of the light of the stars in that direction. To all appearances fewer stars seem to be within the boundaries of this nebula, as if the fainter stars were

blotted out and the brighter ones diminished in luster. If this is not a delusion of some kind, it would appear that this nebula is between us and the groundwork of small stars here.

The connection of nebulosities with vacancies, and the apparent mingling of the outer portion of the nebula with the darkness of the sky, as if that darkness were something really tangible, as suggested in the case of the nebula of ρ *Ophiuchi*, is an extremely important feature, from which I believe there will some day develop facts of the greatest importance in explaining the real structure of the heavens. It would therefore be a very valuable work to locate all these regions and to secure the best long-exposure photographs of them. In this way a consistent study of their peculiarities may be made by those interested in their nature. Of late years I have endeavored to find as many of these places as possible. This has resulted in the development of the extraordinary regions of *Ophiuchus* and *Scorpio*, where these singular features are perhaps best shown.

It was in the order of this investigation that I made the photographs given in the present reproductions.

The region here shown is extraordinary. The narrow vacant lanes are as singular examples of the peculiarities I have mentioned as any that I know of, and they show perhaps even better the fact that the lanes actually exist in the sky independent of the stars. Besides the lanes referred to, the photographs show a large nebula apparently in a hole almost devoid of stars, from which one of the lanes runs away to the southeast for several degrees.

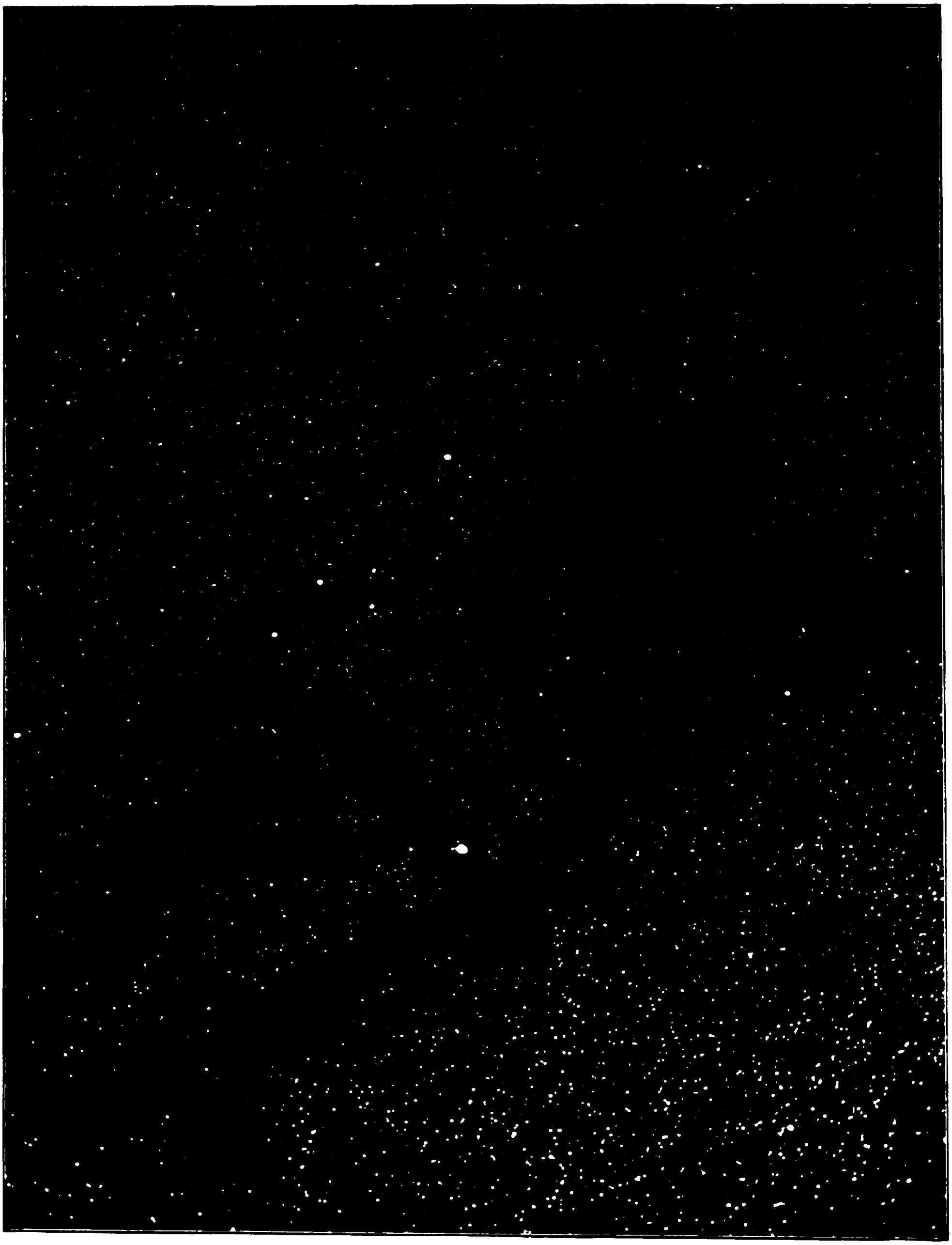
The pictures seem to show that the brighter part of this nebula is only a small portion of it, and that the nebula is feebly luminous over most of the vacancy; a longer exposure will perhaps prove this to be true. The feebler portions of the nebula would almost suggest the idea that a large nebula exists here, but that the major portion of it is dead or non-luminous, and that it actually causes the apparent vacancy by cutting out the light from the stars, while the few stars visible are perhaps on this side of the nebula. I give this simply as what the picture would suggest to one, and not as what may really be the truth.

On the original negative with the 10-inch lens, in the brightest part of the nebula there is a perfectly circular disk slightly brighter

PLATE XI

N

E



W

VACANCY AND NEBULA IN *TAURUS*

10-inch Lens. 1907, January 9, 12^h 27^m to 17^h 55^m G. M. T. Enlarged 1.6 times. Scale: $1° = 35$ mm.

than the rest, as if a large planetary nebula were involved. This is so sharply defined, however, that I doubt its reality. For verificaton it must wait until the end of the year when more photographs can be secured. The scale of the 6-inch plate is too small to aid in the verification.

I have been slow in accepting the idea of an obscuring body to account for these vacancies; yet this particular case almost forces the idea upon one as a fact. There are portions of this apparent vacancy that are certainly darker than the adjacent sky. There is no question that this is real, and not a subjective effect, because, as will be seen later, the dark lane running from it shows similar markings, which are certainly darker than the sky, and this is the case with the lane itself. If we examine this lane, we shall find that it comes to an abrupt stop at a point in

$$\alpha = 4^h\ 16^m, \quad \delta = +26.^\circ 0.$$

But there is no change in the sky here; it is uniform with the rest of the sky among the stars, and is decidedly brighter than the lane. A half degree farther on and the lane reappears in a broken and straggling manner.

These lanes are best shown in the small picture (Plate XII) taken with the 6¼-inch lens. This plate is simply to show the extent and general appearance of this remarkable system of lanes. Other photographs which I have made, especially those on February 7, 1907, show that this plate includes the full extent of these vacancies, the rest of the sky nearby being free from them. The region is comprised between the limits

$$\alpha = 4^h\ 0^m \text{ to } 4^h\ 54^m \text{ and } \delta = +24^\circ \text{ to } +28\tfrac{1}{2}^\circ.$$

There is a larger vacancy, similar to the one in which the large nebula occurs, in about the position

$$\alpha = 4^h\ 31^m, \delta = +25.^\circ 7.$$

It shows considerable dark detail. In it is a small nebula which looks as if it might be only the brighter portion of a larger nebula filling or partly filling the vacancy. The position of this nebula is:

$$\alpha = 4^h\ 31^m, \quad \delta = +25^\circ\ 25'.$$

Perhaps the most singular of these lanes is the one that straggles along almost east and west

$$\text{from } \alpha = 4^{\text{h}}\ 0^{\text{m}},\quad \delta = +24\tfrac{1}{2}^{\circ} \text{ to } \alpha = 4^{\text{h}}\ 26^{\text{m}},\quad \delta = +24^{\circ}$$

near the lower part of Plate XII. It is quite definite. A long strip of it, from $4^{\text{h}}\ 8^{\text{m}}$, $+25^{\circ}$, to $4^{\text{h}}\ 16^{\text{m}}$, $+24\tfrac{1}{2}^{\circ}$, is singularly well defined, especially at the east end, where it abruptly stops and disappears for a space of nearly a half-degree, after which it reappears in a very broken manner farther on, until it enters a considerable region almost devoid of stars, where it appears as a series of irregular black spots. Where this lane is interrupted it shows still more clearly that the lane itself is darker than the sky. These lanes and holes, therefore, would still be seen if the stars were all removed. This of course implies that a substratum of some kind exists all over this part of the sky, and that these are lanes and holes or rifts in it. A very singular feature in this connection is that the stars also are absent in general from these lanes.

Now, the question is: Does this substratum consist of nebulosity, which would be the natural conclusion, or is it something else, as to the nature of which we do not yet know anything?

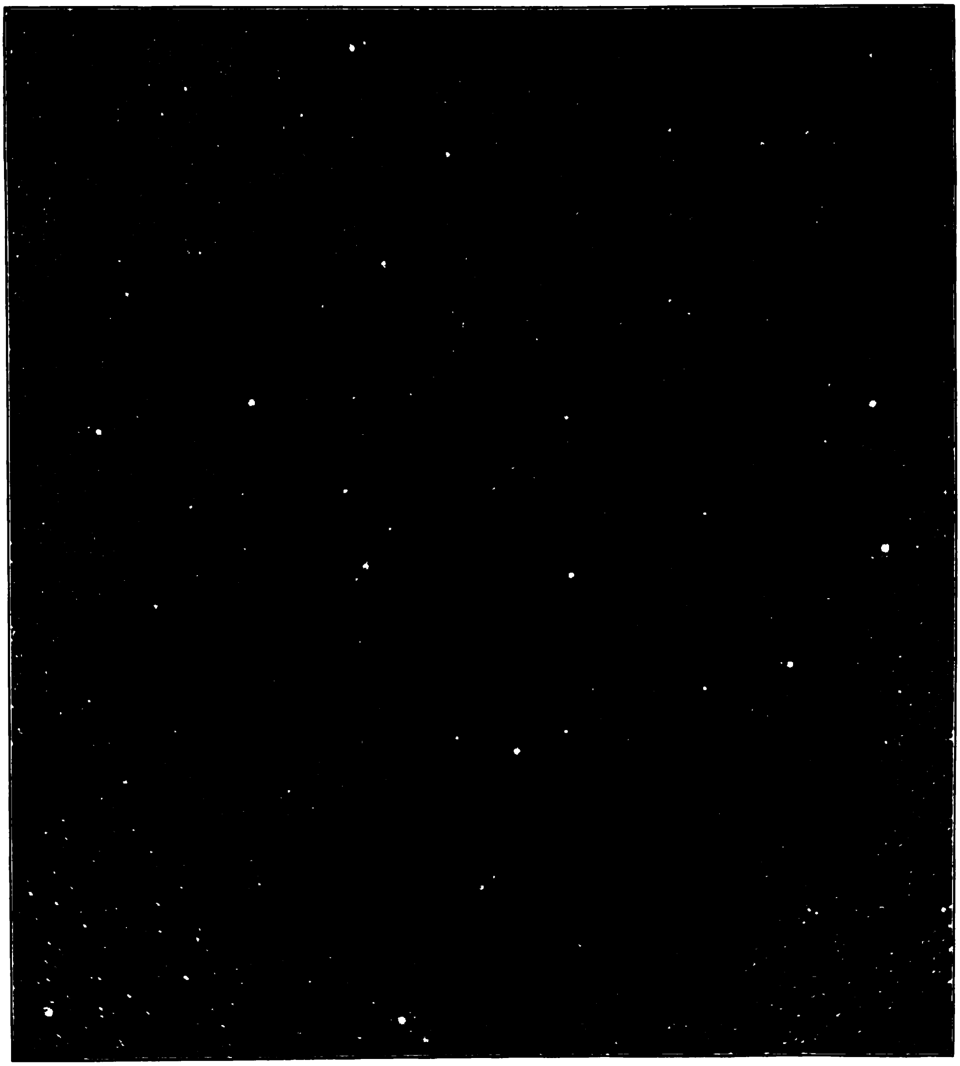
It almost seems to me that we are here brought face to face with a phenomenon that may not be explained with our present ideas of the general make-up of the heavens. The conditions exemplified here also exist in the region of the dark holes and lanes near θ and ρ *Ophiuchi*, but they perhaps do not so clearly show the presence of a groundwork other than that due to stars, as the present case does, although the phenomena there are more remarkable, or rather they are more spectacular.

Among the most surprising things in connection with these nebula-filled holes are the vacant lanes that so frequently run from them for great distances. These lanes undoubtedly have had something to do with the formation of the holes and with the nebulae in them. The idea of the dying-out of a nebula, previously mentioned as a possible explanation of the vacancy, is not strengthened by the presence of the lanes, for we do not find in general any great streams of nebulosity extending away from the nebulae. A possible exception is the case of the great curved stream of nebulosity which seems to

PLATE XII

N

E



W

REGION OF VACANCIES IN *TAURUS*

6-inch Lens. 1907, January 9, $12^h\ 27^m$ to $17^h\ 55^m$ G. M. T. Not enlarged. Scale: $1° = 13$ mm.

emanate from the great nebula of *Orion*, and which stretches away for many degrees northward over a large part of the constellation.

The position of the center of the large plate (Plate XI) is

$$\alpha=4^h\,11^m\!.5\,,\quad \delta=+27^\circ\!.9\,;$$

that of the small plate (Plate XII) is

$$\alpha=4^h\,15^m,\quad \delta=+27^\circ\!.6\,.$$

The brightest portion of the large nebula is at

$$\alpha=4^h\,10^m,\quad \delta=+27^\circ\,55'.$$

As a ready means of identification, the following refer to the small plate.

The two stars near the south edge are

B. D.	$+22^\circ\!.696$,	$4^m\!.5$,	$\alpha=4^h\,17^m\,38^s\!.6$,	$\delta=+22^\circ\,28'\!.3$
	$+22.699$,	6.2,	$\alpha=4\ \ 18\ \ 37.5$,	$\delta=+22\ \ 39.3$.

The star at the southeast corner of the plate is

B. D. $+22^\circ\!.712$, $7^m\!.4$, $\alpha=4^h\,26^m\,5^s\!.3$, $\delta=+22^\circ\,22'\!.5$.

The two conspicuous stars near the right-hand edge are

B. D.	$+27^\circ\!.633$,	$5^m\!.1$.	$\alpha=3^h\,57^m\,43^s\!.7$,	$\delta=+27^\circ\,13'\!.6$;
	$+28.619$,	5.4,	$\alpha=3\ \ 58\ \ 3.9$,	$\delta=+28\ \ 36.8$.

The two close to the upper edge are:

B. D.	$+32^\circ\!.805$,	$7^m\!.3$,	$\alpha=4^h\,20^m\,46^s\!.3$,	$\delta=+32^\circ\,11'\!.8$;
	$+32.806$,	6.5,	$\alpha=4\ \ 21\ \ 20.3$,	$\delta=+32\ \ 8.0$.

In the large plate, the conspicuous star below the middle is

B. D. $+27^\circ\!.655$, $\alpha=4^h\,11^m\,27^s\!.0$, $\delta=+27^\circ\,0'\!.1$.

The less conspicuous star above the middle is

B. D. $+28^\circ\!.642$, $\alpha=4^h\,11^m\,37^s\!.9$, $\delta=+28^\circ\,32'\!.9$.

These plates contain the trail of an asteroid, which also shows on the plates of January 5.

Following are the approximate positions of this object, obtained from the *B.D.* charts. They, like all the positions in this paper, refer to the epoch 1855.0.

1907, January 5, $14^h\,23^m$	G. M. T.	$\alpha=4^h\,13^m\!.5$,	$\delta=+28^\circ\,35'$;
9, 15 11	G. M. T.	$\alpha=4\ \ 12.5$,	$\delta=+28\ \ 24$.

The trails are very strong, showing the asteroid to be a bright one. The trail will be found on the plate made with the 10-inch (Plate XI) at a distance from top, 66 mm (2.6 inches), from the left, 56 mm (2.2 inches)

A much brighter asteroid is shown on the plate of this region on

$$1907, \text{ February } 7, 14^{h} 40^{m} \text{ G. M. T.}, \alpha=4^{h} 17^{m}, \quad \delta=+22^{\circ} 55'.$$

Should it be desired, more accurate places can be obtained by measuring the plates.

On the photographs of February 7 the sixth-magnitude star *B.D.* $+22^{\circ}.699$ has a thin nebulous wisp running from it to the northeast some 6′ or 8′.

The position of this star from the *B. D.* is

$$6^{m}.2, \quad \alpha=4^{h} 18^{m} 37^{s}.5, \quad \delta=+22^{\circ} 39'.3.$$

The star *B.D.* $+28^{\circ}.645$, $9^{m}.1$, $\alpha=4^{h} 12^{m} 55^{s}.8$, $\delta=+28^{\circ} 6'.1$, has two nebulous tails. The brightest of these is in the direction of position angle 330°. The other one is at right angles to this, in position angle 60°. These comet-like tails are about 6′ or 7′ long. They are shown on the larger reproduction, where the star will be found 76 mm (3.0 inches) from the top of plate and 51 mm (2.0 inches) from the left-hand side.

The large scale photograph (Plate XI) was made with the 10-inch Brashear lens of the Bruce telescope, and the smaller one (Plate XII) was made with the $6\frac{1}{4}$-inch Voigtländer of the same instrument. The exposures were from $6^{h} 27^{m}$ to $11^{h} 55^{m}$, Central Standard Time, 1907, January 9, the duration of exposure being $5^{h} 28^{m}$.

The defect at the left-hand side of the bright star *B. D.* $+27^{\circ}.655$ on the large plate was due to trouble with the driving-clock, caused by the cold.

In the smaller scale picture there are certain inequalities of illumination due to the reproduction, which will deceive no one.

NOTE ADDED TO PROOFSHEETS

In observing Swift's periodic comet on 1892 January 18 with the 12-inch refractor of the Lick Observatory, I found a ***very, very*** faint nebula in the position

$$1855.0, \quad \alpha=4^{h} 31^{m} 9^{s}\pm, \quad \delta=+25^{\circ} 26'.3.$$

I have a note which says:

"Both the nebula and the comet are seen on a nebulous background—a vast nebula. The position [of the small nebula] cannot be far from the brightest portion of this great nebula. The small nebula is most excessively difficult."

This description agrees with the small nebula described previously in the position $\alpha = 4^h\ 31^m$, $\delta = +25^\circ\ 25'$. It also shows that the great nebulosity is easily visible in a 12-inch telescope.

YERKES OBSERVATORY
March 9, 1907

Minor Contributions and Notes

AGNES MARY CLERKE

Agnes Mary Clerke was born on February 10, 1842, at Skibbereen, a small country town in a remote part of the County Cork. Her father was John William Clerke, and her mother was a sister of the late Lord Justice Deasy.

Constitutionally delicate, Agnes Clerke from her earliest years, as so often is to be noticed in cases of frail health, found her chief delight in literary study and in music. From quiet talks often enjoyed with her in her later life it was clear that her thoughtfulness and her liking for probing difficult problems must have developed early.

In 1861 the Clerke family moved to Dublin, and in 1863 to Queenstown. The winters of 1867 and of 1868 were spent at Rome, that of 1871 at Naples, and the next five winters at Florence; the summers of 1874–76 being passed at the Bagni di Lucca. The sisters, Agnes and Ellen, both profited to the full from this sojourn in Italy, as their subsequent writings show; but Agnes at Florence worked specially hard, reading constantly in the public library there, and always, I believe, with one great object before her.

It is a question of much interest to examine into the early leanings and aspirations of those who distinguish themselves later, and Agnes Clerke early determined her life-work. Before leaving Skibbereen, at about the age of fifteen, she had clearly before her the intention of writing a history of astronomy, and it is thought had actually written a few chapters. Her first article in the *Edinburgh Review* is in harmony with the above facts.

Agnes Clerke's literary life may be said to have begun in 1877 with the acceptance of her article "Copernicus in Italy," by Henry Reeve, then editor of the *Edinburgh Review*, who recognized the value of his new contributor and kept her at work. The number of her contributions to the *Edinburgh* is fifty-five; and they are all of the highest order.

PLATE XIII

AGNES MARY CLERKE

Agnes Clerke, with her family, returned to England in 1877, and settled in London. With the publication of the *History of Astronomy* in 1885 may be said to have begun her astronomical life. She read systematically, and cultivated personal relations with a wide circle of astronomical workers, in person or by correspondence. I consider that these relations had much to do with the success of her work. Her sympathies were so keen, her interest so warm, her longing for further truth so intense, that everyone liked to offer her all he could.

In 1890 appeared her second book, *The System of the Stars*. The progress of science and the growth of its literature during the last quarter of a century have been so enormous that a new order of worker is imperatively called for; and Agnes Clerke was an admirable example of such a worker, devoting herself to astronomy, which is at once the oldest and, in its new developments, the youngest of the sciences—the science which Poincaré has lately so eloquently declared to have given the conception of *law* to all the others. The mission of these special workers is to collect, collate, correlate, and digest the mass of observations and papers; to chronicle, in short, on one hand, and, on the other, to discuss and suggest, and to expound; that is, to prepare material for experts, to inform and interest the general public. There is urgent need of a better-educated public opinion in this country. That such a mission may be a splendid and fruitful one has been shown by Agnes Clerke; what careful preparation it requires, and how much it demands of those who would enter upon it, her career also shows.

The immense increase in astronomical literature is hardly realized except by those engaged in dealing with it. To give but one instance, *The Annual Index of Astronomical Literature* for 1905, published under the auspices of the *Astronomische Gesellschaft*, contains over two thousand references collated from three hundred separate publications.

The strain of such work as I am indicating is great indeed, involving as it should the power of holding loose in the mind, so to speak, an immense mass of facts, and also a power of rapidly associating or dissociating them as work and discovery may suggest. In one of her latest works, *Modern Cosmogonies*, Agnes Clerke herself dwelt upon this strain. "Year by year," she says (p. 160), "details accumulate,

and the strain of keeping them under mental command becomes heavier." Pathetic words, written almost in blood! For, not long before had been published her last large work, *Problems in Astrophysics*, a work she feared she could not live to complete; a work which she was able to toil at for only half an hour at a time.

All through her life Agnes Clerke was a student. Lectures and Friday evening dicourses at the Royal Institution, which bore upon her work, she was careful to attend. A three-months' visit to Sir David and Lady Gill at the Cape in 1888 gave her some observatory opportunities which increased her power of clearly realizing the records of observatory and laboratory work.

She was awarded by the Royal Institution, in 1892, the Actonian Prize of 100 guineas for her works on astronomy; and in 1903, she received the distinction of being elected an honorary member of the Royal Astronomical Society—an honor and title held previously only by Mrs. Somerville, Caroline Herschel, and Ann Sheepshanks. A frequent attendant at the meetings of both the Royal Astronomical Society and the British Astronomical Association, she was always an interested one. Occasionally she spoke; but she had no liking for speaking in public nor indeed was she well suited for it.

A complete list of Agnes Clerke's papers it would be difficult to compile. They were in truth innumerable. Her articles on astronomers for the *Dictionary of National Biography*, articles for the *Encyclopaedia Britannica* and for other encyclopaedias, were many, and all of them were models of painstaking inquiry and of clear, concise statement. The more important of these are of lasting interest and value.

Her larger works are: *History of Astronomy in the Nineteenth Century* (four editions); *The System of the Stars* (two editions); *Familiar Studies in Homer; The Herschels and Modern Astronomy; Concise History of Astronomy; Modern Cosmogonies; Problems in Astrophysics*.

I venture to think that the *History of Astronomy in the Nineteenth Century* is the most important of her works. It is admirable in its completeness of references, its wide inclusiveness, and in its lucidity. It deserves to live, and it assuredly will live—the invaluable continuation of Grant's fine work. *The System of the Stars* and the *Problems*

in Astrophysics are works of a different order. Treasuries of knowledge and of suggestion they certainly are.

The *Homeric Studies,* except in one chapter, are not specially astronomical, but they are evidence of breadth of culture and of wide intellectual interest, and are full of delightful touches of wit and of humor.

It seems to me a mistake to regard Agnes Clerke's smaller works as of less importance than her larger ones. I have said that I consider the *History* her greatest work. But in some respects I venture to think that her greatest achievement is *Modern Cosmogonies.* I claim for this book that it is not only a history, but a work of philosophical thinking and of imaginative insight of a very high order. Its small size is an accident. It is a work essentially great. In these brilliant sketches Agnes Clerke's style is at its best. But the writing in *Modern Cosmogonies,* good as it is, is a small matter compared with the masterly grasp of, I may say, all things and of their interrelations which the work reveals. And where else is shown, in recent philosophical writing, such vision and faculty divine for seizing and pointing out the reasonable spiritual clues, set in what we call Nature—clues helping to sustainment of soul in the midst of the majestic mysteries surrounding us?

No sketch of Agnes Clerke would be complete without reference to her love of music. To her music was in the highest sense of the term a recreation. She turned to it for very life. Her piano-playing was truly musicianly, and her repertory was large. Perhaps her playing was at its best in rendering Chopin. As an accompanist she excelled. Her teachers were, in Dublin, Miss Flynn; in Florence, Buonamici.

Remarkable as were the intellectual powers of Agnes Clerke, her moral endowments were equally so. It was a question we frequently debated—the influence of character on work; and as I write the memory of certain talks is hauntingly present. As is the heart, is the work. The best work is and must be associated with lofty character. It was so with Agnes Clerke. No purer, loftier, and yet more sweetly unselfish and human soul has lived. She was so incapable of meanness that she even incurred danger as a historian in crediting too readily all workers with her own high ideals.

As a friend and companion she was faithful and true, and full of charm; and without her the world to those who had her friendship seems darkened and empty.

But her mission, I believe, had been fulfilled. For twenty years she had been to modern astronomy an admirable historian, and had kept before working astronomers clear charts, so to speak, of what was being done, and of what should and might be done. In so doing she rendered splendid service, and inaugurated a kind of work which must be more and more needed—a kind of work which not only advances astronomy, but promotes a universal brotherhood and co-operation, golden indeed.

Agnes Clerke's death comes as a shock to many. A cold—I fear not sufficiently nursed at first—led to pneumonia and complications, and, in spite of all that devoted love and skill could do, she passed gently to the next life, peaceful and fully conscious almost to the last, on the morning of January 20, 1907.

MARGARET LINDSAY HUGGINS

FEBRUARY, 1907

REMARKS ON HULL'S OBSERVATIONS OF THE DOPPLER EFFECT IN CANAL RAYS

Professor G. F. Hull recently published in this *Journal* an investigation of the influence of electrical fields upon spectral lines.[1] In his paper he also communicates his observations on the spectra of the canal rays, which led him in several points to negative results, where I had obtained positive results.

In the summer of 1906 I made a brief announcement that I had observed a slight polarization in the line-spectrum of rapid canal rays.[2] Mr. Hull does not find this polarization. I shall later report in an extensive paper as to the phenomenon which I announced.

I have already published *in extenso* my observations upon the Doppler effect on canal rays. Mr. Hull succeeded as I did in finding the Doppler effect in the case of hydrogen.[3] I further demonstrated

[1] *Astrophysical Journal*, **25**, 1, 1907.

[2] *Verhandlungen der deutschen physikalischen Gesellschaft*, **8**, 105, 1906.

[3] *Annalen der Physik*, **21**, 401, 1906; *Astrophysical Journal*, **25**, 23, 1907.

the presence of the Doppler effect for a number of mercury lines;[1] but Mr. Hull does not find the effect in the case of mercury or helium.

Regarding this difference it should be first remarked that the problem of demonstrating the Doppler effect in canal rays is rather electrical than spectroscopic. It is not necessary to employ a spectrograph of a large dispersion; it is sufficient, and indeed preferable, to use a grating or prismatic instrument of moderate dispersion and great light-power, since the Doppler effect, if it occurs at all, is always of considerable magnitude in the case of canal rays. But, on the other hand, it is necessary, to satisfy the condition that the cathode-drop producing the canal rays shall not be less than a certain limiting value.

When the line-spectrum of the canal rays has been photographed in the direction of translation, the experimenters have found, in all cases yet investigated,[2] an undisplaced ("stationary") line, and beside it a band of displaced ("movable") lines; while between the line at rest and the band of moving lines the density of the photographic plate shows with sufficient dispersion a minimum of intensity.

It is singular that Mr. Hull has overlooked, or at least not described in his paper, these two important things—the simultaneous appearance of stationary and movable intensity, and the appearance of the minimum of intensity. He does not seek in his spectrograms for the movable line beside the stationary line, which always appears with more or less intensity, but he compares the position of the maximum of intensity of the line on the photograph of the canal rays with the position of the maximum on the photograph of the positive column of an ordinary mercury tube or helium tube. Inasmuch as the stationary intensity is greater than the movable intensity, even for large velocities, certainly for mercury, and probably also for helium, it would appear that Mr. Hull had made his comparison between the stationary line in the spectrum of the canal rays, and the stationary

[1] J. Stark, W. Hermann und S. Kinoshita, *Annalen der Physik*, **21**, 462, 1906.

[2] Hydrogen: J. Stark, *loc. cit.;* B. Strasser and M. Wien, *Physikalische Zeitschrift*, **7**, 744, 1906; F. Paschen, *ibid.*, **7**, 924, 1906.

Mercury: Stark, Hermann, and Kinoshita, *loc. cit.*

Nitrogen: W. Hermann, *Physikalische Zeitschrift*, **7**, 567, 1906.

Hydrocarbon: S. Kinoshita, *ibid.*, **8**, 35, 1907.

lines of an ordinary Plücker tube, without noticing this fact. It can then be understood why he has not found the slightest displacement between the two stationary lines. He ought to re-examine his spectrograms of the canal rays and see if he cannot detect beside the stationary lines some very faint lines (diffuse bands), the movable lines, or the Doppler effect.

If he should thus find that there was certainly no Doppler effect in his canal rays in mecury vapor, then this genuine negative result can be explained, for he does not seem to have satisfied the following important condition for the appearance of the Doppler effect.

If, during the greater portion of the exposure time, the cathode-drop is smaller than that which communicates to the canal rays just the velocities which correspond in the Doppler effect to the width of the intensity-minimum, then the displaced intensity is too slight, and hence no Doppler effect will appear, only the stationary or undisplaced line being observed. In order, therefore, to obtain the Doppler effect at all, the cathode-drop should not be allowed to fall below a certain minimum value during any considerable part of the whole exposure time.

I have further shown that the ratio of the displaced to the stationary intensity increases with the velocity of the canal rays, or hence with the amount of the cathode-drop. For mercury lines this ratio is very small for small values of the cathode-drop, whence the stationary intensity is large in proportion to the displaced intensity. A moderately long exposure for a relatively small cathode-drop can therefore probably bring out the stationary line in considerable intensity on the photographic plate, while the movable line (Doppler effect) remains invisible. For two reasons, therefore, careful attention must be given that the cathode-drop does not fall below a certain value during the whole of the exposure. There is therefore no value in establishing that the potential (cathode-drop) had a very high value during the exposure, if at the same time a precaution was not taken that it did not during the exposure sink below a certain value for some time. After I had recognized the relation between the velocity of translation and the intensity of radiation, I and my collaborators (Messrs. Hermann, Kinoshita, and Siegl) have in all photographs of the Doppler effect (in *H*, *Hg*, *Na*, *Ka*, *N*, and *C*), in the full apprecia-

tion of its necessity, attended to the fulfilment of this condition with painstaking care. In our papers we have always stated that during our exposures the cathode-drop was not allowed to fall below a definite and quite high value; and I may here add the remark that for us the problem of the proof of the Doppler effect in a line-spectrum has reduced itself to the satisfaction of that electrical condition. The spectroscopic technique may be regarded as secondary to the satisfaction of this condition. Mr. Hull does not anywhere mention in his paper that during exposure he did not permit the cathode-drop to fall below a certain value for some time. He does say in several places that the tension in his experiments with the induction coil reached a high value, and he also estimates with the aid of a spark-gap the tension at the canal-ray tube. A spark-gap, however, gives only a maximum value of the tension, and does not control the fall below a definite minimum value. Nothing is said in Mr. Hull's paper as to whether he has done this; and if a revision of his spectrograms should again lead to actually negative results, it is to be assumed that he has not satisfied that condition. I myself have made no investigations on helium, but for mercury lines at λλ 4358, 4078, 4047, 3663, 3655, 3650, 3341, 3131, 3125, 3021, 2967, and 2536 the Doppler effect in the canal rays has been unquestionably demonstrated by myself and Messrs. Hermann and Kinoshita. We have three photographs taken with the concave grating in the first, second, and third orders, and six plates taken with the prism spectrograph, on all of which the Doppler effect in the mercury spectrum is distinctly visible and measurable. Herr A. Krüss, of Hamburg (Adolphusbrücke), sells a reproduction of one of our plates, which, although inferior to the original negative, distinctly shows the Doppler effect for the lines λλ 4358, 4046, 3663, 3655, 3650, 3341, 3131, 3125, 2967, 2536. Herr Rau will publish an investigation on the Doppler effect for canal rays in helium.

The investigation of the Doppler effect in canal rays gives us information as to the carriers of the line-spectra. To me, however, another result seems to be more important, namely, that a relation exists between the intensity of radiation and the velocity of translation, the displaced intensity increasing first slowly and then rapidly with the square of the velocity of translation. The investigations by my-

self for mercury show that for a large velocity the displaced intensity is photographically demonstrable. Hence, if Mr. Hull's results should actually turn out to be negative on a revision of his spectrograms, this probably could be interpreted in a positive sense as showing that for a low velocity of the canal rays the displaced intensity is very small in comparison with the stationary intensity.

J. STARK

HANNOVER
February, 1907

DOPPLER EFFECTS AND POLARIZATION IN CANAL RAYS

With regard to Professor Stark's suggestions I wish to make the following comments:

1. In looking for polarization in the light from the hydrogen canal stream I took the precautions to eliminate the effects due to the passage of the light through the strained glass wall of the tube and also due to reflection from the glass surfaces. When these precautions were taken, no polarization was observed even with a very sensitive detector. As far as I know, Professor Stark did not take these precautions.

2. Professor Stark remarks that "the problem of demonstrating the Doppler effect in canal rays is electrical rather than spectroscopic." In the case of hydrogen, however, there is neither an electrical nor a spectroscopic difficulty. The effect is very easily obtained. But, owing to the fact that the spectroscopic apparatus at my disposal was not such as I should care to use for an *accurate measurement* of the displacement of the hydrogen lines, I did not include in my paper the observations made on the Doppler effect in that gas. I may state here, however, that some of my plates show the stationary line on the red edge of a uniform broad displaced band, others show the displaced band without the stationary line, and still others show the displaced band with the minimum of intensity between. But I should not like to attach any great importance to the last phenomenon; for it is extremely difficult to keep the potential and intensity of the discharge constant during the exposure. If these

vary, we should expect just such a phenomenon as we both have found.

3. Professor Stark suggests that "it is necessary to satisfy the conditions that the cathode-drop producing the canal rays shall not be less than a certain limiting value." The inference is that Professor Stark has found that limiting value, and that when he worked with canal rays with a cathode potential-drop less than that limiting value he found no Doppler effect. It certainly would have made for definiteness of our ideas if Professor Stark had given us the limiting values for hydrogen and mercury. But I am under the impression that the discrepancies in our results are not due to a lack of cathode-drop in my experiments. In the case of helium, for example, a potential of 20,000 volts between plates 10 or 15 cm apart ought to produce a sufficient cathode-drop. Nor are the discrepancies to be accounted for on the assumption that I have confined my attention to the "stationary" line. For in the case of mercury the plates show the various satellites as well as the line of strongest intensity. If there were a faint displaced component of this strong line, the satellites in all probability would be entirely obscured. But they are almost as clear in the canal stream as in the Plücker tube.

I am inclined to the view that the apparent absence of motion in the mercury and helium canal streams is due to other particles, possibly non-luminous, carrying the positive charges. From the ease with which hydrogen particles are set in motion, it looks as though they were those carriers. There is other experimental evidence in favor of this hypothesis.

In conclusion it should be noted that I used an induction coil as a potential source. This source, though more or less variable, is sufficient to give the Doppler effect in the canal stream of hydrogen, but no effect, according to my experiments, in the cases of mercury and helium.

G. F. Hull

Dartmouth College
March 4, 1907

THE SPECTRUM OF *MIRA CETI*

The spectrum of *Mira Ceti* reproduced in Plate XIV was directly enlarged 5.1 times from a spectrogram exposed January 11, 1907, in

the single-prism spectrograph. Below this is reproduced also a broadened enlargement of the same plate, which displays more clearly the nature of the star spectrum than does the narrow direct enlargement. The region of spectrum covered by the plate includes the four hydrogen lines, $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$, all of which are bright. It shows the increase in the intensity of these bright lines from the red to the violet end of the series. $H\delta$ is less intense on the plate than $H\gamma$, because the $H\delta$ star image was out of focus on the slit-plate and only a part of that light was allowed to enter the slit; the effect of this is also seen in the fading-away of the continuous spectrum between $H\gamma$ and $H\delta$.

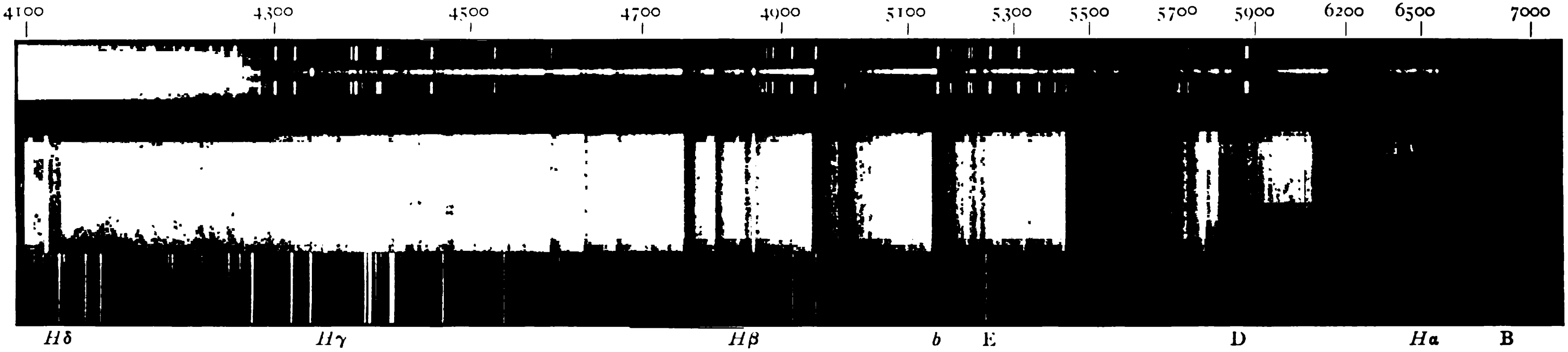
There are other points where the spectrum appears to contain bright lines, but at present it is not certain whether these are emission lines or only narrow sections of continuous spectrum unaffected by absorption. The series of absorption bands, which are sharp above and diffuse below, begins toward the violet at λ 4584, and possibly at λ 4463, and extends to the red end of the spectrum. The star spectrum stops so suddenly at λ 7040 as to leave little doubt that another of these bands begins at that point and outruns the sensitiveness of the plate into the red. The prominence of vanadium absorption in this star is indicated by the close agreement between the group of strong comparison lines at λ 4400 which are mostly due to vanadium, and the group of dark lines in the star.

Comparison of this plate with those made in 1902 by Dr. Stebbins, and published in *Lick Observatory Bulletin*, No. 41, shows that the series of dark bands, in 1902, extended farther into the violet to λ 4314, and that the bands were then more intense; and also that the bright hydrogen line $H\beta$ (and, by inference, $H\alpha$, likewise) was not so intense then as it was during the recent maximum of the star. The region about this line is covered by one of the dark bands, the density of which must have some effect on the brightness of the emission line.

V. M. SLIPHER

LOWELL OBSERVATORY
February 9, 1907

PLATE XIV



SPECTRUM OF *MIRA CETI*, JANUARY 11, 1907

The upper spectrum is a five-fold direct enlargement of the original negative; the lower one is a vertical enlargement of the same on a slightly smaller horizontal scale. The comparison lines are of *V Fe* and *Na*.

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME XXV | MAY 1907 | NUMBER 4

INVESTIGATIONS ON PLANE REFLECTION GRATINGS WITH REFERENCE TO THEIR USE IN THE ABSOLUTE DETERMINATION OF WAVE-LENGTHS[1]

By ERNST GIESING

I. INTRODUCTION AND DESCRIPTION OF THE GRATINGS

In 1894 A. A. Michelson succeeded in determining the wave-length of the red cadmium line at λ 6438 with his interferometer so accurately that the results of the separate experiments differed at most by 0.0066 tenth-meters, or 1 : 1,000,000 of the value.[2] Relatively to these absolute measurements, MM. Ch. Fabry and A. Perot,[3] by the same method and probably with the same accuracy, determined among others the wave-lengths of the mercury lines. They obtained the following values, reduced to 15° C. and 760 mm, which are important in connection wth the experiments to be described in this paper:

Hg Yellow lines	5790.659 5769.598
Green line	5460.7424
Indigo line	4358.343

[1] Translated from *Annalen der Physik*, **22**, 333, 1907. The paper was an abstract from the author's dissertation at Tübingen, 1906.

[2] "Détermination expérimentale de la valeur du mètre en longueurs d'ondes lumineuses," *Travaux et mémoires du Bureau international des poids et mesures*, **11**, 1895.

[3] "Détermination de nouveaux points de rep re dans le spectre," *Comptes Rendus*, **130**, 492, 1900.

They further determined the wave-lengths of D_1 to be 5895.932 for 15°, or 5895.960 for 20° C. The use of diffraction gratings for the absolute measurement of wave-lengths has been abandoned since these results appeared, aside from an investigation by Thalén,[1] which, according to the criticisms of H. Kayser[2] must be regarded as unsuccessful.

For purposes of comparison I give in the following summary the results for the sodium line D_1 obtained with gratings by different observers since 1886.

Observer	Maker of Grating	Material	Approximate Width of Grating in mm	No. of Rulings	D_1 for 20° C. and 760 mm
F. Kurlbaum*	Rutherfurd	Metal	43	29521	5895.86
(1886–87)	Rowland	Metal	42	23701	5895.98
L. Bell† (1885–87)	Rowland	Glass	30	12100	5896.18
	Rowland	Glass	30	8600	5896.23
	Rowland	Metal	100	29000	5896.15
	Rowland	Metal	100	40000	5896.17
R. Thalén (1898)	Rowland	Metal	51	29101	5895.976

* "Bestimmung der Wellenlänge Fraunhoferschе Linien," *Wied. Ann.*, **33**, 159, 381, 1888.

† "On the Absolute Wave-Length of Light," *Am. Jour. Sci.* (3) **33**, 167, 1887; **35**, 265, 1888.

A study of the investigations of these observers shows that the differences between their results are considerably larger than they ought to be, as inferred from the errors of the instruments and the observations. According to the view of Kayser, the reason for this lies solely in the inadequate determination of the constants of the gratings: the width of the grating divided by the number of rulings, without taking into account the undoubted lack of uniformity of the rulings on account of technical defects. At the end of his discussion Kayser thinks the conclusion is justified that "it is impossible to determine the wave-length with gratings to an accuracy of 0.1 Ångström units."

The causes for the lack of uniformity in the distance of two successive rulings are doubtless due to errors of the dividing engine; to variations of temperature during the process of ruling; to change

[1] "Sur la détermination absolue des longueurs d'onde de quelques raies du spectre solaire," *Nova acta Upsal.*, 1899.

[2] *Handbuch der Spektroskopie*, 1, 707.

or wear of the diamond, which surely can occur in ruling 29,000 or even 40,000 grooves, as in the case of the gratings used by Bell; and finally to the disturbance of a groove already ruled by the following one when the space is so small; for it seems inevitable that with from 300 to 700 grooves per millimeter the diamond will meet with less resistance on the side of the preceding groove than on the other side. The groove must to a certain degree injure the edge of the preceding groove, and itself also be injured. Finally, there might come into consideration a mechanical deformation of the substance of the grating in the course of time. (See papers by Bell and Thalén.)

If we share Kayser's view that all of the determinations above tabulated "were executed with about the same care, errors cannot be attributed to any of the observers, and they all refer to the same standards of length," then the question arises: If a grating is ruled under the most favorable conditions; in particular, if the sources of error just named are avoided as far as possible, will it then be possible to obtain a better agreement than heretofore between such gratings? And how will the values obtained compare with those of Michelson, and of Perot and Fabry? It must further be of importance and interest to discover as closely as possible the degree of error to be assigned to a grating. It will be necessary to show that the lack of uniformity of the space between the rulings may not exceed a definite limit according to theory, if it is not to render illusory the precision of the measures with the spectrometer and the comparator.

On the basis of these considerations I have, at the suggestion of Professor F. Paschen, carried out the following investigations.

I had for my experiments two reflection gratings made by Professor H. A. Rowland in accordance with Professor Paschen's wishes. The material of the two gratings is speculum metal, as employed in the other Rowland gratings. On a surface of 10.5 cm^2 a circle of about 9.5 cm was plane-polished by J. A. Brashear, of Allegheny, and on this plane surface, which was of the same size for the two gratings, the rulings were executed with Rowland's best machine. The two gratings were made in 1899 at the same time, and therefore are among the last ruled by Rowland. The ruled surface of the one which the experiment showed to be the better, which I shall designate hereafter as G_1, is 79 mm wide, and contains 3120 grooves, the height

of which is about 47 mm. The other grating, which will be designated as G_2, is 78 mm wide, and contains 3085 grooves of the same height as for G_1. Thus the two gratings have approximately the same constant. It is obvious from these facts that the errors above mentioned as probably the most serious are avoided to a greater degree than formerly. In spite of the small number of rulings, the width of my gratings is exceeded only by that of the two used by Bell. This gives the advantage that we can determine the constant of the grating—width divided by number of rulings—with an accuracy corresponding to that of the spectrometer measures; and, moreover, we increase the resolving power of the grating, as is shown by the relation

$$r=\frac{\lambda}{d\lambda}=\frac{b}{\lambda}\sin\delta,$$

where r is the resolving power, λ and $\lambda+d\lambda$ are the two wave-lengths which are to appear just separated, b the width of the grating, and δ the angle of diffraction. Further details and characteristics of the two gratings will be mentioned in connection with the separate experiments.

My experimental problem therefore included: (1) a precise investigation of the two gratings for their errors, in respect to the curvature of the surface and to the lack of uniformity of the space between the rulings; (2) a new absolute determination of the wave-length of some suitable spectral line by measurements with the comparator and spectrometer. (The latter measurements were actually made in advance of the investigation of the errors.) My experiments were carried out in the physical laboratory of the University of Tübingen in the period from December 1904 to Easter 1906.

II. INVESTIGATION OF THE CURVATURE OF THE SURFACE OF THE TWO GRATINGS

The most reliable method for this, which permits the test of the whole surface at once, is doubtless the use of interference fringes in the manner of the following experiments. The firm of Carl Zeiss, of Jena, placed at my disposal a quartz plate of cylindrical shape having a diameter of about 10 cm and a height of about 2.5 cm, one surface of which was guaranteed to be plane. The plane surface

of this quartz plate was laid upon three steel needles resting at equal intervals on the surface of the grating. The needles permitted me to vary the distance of the two surfaces and exclude any adhesion which might injure the surfaces. The objective of the telescope of the spectrometer was laid directly on the quartz plate, the whole apparatus being solidly built up on a horizontal surface. At the focal length of the objective a circular mirror of a few millimeters diameter was placed a trifle to one side of the optical axis of the objective, and this reflected the light of a sodium flame upon the objective, quartz plate, and grating. The interference phenomena in the sodium light reflected from the plane surface of the quartz plate and the grating could be seen directly beside the mirror. If the grating surface was accurately plane, it should be possible to make all the fringes disappear. For G_2 this was possible except for one fringe, which had a width of about 1 cm, and lay approximately parallel to the rulings; this was slightly curved above and below. The remainder of the field of view was wholly bright. The whole apparatus was left to itself for several hours for the purpose of eliminating any possible differences of temperature, but the results were the same. We may therefore conclude that the ruled surface of grating G_2 exhibits no differences of more than $\frac{1}{2}$ of a wave-length of sodium light (λ 5893, mean for D_1 and D_2).

Precisely the same experiment gave the following result for G_1: A position could be found at which a dark shimmer appeared solely close to the left and right edge of the ruled surface, while the remaining surface was uniformly bright. This justified the conclusion that no differences of more than $\frac{1}{4}$ of a wave-length occurred in the ruled surface of G_1.

Of course, this tells us a little as to the nature of the curvature; but we may assume it to be approximately cylindrical for G_2 with the axis parallel to the rulings, as may be perceived from the single interference fringe; we can only suspect the same to be true for G_1. We do not know, however, whether the grating is convex or concave, whence it is practically impossible to employ the theory of the curvature of diffraction gratings developed by Cornu.[1] But I will give

[1] "Sur la diffraction; propriétées focales des réseaux," *Comptes Rendus*, **80**, 645, 1875; "Etudes sur les réseaux diffringents;" "Anomalies focales," *ibid.*, **116**, 1215,

a geometric representation which permits us to judge of the amount of the error which might be caused, particularly for my two gratings, by this curvature. Let the curvature of the grating be considered cylindrical, and the section $A_1 A_2$, an arc, represent the ruled surface in Fig. 1. Now, if we assume the grating to be free from errors in regard to surface, and consider it first at the point A_1 and then at the point A_2, perpendicular to the radius of curvature of the concave grating, then the two directions will form the angle γ. If we now let parallel light fall on the grating, that is E_1A_1 be parallel to E_2A_2, the rays will make an angle of incidence $E_1A_1N_1 = \alpha_1$ at the first position of the grating;

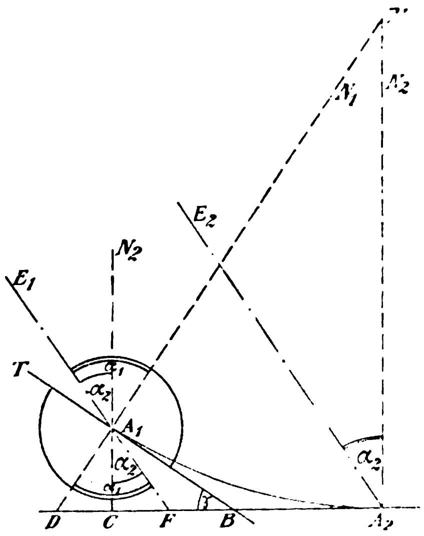


FIG. 1

and the angle of incidence $E_2A_2N_2 = \alpha_2$ at the second position. We then have the following relations:

$$E_1A_1N_2 = FA_1C = \alpha_2 ,$$
$$E_1A_1B = TA_1F = R + \alpha_1 = R + \alpha_2 + \gamma ,$$
$$\alpha_1 = \alpha_2 + \gamma .$$

Applying this now to the actual conditions, the greatest differences of the surface of the grating will be: for G_1, $\frac{\lambda}{4}$; for G_2, $\frac{\lambda}{2}$; whence $A_1C = \frac{\lambda}{4}$ or $\frac{\lambda}{2}$.

If we then make A_1B equal to one-half the width of the grating we obtain for G_1,

$$\sin \gamma_1 = \frac{2\lambda}{4b} = \frac{\lambda}{2b} ,$$

1893: "Sur diverses méthodes relatives à l'observation des propriétées appelées 'anomalies focales' des réseaux diffringents," *ibid.*, 1421, 1893; "Vérifications numériques relatives aux propriétées focales des réseaux diffringents plans," *ibid.*, **117**, 1032, 1893.

and for G_2,

$$\sin \gamma_2 = \frac{2\lambda}{2b} = \frac{\lambda}{b} .$$

We thus compute γ for G_1 to be about $\frac{3}{4}''$; for G_2, about $1\frac{3}{4}''$. The angle of incidence therefore may differ in an extreme case for the whole surface of the grating by this amount, whence we may safely draw the conclusion that the two grating surfaces depart so slightly from the plane that they may be regarded as perfectly plane in practice; that is, within the accuracy of the measurements.

III. INVESTIGATION OF THE TWO GRATINGS FOR IRREGULARITIES OF THE SPACE BETWEEN THE RULINGS

The practical problem is to establish quantitatively the amount of the irregularity in the separation of the rulings, whether it be caused by the curvature of the surface or by the other defects mentioned previously. It is necessary for this purpose to examine a few rulings at a time successively over the whole ruled surface. Bell[1] evidently had in mind the method that I have employed, but, as his description indicates, he does not seem to have accomplished his purpose in consequence of technical difficulties. He therefore calibrated his gratings with a comparator, with the consciousness that the calibration only approximately disclosed the errors. I have succeeded, however, in obtaining with an angle of 26°, and using the method of auto-collimation, a diffraction image from as few as 40 rulings, which appeared sufficiently well measurable when photographically recorded. The 40 rulings were cut out by a slit parallel to the rulings, which was moved along in front of the surface of the grating. The errors of spacing then appeared to the eye as the oscillations of the spectral line with respect to the cross-hair in the eyepiece when this slit was moved with sufficient rapidity. Thus my method of detecting the errors of the grating shows directly their effect on the pattern, which is the essential matter, and permits me to determine, for each desired limited region of the grating, the differences of spacing. Thus the method should be preferable to that of Bell.

The spectrometer used was the same one with which the measures

[1] *Am. Jour. Sci.* (3) **35**, 357, 1888.

of the angle of diffraction were made. It was made by R. Fuess, of Berlin-Steglitz, according to Professor Paschen's designs, and for the particulars I would refer to my dissertation. The essential thing to be remarked is that the two Porro microscopes read directly on the graduated circle to 1″, and permit an estimate to $\frac{1}{10}$″. The original telescope and collimator could not be used on account of too small aperture, and so they were replaced by others made by Zeiss with aluminum tubes. The objectives were apochromatic triplets with an aperture of about 70 mm and focal length of 43 cm. The ordinary eyepiece of the telescope could be replaced by a slit-eyepiece which I used for the majority of my experiments. The lower half of this eyepiece is occupied by the slit and the total-reflection prism, which throws the light through the objective upon the grating; a reticle occupies the upper semi-circle.

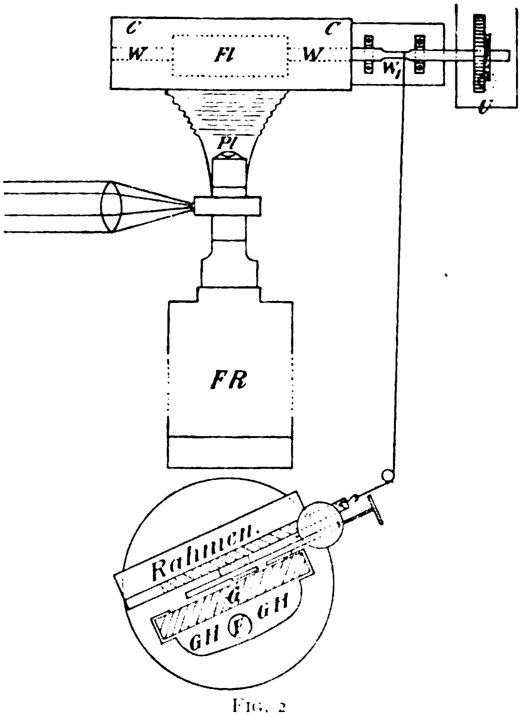


FIG. 2

A mercury arc *in vacuo* served as the source of light. Professor Paschen had several lamps constructed according to Arons' principle, which gave an excellent light with 60 volts and 4½ amperes, and could run for 20 minutes without interruption for cooling.

Fig. 2 shows the arrangement for cutting out a small number of rulings and for photographically recording the resulting oscillations of the spectral line employed. The adjustment of the photographic experiment is carried out in the following steps. The grating G is

placed in the holder *GH*, with the spectrometer table removed. A thermometer *T*, graduated in tenths of degrees, and permitting estimates within a few hundredths of a degree, was attached to this grating-holder in a mercury capsule. Care was taken that the variations of temperature did not exceed 0°1 during an experiment, which often lasted through several hours. The grating was now adjusted for auto-collimation. The slit for cutting out the small number of rulings was adjusted parallel to the rulings. The graduated circle was turned until the line to be photographed appeared in the desired order beside the vertical mark in the eyepiece which replaced the cross-hair. The films employed were hardly sensitive to the lines λ 5460 and 5790, so that I was compelled to use the blue line λ 4358, which was photographically effective. The eyepiece was replaced by a Zeiss "planar" lens, 1:4.5, of focal length 35 mm. The camera *C* was displaced in the direction of the telescope's axis, so that subsequently the film was moved past in the focus of the planar lens. Everything was screened off in front of the film by black cardboard, except a horizontal strip 1.5 mm high, in which appeared the mark and the spectral line. The film, of English manufacture, 4 mm wide, was placed in the camera on the roll *W*, and then the experiment could be begun. In principle it consisted in moving the slit directly in front of the grating by an improvised clockwork *U* turning the shaft *W*, and simultaneously unrolling the sensitive film. At the beginning and end of each experiment the image of the uncovered grating is photographed; and at the conclusion it is repeated after a change of angle of 60″, so that the variations could afterward be measured according to their amount in seconds. In my dissertation I have treated *in extenso* of certain possible outside errors which might equally produce apparent variations of the spectral image on the film, not due to the irregularity of the grating-space. I have there described how I avoided such errors, principally by the greatest possible variations in the experiments, as they might produce very serious confusion.

At an angle of incidence of 26° 1′, the limiting measurable visibility was about 40 rulings; and for this the necessary exposure was so long that I had to content myself with taking short sections of equal distances successively over the grating, while with the wider

slit, for which a decidedly shorter exposure was sufficient, I could readily cover the grating in a few hours.

In the measurement of the films the point was to be able to measure the variations of distance between the spectral image and the mark corresponding to the displacements of the moving slit. For this purpose I calibrated the films perpendicularly to the image of the mark on a dividing engine. I had for the measurement an ordinary microscope with micrometric stage and rotating plate. The ratio of enlargement was reduced by a suitable combination of lenses to about 1:2.5. The micrometer screw had a pitch of 0.2 mm, and one division of the head was equal to 0.002 mm. By the use of the well-known method of adjustment the shortest distance of the spectral line and mark was always measured. Subsequently the measurements were arranged graphically, the displacements of the slit being platted as abscissae, and the corresponding distances between the mark and spectral line gave the ordinates, for which the variations were exhibited by a curve. The most important of my experiments were the following:

A. Six measurements by sections with a narrow slit, 1 mm being about equal to 40 rulings for grating G_1.

$\lambda = 4358$. 51st order, to the left, $i = 26° 1'$. To each section corresponded an exposure of 40 minutes and 360 revolutions of the smallest wheel of the clockwork, whence 3.5 mm of displacement of the slit was equal to about 141 rulings, so that the whole number of rulings that were affected was $141 + 40 = 181$, the displacement of the slit of about 9.3 mm corresponding to one revolution of the largest wheel of the clockwork. The slit was displaced about 10 mm between every two sections.

I will content myself with publishing the numerical data of section 4 for illustrating the measures. I repeated all the measurements as shown in column 5*a*; each figure of this column denotes the mean of three settings on the center of the spectral line: column 5*b* gives the mean of these two measures. The significance of the other columns is as follows: The first gives the temperature during the exposure; the second, the length of the strip of film used; the third, the displacement of the grooves for one interval of calibration; the fourth, the interval of calibration at which the measures were made;

column 6*a* gives the largest difference of distance between the mark and the diffraction pattern within one section, which is converted into seconds of arc in column *b*, while *c* contains the corresponding difference of the constants of the grating.

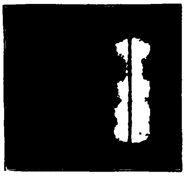


FIG. 3

The accompanying Figure 3 is a copy of the section with the mark at the right and spectral line at the left; but unfortunately the copies do not satisfactorily represent the originals.

EVALUATION OF THE DISTANCE OF 60″

THE ENTIRE GRATING, JANUARY 17, 1906

Position	Mark Right Edge	Spectral Line			Distance from Mark to Center of Line	Difference II−I
		Right Edge	Left Edge	Center		
I	2.081	7.936	8.858	8.397	6.316	0.817
II=I+60″	2.081	8.752	9.676	9.214	7.133	

The difference between the mark and the mean diffraction pattern amounts to 6.316 mm on the film. A displacement of the spectral image on the film of 0.817 mm corresponds to a rotation of the graduated circle of 60″.

MEASUREMENT OF A SECTION

1 *t* *a*	1 *t* *b*	2	3	4	5. Distance of Mark at Right Edge from Center of Line in mm *a*	5. *b*	Greatest Difference *a*	Greatest Difference *b*	Greatest Difference *c*
		mm							
20°.93				1	6.544 402	6.518			
				2	6.578 572	6.575			
				3	6.478 432	6.455			
				4	6.290 228	6.259			
20.95				5	6.246 266	6.256			
				6	6.274 260	6.267			
	20°.95			7	6.338 340	6.339			
		15.1	8	8	6.274 260	6.267	2–15 = 0.367 mm		70.6 Å. U. = 1 : 3600
				9	6.280 290	6.285		28″	
20.97				10	6.273 309	6.291			
				11	6.353 361	6.357			
				12	6.362 350	6.356			
				13	6.354 358	6.356			
				14	6.266 268	6.267			
				15	6.199 217	6.208			
20.93				16	6.218 232	6.225			
				17	6.217 218	6.217_5			

The following six curves exhibit the variation of the ruling-space for all the six sections of the experiment. The fourth curve from the top in Fig. 4 corresponds to section IV, the measurements of which are given in full. The straight line corresponds in all cases to the diffraction pattern of the full surface of the grating. The accuracy

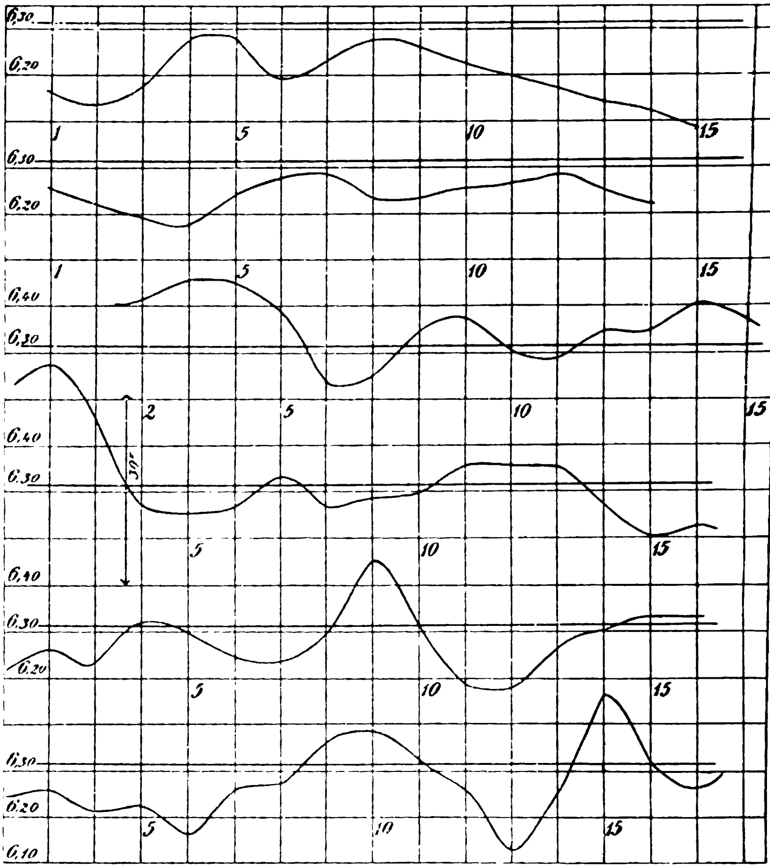


FIG. 4

of measurements is seen to be not very great on account of the indistinctness of the spectral line; the error can amount to a few hundredths of a millimeter, equal to about 4″, but in general would hardly exceed 2″. The previous experiment shows that by the isolation of a small number of rulings we have to do with very considerable variations;

that is to say, ***we cannot properly speak of a constant of the grating.*** The largest difference between mark and spectral line in the whole experiment is Fig. 4, fourth curve; 2 – first curve; 14 = 33″.5 = 84.5 Å.U. (1 : 3000) difference of the space between rulings. It is a striking fact that sections I and II lie throughout below the line of the image of the whole grating, whence it would follow that at this edge of grating G_1 the constant was too great throughout. The later experiments with wider slit confirmed this fact. In fact, Bell found something similar in his calibrations.

According to this experiment periodic errors could not be proven to be present; if they actually exist, they are apparently so affected by other errors of the rulings, which operate first to intensify the phase (see fifth curve at 9) and then to weaken it, that they no longer have any validity as such, and have no practical signification. Rowland's theory for the periodic errors of the spacing of gratings therefore cannot be applied.[1]

An experiment *B* then followed, in order to determine in what ratio the oscillations decrease on increasing the number of rulings which are simultaneously effective. The section from the middle of the grating G_1 was used in precisely the same position as in experiment *A*. For a definite section of about 25 rulings—to select at random an example of the figures—the oscillation changed as follows:

No. of Rulings Effective	Oscillations
100	0.021 mm = 1″.6 = 4.0 t.-m. difference from *c*
80	0.043 = 3.2 = 8.1
60	0.110 = 8.3 = 20.9
40	0.140 = 10.5 = 26.5

No relation can be proven to exist between these figures, but it could scarcely be present as long as the spacing does not vary according to some law.

I then made for grating G_2 two experiments similar to *A* and *B*. The result was quantitatively and qualitatively essentially the same as for G_1. I therewith gave up experimenting with the narrow

[1] "Gratings in Theory and Practice," *Astronomy and Astrophysics*, **12**, 129, 1893.

SUMMARY OF EXPERIMENTS WITH WIDE SLIT PASSING OVER ENTIRE GRATING

Grating	Date	Designation	Width of Grating-Slit	Range of Temp. During Exposure	Position of Grating	Diameter of Roll †	Portion of Grating Used	Length of Film	60″ = mm Difference	Position of Spectral Image for Open Grating	Position of "Center" Line ‡	Greatest Variation		
												In mm	In ″	Effect on c
G_1	11, III 1906	C	mm 5	0°.08	R_2 right 51st Order right $i=26°\ 1'$	mm 8.	mm 57.54	mm 211.8	0.831	2.481	2.491	0.115	8″.3	20.9 Å. U. (1:12000)
G_1	11, II 1906	D	5	0°.09	R_1 right 51st Order right $i=26°\ 1''$	8	53	188.3	0.81	4.350	4.350	0.137**	10″.2	25.8 Å. U. (1:9700)
	10, III 1906	E	5	0°.05	R_2 right 52d Order right $i=26°\ 34'$	4	52	120.5	0.81	1.948	1.981	0.129	9″.6	24.2 Å. U. (1:10000)
G_1	15, III 1906	C_1*	5	0°.04	Position as for C	8	....	....	0.79	1.866	.....	.....	....	....
G_1	20, III 1906	E_1*	2.5	0°.10	Position as for E	4	....	....	0.79	1.960	.....	.....	....	....
G_2	1, III 1906	F	5	0°.14	R_2 right 39th Order right $i=19°\ 34'$	8	55.1	194.6	0.86	2.934	2.956	0.145**	10″.1	35.0 Å. U. (1:7000)
G_2	23, III 1906	G	3	0°.07	R_1 right 39th Order left $i=19°\ 35'$	4	48.6	135.6	0.76	3.224	3.250	0.116	9″.2	31.8 Å. U. (1:7800)

* Edges of grating; entire grating in other cases.
† Axis on which film is rolled.
‡ Mean of all values of distance of mark from line.
** Edge values omitted.

slit, hence with the small number of grooves. But it was of importance, and in practice of interest, whether on selecting a considerable number of rulings (I usually took 200), differences as compared with the spectral image of the entire grating would appear, and in what amount. This experiment, on account of the short exposure required, could be extended over the whole grating, which was of course important. I made such experiments for both gratings in the most varied positions, in order to be sure of the outside errors previously described as possible. The spectral line was so clearly and sharply defined that I could very readily set on its edges. The accuracy of the setting is therefore decidely greater than for *A* and *B*. These experiments are designated in the original by "C" to "G" with their results in the accompanying table. A peculiarity appeared for $G_{,}$, that at one edge the spectral line appeared to be split and superposed upon itself. "F" showed this so strongly as to affect the measures. The experiments give the result that for both gratings, even with a wide slit where as many as 200 grooves are affected, still we have to do with noticeable oscillations of the diffraction pattern. I shall therefore appropriately discuss this further in speaking of the results for the next experiment, the absolute measurement of the wave-lengths of the mercury lines λ 5460 and 5791.

The fundamental formula for the determination of a wave-length by the grating method is given by the theory of diffraction of light as

$$m\lambda = c\big(\sin(\delta+i) - \sin i\big) .$$

Here λ denotes the wave-length to be determined, c the grating-constant, i the angle of incidence, δ the diffraction angle of the light employed, m the order.

IV. THE CONSTANT OF THE GRATING

In spite of the results of the previous section, I was at first obliged to avail myself of the customary practical definition of the constant as the width of the grating divided by the number of spacings. I had a small Zeiss comparator for measuring the width; with a magnifying power of about 60, two rulings (of which there were about 40 to one millimeter) were well separated, and permitted me to detect the details of their form. With suitable illumination the unruled portion

of the surface appeared dark, and upon it each ruling was projected in the form of the diamond which had ruled it as a system of four bright parallel lines; on closer examination these had a trough-like appearance. The grooves of G_1 appeared more brilliant and sharper than those of G_2. It was easy with this large grating-space to count the grooves, and I found

3119 spaces on 79.0 mm for G_1;
3084 spaces on 78.2 mm for G_2.

In the course of my experiments it appeared, however, that the small Zeiss comparator was not adapted to the large degree of precision required, whence at the conclusion of the experiments, both gratings were sent to the Normaleichungskommission at Berlin, where the widths were measured with the apparatus there available. The result of the four series of measures made, each of which included 32 experiments, is as follows:

Grating	Width Reduced to 17°	Deviation from Mean of All	Largest Range of Temperature in a Set of Measures
G_1........	79 mm + 50.1 μ	0.5 μ	17°66 – 16°01
G_1........	+ 50.7	0.3	17.14 – 15.97
G_2........	78 mm + 162.2	0.4	17.96 – 16.35
G_2........	+ 163.1	0.4	17.58 – 16.41

The commission remarked: "The uncertainty of the absolute amount of the widths of the gratings is to be taken at about $\pm\frac{1}{2}\,\mu$ for room temperature of $+17°$ C. If observations are made at another temperature, there will be added to this uncertainty the possible error in the assumed coefficient of expansion, which we should estimate as about $\pm 0.5\,\mu$ for 1° and one meter's length."

The final result for the width of the gratings, that is, for the distance of the center of the first to the center of the last ruling, measured perpendicularly to the rulings, is:

$G_1 = 79.0504$ mm at 17° C.,
$G_2 = 78.1629$ mm at 17° C.

Therefore the grating-constant is:

for G_1, $c = 25.34479\,\mu$ at 17°,
for G_2, $c = 25.34465\,\mu$ at 17°.

The uncertainty of these two absolute values would therefore probably not be greater than $\pm 0.0002\ \mu$ for 17°.

The coefficient of expansion ϵ of the substance of the grating I determined by comparison of the widths at different high temperatures; and again by the method given by F. Kurlbaum,[1] where I adopted $n = 1.0002763$ from the determinations of Kayser and Runge[2] as the index of refraction of the not dry air at 16°, and 760 mm for the wave-length of the light I used, *Hg* λ 5460. The coefficient of expansion for the two gratings resulted in the mean $\epsilon = 18.26\ \mu$ for 1° C. and one meter, with a mean error of $\pm 0.10\ \mu$.

The spectrometer measurements which I made use of were conducted within the range of temperature 15° to 19° for G_1, and 13° to 23° for G_2 (once at 11°5); of these the measurements of the width of the grating were made between 15°7 and 17°60 for G_1 and between 16°35 and 17°96 for G_2. Accordingly the greatest possible difference of temperature is in round numbers 6° between the measurements of the width of the grating and those with the spectrometer.

But an error of 0.5 μ in the coefficient of expansion with such a maximum difference of 6°, would involve an error of the grating of only 3 parts in a million. This error is therefore smaller than the uncertainty arising from the errors of observation, whence we may regard the coefficient of expansion as determined with sufficient accuracy.

V. MEASUREMENT OF THE DIFFRACTION ANGLE

The spectrometer, grating-holder, and thermometer which I used have already been described in Section III. The equation for determining the wave-lengths from grating measurements, derived from Huygen's principle, is:

$$m\lambda = c[\sin(\delta + i) - \sin i] \tag{1}$$

If the surface of the grating is perpendicular to the incident ray, $i = 0$ and (1) becomes

$$m\lambda = c \sin \delta \; . \tag{2}$$

If we observe on the other side of the direct image the order corresponding to that of equation (1), we may write it

$$m\lambda = c[\sin i + \sin(\delta - i)] \; .$$

[1] *Wied. Ann.*, **33**, 393, 1888.

[2] *Ibid.*, **50**, 203, 1893; also *Sitzungsberichte der Berliner Akademie*, 1893.

In the use of the slit-eyepiece, the incident and the diffracted rays coincided, that is

$$\delta=2i\,.$$

Then the previous equation becomes:

$$m\lambda=2c\sin i\,. \tag{3}$$

which position corresponds to the minimum deviation.

The two methods of observation depending on equation (2) were exclusively used by me, particularly the slit-eyepiece. It required no setting on the direct image, which was impossible with my objective on account of parallax. I therefore in my measurements always set on the same order to the left and right, and took half of the resulting angle. The adjustment for the method of auto-collimation was the simplest imaginable. The axes of telescope and collimator coincided. After parallax had been eliminated between the slit and the green mercury line, the position of the grating was regulated by the two screws of the spectrometer table, until on turning the grating by the graduated circle (the telescope always remaining fixed by this method) the upper edges of the spectral images of the different orders, both left and right, all touched the upper edge of the slit as they passed through the field. Thus the rulings were parallel to their axis of rotation, that is, to the axis of rotation of the spectrometer table. Then the grating was slightly tilted by a very slight turning of the two screws of the spectrometer table, so that the spectral image reached over upon the slit, which could therefore be used as a pointer, and thus the adjustment was complete.

The adjustment of the spectrometer for using the collimator is decidedly more complicated, but the steps are familiar, and moreover were fully described by Kurlbaum (*loc. cit.*).

I first made a series of experiments for acquiring general acquaintance with the arrangement for auto-collimation.

The formula

$$m\lambda=2c\sin i$$

shows that the spectral images of a definite wave-length of different orders must follow rapidly after each other, when we consider that the grating-constant is about 25 μ; in a theoretical case for $\sin i=1$, even a 90th order should be possible for λ 5460. In these

experiments grating G_1 was superior to G_2 in the brightness of the images. Both gratings showed ghosts, but they were so faint that they did not in any case disturb the principal spectra. Both showed that the actual grating in no wise fulfils the ideal case, according to which the intensity of the spectra must decrease with increase of order. As I always used for my angular measurements two corresponding orders left and right, there were few of the great number of images present which could be used for my purpose, having an equal intensity right and left. With G_2 the 75th order was distinctly perceptible, while with G_1 I could go beyond the 80th. Aside from the lower orders, which I did not use on account of the too small angle, sufficient equality was found right and left for the following orders:

For G_1		Per G_2	
Order	i (approx.)	Order	i (approx.)
20	12° 26′	22	13° 42′
39	24 50	41	26 13
53	34 49	72	50 52
65	44 27		
81	60 46		

I then made extensive experiments to determine whether the spectral lines of different high orders and of different orders right and left required a different focal setting; but, in spite of the sensitiveness of the objective, I was unable to detect any parallax in the different cases, a proof of the excellence of the gratings.

In the experiments with the use of the collimator tube, it was not possible, on account of the large diameters of the two tubes, to bring them closer than an angle of about 57°. Thus the lowest order which could be observed was the 38th; the 39th to the 42d orders could be used and were very bright.

THE DEFINITIVE MEASURES

I here used exclusively the mercury lines λ 5460 (green) and λ 5791 (yellow), giving up the comparatively weak line at λ 4358. I did not undertake a calibration of the graduated circle, but, instead of this, I at first turned the circle by 60° six times for each series of measures, and then repeated the measurement. The results

of this experiment soon justified me in contenting myself with a single repetition at 90°, so that the the effect of the eccentricity was eliminated. As is seen from the following table, I almost exclusively employed for the precise measures the method of auto-collimation, as this made it possible to work very quickly and surely.

I give now an example of a series of measures; all the measurements subsequently used follow this scheme. The atmospheric pressure was read from a barometer in an adjacent room. The readings in seconds given in the following scheme one above the other, which are afterward combined in a mean, correspond to each setting on the spectral line, and are in each case the mean of three settings of the double thread on the particular division of the circle concerned.

Grating G_1, *Hg* λ 5460. Auto-collimation. 65th order. Turned once 90°.
March 23, 1905

Circle in	Temp.	Pressure	65th Order	Microscope I: Separate Settings	Microscope I: Mean	Microscope II: Separate Settings	Microscope II: Mean
Position I	14°.79	729.0 at 14°.2 = 727.21 at 0	left	18° 16′ 37″.1 37.0 37.6	37″.23	198° 13′ 53″.7 53.1 53.8	53″.53
	14.77		right	289 22 42.9 42.9 43.1	42.97	119 19 57.4 57.4 58.1	57.63
	14.78		left	18 16 38.1 38.4	38.25	198 13 54.0 54.4	54.25
Position II Turned 90	14.79	729.0 at 14°.3 = 727.20 at 0	left	108° 16′ 34.0 34.2 34.0	34.07	288° 13′ 54.9 55.1 55.0	55.00
	14.79		right	19 22 40.0 39.7 39.3	39.67	209 19 57.9 57.2 57.4	57.50
	14.79		left	108 22 33.9 34.1	34.00	288 13 55.3 55.0	55.10

Position I: $2i=88°\ 53'$ 54″.26, 55.90, 55.28, 56.62 — 55″.51₅

$i=44°\ 26'\ 57''.76$ at 14°.77 and 727.21 mm pressure

Position II: $2i=88°\ 53'$ 54″.40, 57.50, 54.30, 57.00 — 55″.06

$i=44°\ 26'\ 57''.08$ at 14°.79 and 727.20 mm pressure.

I computed the wave-length λ from each of these two determinations separately, and I take the mean from the two values thus obtained as the definitive result of the series. The corrections applied in the following tables to the wave-lengths for the effect of temperature and pressure were reduced from the relation

$$\lambda\left(1+0.00000039\frac{p}{1+at}\right)=\text{constant}^{1}$$

where λ signifies a definite wave-length, p the pressure in millimeters, and t the temperature. Accordingly for $+1^\circ$, λ increases by about $0.95\times10^{-6}\lambda$; and for -1 mm change of pressure, by $0.36\times10^{-6}\lambda$. The significance of the columns of the following tables, in so far as not directly stated, is as follows:

Two gives the line used; 3, the method of measuring with the spectrometer; 4, the order; 5, the number of repetitions; 6, the observed temperature (C), α being the mean and β the largest difference occurring during the measurement; 7, the air pressure reduced to 0°; 8, the angle of diffraction i, α being the mean of four differences, and β the largest difference in seconds; 9, the grating constant expressed in microns and reduced to the temperature of the angular measurement; 10, the wave-length λ for temperature and pressure of the angular measurement; 11, the corrections to λ on account of temperature and pressure for reduction to 20° and 760 mm; 12, λ plus reduction for the separate measures; 13, the final resulting λ for each series from the mean of the separate measures; 14, the residuals of the separate measures from this mean; 15, the residuals of each series from the final mean of all the series, hence from the definitive wave-length to be given below; 16 further gives the effect of the error of 1° C (a), 1 mm pressure (β), and under γ the effect of 1″ of angle of i or δ on the wave-length λ. All the quantities in columns 10 to 16 are expressed in Å.U.

The comparative measurement in the different orders for grating G_1 is more comprehensive than it could be by any earlier observer; it yielded a perfectly satisfactory agreement, all the differences falling within the limits of error mentioned. The same is to be expected for G_2, since the measures by auto-collimation and the measures with

[1] This is the formula which Kohlrausch gives (*Prakt. Phys.*, 1901, **9**, 588); it was satisfactory for my purposes, as I have proved.

the collimator tube, where the diffraction angle is about 2½ times greater in the latter case than in the former, also were in entire agreement. The final result of the spectrometer measurements, using the widths of the grating as determined by the Commission, is given in the tables as follows for 20° C. and 760 mm pressure.

	G_1	G_2	Mean
Green mercury line............	5460.78_5	5460.75_3	5460.76_9
Yellow mercury line...........	5700.71_0	5790.67_9	5790.69_0

The inaccuracy of the results for each grating, on account of the errors of observation and of the instruments, can scarcely exceed a few hundredths of an Ångström unit for the line λ 5460; for λ 5791 it is doubtless larger, as I have contented myself with only one series of measures with that line. The object of this research came to be the measurement of one line accurately. As to the errors of observation and those of the graduated circle, I could probably have attained a certainty of approximately 0.01 Å. U. by increasing the number of spectrometer measurements; but I contented myself with the previous experiments, because, from what was said at the beginning of this paper, it would have been a fruitless undertaking to attempt to attain with gratings the accuracy of the results obtained by Messrs. Perot and Fabry with the interferometer. The question was rather this with respect to the results of Kurlbaum and Bell: Will my results agree among themselves, and with those of Perot and Fabry within the limits set by the errors of observation, of the graduated circle, and of the comparator? We see that my gratings exhibited between themselves differences similar to those in the gratings used by Bell; compared with the value found by Perot and Fabry at 20° and 760 mm for the green line, viz. 5460.768 Å. U., my results showed the following differences:

Pérot and Fabry $-G_1 = -0.01_7$ Å. U.
" " " $-G_2 = +0.01_5$ "

The differences of the results of Kurlbaum and of Bell, respectively, from those of Perot and Fabry, for D_1 (λ 5896) are

Kurlbaum – Pérot and Fabry: −0.10 and +0.02 Å. U.
Bell " " " +0.22 " +0.27 "
" " " " +0.19 " +0.21 "

Hence we see that my results are in thoroughly satisfactory agreement with those of Pérot and Fabry within the limits of accuracy. Kurlbaum's results approach this agreement, but it must be admitted that, while Bell's experiments were carried out with the same care, and furthermore with better gratings than Kurlbaum's, nevertheless, or perhaps on that account, the above comparison compels us to assume that Bell was subject to an error which cannot be attributed to the defects in his gratings, as treated by me in Sections II and III.

The question now is: How do my results compare with the conclusions of Section III? We must here bear in mind that the calibration only partially explains the action of the whole grating, inasmuch as the larger intervals of the grooves (hence from 0—3119 or 0—3084 rulings) are effective for the whole grating, and these were not investigated in the calibration. This explains why the line given in the table on page 250 as "center of gravity" line need not fall in coincidence with the spectral image of the entire grating. My experiments with the oscillations, however, give a basis for the following considerations, which are due to Privatdozent Dr. R. Gans, of Tübingen, and the computations based upon them.

The theory of the grating gives for the distribution of intensity of the diffraction pattern the following relation.[1]

$$J_0 = J'_0 \frac{\sin^2\frac{\mu a}{2}}{\left(\frac{\mu a}{2}\right)^2} \cdot \frac{\sin^2\frac{\mu(a+d)}{2}p}{\sin^2\frac{\mu(a+d)}{2}}, \tag{1}$$

in which

$$\mu = \frac{2\pi}{\lambda}(\sin\phi + \sin\psi), \tag{2}$$

where λ denotes the wave-length, ϕ the angle of incidence, and ψ the angle of diffraction. J'_0 denotes the intensity for the diffraction angle zero ($\mu = 0$), as it would be for a slit; p is the number of rulings, a the reflecting portion between two rulings, and d the width of a ruling, whence $a+d$ is the grating constant.

The intensity regarded as a function of μ is a maximum, if

$$\frac{dJ_0}{d\mu} = 0. \tag{3}$$

[1] See Mann and Millikan's translation of Drude's *Optics*, 1900, p. 222.

This occurs for

$$\mu_0 = \frac{2\pi}{a+d} m \quad (m = 0, \quad 1, \quad 2 \ldots .) \,, \tag{4}$$

m being the order of the diffraction image.

Now, if there is a small error of any given sort in the grating, the intensity of the diffraction pattern will no longer be given by J_0 from equation (1), but it will be

$$J = J_0 + J_1 \,, \tag{5}$$

where J_1 is a small quantity to be added to J_0, and is also a function of μ.

The position of maxima is now found from

$$\frac{dJ}{d\mu} = \frac{dJ_0}{d\mu} + \frac{dJ_1}{d\mu} = 0 \,. \tag{6}$$

This equation is no longer satisfied by μ_0 from (4) but by

$$\mu = \mu_0 + \delta\mu \,, \tag{7}$$

where $\delta\mu$ indicates the small error in position of the diffraction pattern. Neglecting the higher powers of $\delta\mu$, (6) and (7) give

$$\left(\frac{dJ_0}{d\mu}\right)_{\mu=\mu_0} + \left(\frac{d^2 J_0}{d\mu_2}\right)_{\mu=\mu_0} \delta\mu + \left(\frac{dJ_1}{d\mu}\right)_{\mu=\mu_0} = 0 \,. \tag{8}$$

From (3) and (4) the first term of (8) disappears, and we obtain for the error in position of the diffraction pattern

$$\delta\mu = -\left(\frac{\dfrac{dJ_1}{d\mu^2}}{\dfrac{d^2 J_0}{d\mu^2}}\right)_{\mu=\mu_0} \,. \tag{9}$$

Consider now the amplitude. For a grating without error, let the amplitude of the diffraction pattern in a definite order be expressed by A_0, if only the nth part of the grating is effective. If the whole grating is effective, so that there are n elements of the wave in the same phase, the amplitude will be nA_0. Correspondingly the intensities $J_0 = A_0^2$ or $J_0 = n^2 A_0^2$, according as the nth portion or the whole grating is effective. This may also be derived from (1).

For an imperfect grating the amplitude is given by

$$A = A_0 + A_1 \,, \tag{10}$$

in case the nth part of the grating is effective. A_1 is here a small quantity in comparison with A_0. If the whole grating is used, the amplitude becomes

$$A = nA_0 + \sum A_1 . \tag{11}$$

$\sum A_1$ denotes the sum of the amplitudes, which are added on account of the errors of the whole grating. But in case the errors are not of a systematic nature[1] the quantities A_1, on account of the different phases of the erroneous elements, will regularly alternate in quantity, magnitude and sign, so that $\sum A_1$ is of the order of A_1 and not of the order nA_1.

The intensity of the nth portion of the erroneous grating is therefore

$$A^2 = A_0^2 + 2AA_1 , \tag{12}$$

since we may neglect A_1^2.

But the intensity of the whole grating is

$$\left.\begin{aligned} J &= J_0 + J_1 = \left(nA_0 + \sum A_1\right)^2 \\ &= n^2 A_0^2 + 2nA_0 \sum A_1 \end{aligned}\right\} . \tag{13}$$

Therefore

$$J_1 = 2nA_0 \sum A_1 . \tag{14}$$

The numerator of (9) will therefore be of an order n times as great if the whole grating is used instead of its nth portion, while, on the contrary, the denominator will become n^2 times as large. It follows from this that the order of magnitude of the error in the position of the diffraction image becomes n times smaller if the whole grating acts, than if only its nth portion acts.

This result for the error is not rigorously correct, since A_0 and A_1 depend upon the whole grating-width in the manner given, while the differential quotients with respect to μ depend upon it in a complicated manner, and, moreover, perhaps not only the higher orders of $\delta\mu$ may be neglected. In any case (9), in connection with the reasoning regarding the amplitude, gives an idea of the error committed.

[1] See Lord Rayleigh, *Phil. Mag.* (4) **47**. 193–205. 1874; *Encyc. Brittanica* (9) **24**, 438.

Since my gratings, as has been shown, satisfy the conditions that no systematic errors are present, what precedes can be immediately applied to Section III. I give here as there, for the whole grating also, the order of the deviations in seconds, in the last column.

Grating	Desig. of Experiment	Angle of Diffraction i	Effect in Å. U. of Error of 1″	n	Greatest Departure from "Center" Line	Order of Magnitude of Deviations for Entire Grating
G_1	A	26° 1′	0.054	$\frac{3119}{40}=78.0$	19″.0	0″.24
G_1	C*	26 1	0.054	$\frac{3119}{200}=15.5$	5.1	0.34
G_1	D	26 1	0.054	$\frac{3119}{200}=15.5$	5.6	0.36
G_1	E	26 34	0.053	$\frac{3119}{200}=15.5$	6.4	0.41
G_2	F	19 34	0.074	$\frac{3084}{200}=15.4$	5.3	0.34
G_2	G	19 35	0.074	$\frac{3084}{120}=25.7$	7.9	0.31

* See table, p. 348.

The effect of the error on λ according to this table would therefore be in an extreme case

For grating G_1, 0.022 Å. U.
" " G_2, 0.025 " .

This explains the excellence of my results on page 258 ff.

CORRECTION[1]

Dr. Otto Schönrock, in Berlin, has had the kindness to call my attention to an error in my computation of the curvature of the grating surface (Section II). In Fig. 1 I incorrectly gave for the numerical value for the length A_1C the largest possible departure from the plane, or $\lambda/4$ and $\lambda/2$. But the correct reasoning is as follows, according to Brodhun and Schönrock.[2] If in Fig. 1 we call $NA_1 = NA_2 = R$ the radius of curvature of the grating A_1A_2, assumed to be concave, and d the versed-sine of the arc A_1A_2; then

$$R=\frac{b^2}{8d}+\frac{d}{2}$$

[1] *Annalen der Physik*, **22**, 708, 1907.

[2] *Zeitschrift für Instrumentenkunde*, **22**, 353, 1902.

or, since $d/2$ vanishes as compared with R,

$$R=\frac{b^2}{8d}\,.$$

A simple consideration of the figure shows that

$$\gamma=\frac{b}{R}=\frac{8d}{b}\,.$$

Mr. Schönrock also believes, on the basis of his experience, that it is safe to infer from my observations that for each of the two gratings $d<\lambda/2$; but that it is not at all impossible that d may be nearly equal to $\lambda/2$ for each grating. Hence, in the extreme case, $d=\lambda/2=0.000295$ mm. It is further to be considered that the objective used in the experiment with Fizeau's fringes had an aperture of only 70 mm; hence the radius of curvature will be

$$R=\frac{70^2}{8\cdot 0.000295}=2.08\text{ km}\,.$$

For $b=78$ mm, γ becomes $7.'7$, and is thus considerably larger than as computed by me as $1\frac{3}{4}''$.

With the apparatus described by Brodhun and Schönrock it is furthermore possible, by varying the inclination of the plates producing the interference, to follow the wandering of the fringes so as to derive an idea ultimately as to the nature of the curvature. My improvised apparatus did not permit of any such manipulation.

While I must admit from what precedes that the second section of my paper is defective, I should like to emphasize that this has slight bearing on the essential part of the research. For on the one hand, nothing was known by me as to the character of the quartz plate used. In Mr. Schönrock's opinion the plane surfaces by Brashear are quite as perfect as, if not more so than, the quartz surface by Zeiss. On the other hand, the calibration described in Section III adequately cares for the errors of curvature in practice. They can indeed be only very insignificant, inasmuch as the different high orders exhibited no focal errors.

A PHOTOGRAPHIC METHOD FOR THE DETECTION OF VARIABILITY IN ASTEROIDS

By JOEL H. METCALF

By the method described in the *Astrophysical Journal* for May 1906 (**23**, 306–311) for the discovery of asteroids, the photographic plate is made to follow the mean motion of asteroids in opposition at that season of the year in which the plate is taken. As a result the stars are drawn out into trails, the length of which depends upon the exposure time. The asteroids, on the other hand, are points like the ordinary star images of chart-plates.

In long exposures any departure of the actual asteroid from the mean motion will show itself in the elongation of the images. Practically most asteroids can be followed for 35 or 40 minutes without this departure becoming apparent.

As it would be easy to mistake an imperfection on a plate for an asteroid, two exposures are always taken, separated by a small arbitrary amount in right ascension. Each asteroid is therefore represented by two more or less circular images in close proximity.

If the time is equal in both exposures, the star-trails will be of the same length, and the two images of the asteroid ought to be of the same size, shape, and density.

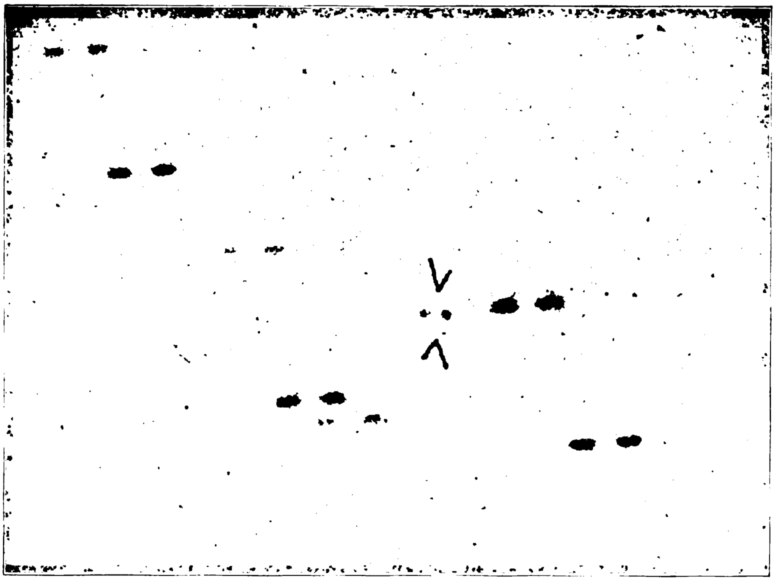
Practically this is not always true, for two reasons: first, the motion of the photographic plate may not have been uniform during the exposure; second, there may have been a change in the transparency of the atmosphere between the first and second exposures due to changing altitude of the asteroids or to a gathering storm.

In both cases, however, the star trails would register this change. If the motion of the plate were not uniform, the star-trails would not be straight and of similar length. If the transparency of the air changed, there would be differences shown in the density of the fainter star trails.

Now, if on any plate the star trails are of the same length and density, showing that the motion of the plate has been uniform and that the atmosphere has not changed, then, if the images of the asteroid

are not of the same size and density, the presumption is that there has been a change in the luminosity of the asteroid.

The only escape from this would be that the photographic plate contained a defect or was of unequal sensitiveness. Defects are possible, but are usually apparent under a high magnifying power. I think the experience of the discoverers of variable stars is against a belief in any great differences of sensitiveness in such close proximity as the adjacent images of an asteroid (about 0.3 mm on my plates).



I therefore believe that any plate fulfilling the above conditions as to atmosphere and uniform motion, and yet showing the asteroid images to be greatly different in size and density, would establish a probable variability.

By the usual method of picking up asteroids the star images are points and the asteroids trail. If a plate was found in which the trail showed greater density in one part than another, there would be no check by which it could be decided whether it was due to variability or the hazing of the atmosphere. Other asteroids even on the same plate would not be a test, for often local cloudiness is apparent even

on the best nights in our climate. If the stars trail, however, they become an infallible index of these changes.

That these remarks are not entirely theoretical I would contend from the accompanying illustration. It is an enlargement of a photograph made on November 6, 1906, of *1906 WE*, discovered at Taunton on October 26, 1906. I believe the star-trails show that the motion of the plate was practically uniform and that there was between the exposures no great change in the transparency of the atmosphere; and yet the two images of the asteroid show a surprising difference in size and density. Unfortunately, due to an accident, this asteroid was observed only four times—on October 26, November 6, December 7, and December 18, 1906.

On every plate there are two images of the asteroid, and yet on three of them—those of October 26, November 6, and December 18—the companion images show differences of size and density, although the differences on October 26 and December 18 are not so marked as on November 6.

On the plate of November 6 the exposure times were 35 minutes each, with 1 minute between exposures. If this change is real and not illusory, it shows that *1906 WE* went through a noticeable change of brightness in 1 hour and 11 minutes.

I believe this method of photographing asteroids, containing as it does such a good check on atmospheric changes, might be extended much farther. A long series of images might be taken, not only on one night, but on several nights, in a climate where good weather prevails, and the corresponding star-trails would in every case be a check on the condition of the atmosphere. If there arose any difficulty in getting absolute magnitudes from the trouble of comparing images with trails, the motion of the plate might be made just half the real motion, and then the asteroid and stars would be of equal length, and there would only be the difficulty of identifying the asteroid trail.

For variations taking place, however, in the course of a few hours, as many circular images as possible should be taken, and the exposure time should be as short as is consistent with obtaining a good image of the particular asteroid to be observed.

TAUNTON, MASS.
April 8, 1907

ON THE STRUCTURE OF THE FINEST SPECTRAL LINES[1]

By O. von BAEYER

In a previous paper E. Gehrcke and O. von Baeyer communicated their observations on the resolution of different spectral lines by the method of interference points, giving at the same time quantitative measures of the distance of the satellites from the principal lines concerned.[2] Meantime the Reichsanstalt purchased from A. Hilger, of London, a plate surpassing in its excellence all of the plates hitherto acquired by the Physikalisch-Technische Reichsanstalt. Investigations on the Zeeman effect in faint magnetic fields were promptly begun with this plate, and reference made to the excellent qualities of the plate in the publication.[3] It therefore seemed desirable to repeat with this improved apparatus the measurements on the most important spectral lines which had thus far been proven to have satellites. The results thus obtained are here communicated.

I. ARRANGEMENT OF EXPERIMENT

The arrangement is entirely similar to that described in the previous paper. For producing the interference points, the Hilger plate was crossed with one of 3 mm thickness by Haecke of Berlin, which had been employed in our previous work. It is identical with the plate[4] designated in the other paper as D, of 0.317 cm thickness, 15 cm length, and of refractive index 1.53. The plate by Hilger is 0.971 cm thick, 30 cm long, 4 cm high, and has a refractive index of 1.58. It exhibits over its whole surface variations of thickness which may be estimated as only one-twentieth of the wave-length 436 $\mu\mu$. In

[1] A communication from the Physikalisch-Technische Reichsanstalt. Translated from the *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **9**, No. 4, 1907.

[2] *Annalen der Physik*, **20**, 269, 1906.

[3] E. Gehrcke and O. von Baeyer, *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **8**, 399, 1906; and *Physikalische Zeitschrift*, **7**, 905, 1906.

[4] This plate was kindly put at the disposal of the Reichsanstalt by the firm of Haecke, of Berlin. It was meanwhile purchased by Professor M. Wien, of Danzig, but was kindly left with us until the completion of these measurements. I would not fail to express to Professor Wien my best thanks for this favor.

consequence of the great breadth and uniformity of the plate, a far greater brightness could be given to the phenomenon, and thus the exposure time of the photographs was considerably diminished. In general, about one-half of the previous exposure time was sufficient.

The revolving power, i. e., the quantity $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is the least difference of wave-length resolvable with the apparatus, is for this plate in round numbers 700,000 (for $\lambda = 0.5\ \mu$). The plate formerly used, Ċ, from Zeiss (5 mm), had a resolving power of about 400,000. The echelon which Mr. Janicki[1] used had a resolving power of 420,000 (for $\lambda = 0.5\ \mu$).

II. MEASUREMENTS

a) *Spectrum of mercury.*—For producing this light, the Arons mercury lamp of Lummer type again served in part. For the yellow and green mercury lines the sharpness attained is entirely sufficient. A mercury vapor lamp by Boas, of Berlin, such as is used for technical purposes of illumination, did not give the same degree of brightness, and even the green line was not as sharp as could be desired. For the blue and violet lines a Geissler tube with mercury electrodes had to be used. Even in the Geissler tube the method of excitation had a great effect on the sharpness of the lines. Thus, for instance, in using a mercury interrupter by Boas, the lines at 434.8 $\mu\mu$ obtained on gentle warming of the Geissler tube were very diffuse; while a platinum interrupter, also by Boas, which furnished a very bright mercury spectrum in the same Geissler tube without exterior heating and with the same induction coil, yielded sharp lines. It would therefore seem as if the increase of the pressure of the mercury vapor by external heating broadened the lines more readily than the increase in temperature produced by the greater density of current.

The results of the measures agree in general with the previous values,[2] as the following table shows.

[1] *Annalen der Physik*, **19**, 36, 1906.

[2] E. Gehrcke and O. von Baeyer, *Annalen der Physik*, **20**, 269, 1906; L. Janicki, *ibid.*, **19**, 36, 1906. Observations on the positions of the satellites of the mercury lines have also been made by Pérot and Fabry, Prince Galitzin, and F. Bates, which are not cited further in this communication.

λ (μμ)	10^{-8} μμ: Plate C and D Gehrcke and v. Baeyer	10^{-8} μμ: Hilger Plate and Plate D. v. Baeyer	I	Echelon. Janicki	I
	−1.13	−1.26	4	−1.11	1/8
404.7	−0.49	−0.59	1	−0.51	3/4
	+0.77	+0.75	3	+0.67	1/2
	+1.38	+1.43	2		
	−0.77	−0.86	2	−0.76	1/6
407.8 weak	−0.44	−0.53	2	−0.46	1/6
		+0.37	4	+0.32	1/10
	+0.58	+0.55	1	+0.49	1/4
	+0.89	+0.82	3	+0.74	1/9
433.9	No satellite visible			−1.2	1/10
				+0.6	1/10
434.8 weak	−0.48	−0.54	1	−0.46	1/5
	+0.55	+0.56	3	+0.53	1/7
	+0.82	+0.89	2	+0.83	1/7
	−1.71	−1.75	2		
	−1.18	−1.18	4	−1.12	2/3
	−1.08	−1.03	4	−0.97	2/3
		−0.48	7	−0.52	1/7
	−0.17	−0.18	1	−0.23	1
435.9		+0.21	5	+0.20	1/5
	+0.27	+0.30	6		
	+0.51	+0.49	5	+0.43	1/4
				+1.05	2/3
	+1.21	+1.26	5	+1.2	2/3
	+2.02	+2.04	3		
491.6	No satellite visible				
	2.41	−2.50	2	−2.32	1/5
	1.03	−1.07	4	−0.99	1/10
		−0.72	3		1/7
	0.55	0.51	5	−0.66	
546.1		−0.25	1		
	+0.93	+0.87	1	+0.88	1/8
	+1.40	+1.32	3	+1.33	1/4
	+2. ?	+2.22	6		
				−1.13	1/4
	0.56	−0.50	2	0.50	1/4
576.9	+0.47	+0.46	1	+0.46	1/3
				+0.87	1/10
				+1.20	1/10
				−2.51	1/10
	−2.02	1.9...	4	−1.87	1/10
	1.28	1.27	1	−1.19	1/5
579.0				+0.84	1/9
	+1.43	+1.30	2	+1.32	1/7
				+1.68	1/10
	+2.41	+2.37	3	+2.30	1/7

Attention should be called in detail to the following points:

Hg λ 407.8. The satellite $+0.37.10^{-2}\ \mu\mu$ occurs, which was previously observed by Janicki. λ 434.4 shows no satellite, in spite of long exposure (4 hours).

λ 435.9 shows, in comparison with the earlier results of Gehrcke and von Baeyer, two new satellites. One at −0.48 was also already observed by Janicki. The other arises from the splitting of the former satellite +0.27, which first became visible with the high resolving power. In accordance with the earlier measurements of Gehrcke and von Baeyer, the satellite at −1.75 appeared, which

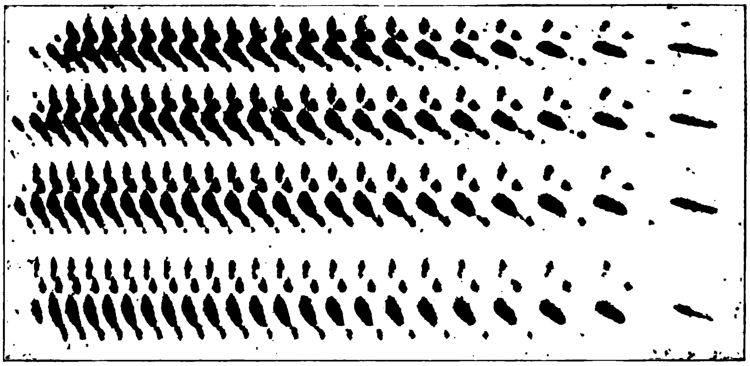


FIG. 1

Janicki did not correctly assign, and therefore considered to be a line at +1.05.

λ 546 $\mu\mu$ gave three new satellites. Fig. 1 shows an enlargement of this plate on which the satellites can be recognized, excepting the one faint one of intensity 6. In order to make the position of the satellites distinct, Fig. 2 gives a schematic representation of a principal line and its associated satellites. The numbers correspond to the intensities. See also the table.

We see at first that the earlier satellite −0.55 is split into two: −0.72 (intensity 3) and −0.51 (intensity 5). This also explains the uncertainty in the measurement which hitherto applied to this satelite. The principal maximum has in its immediate neighborhood a very strong satellite −0.25 (intensity 1). +2.22, which is exceedingly faint, is also new. This was not included in the earlier work of

Gehrcke and von Baeyer, although it was noticed; but, as it was then hardly visible, and moreover fell very close to a principal maximum, we regarded it as false. But the photographs with the Hilger plate show it very clearly. If Janicki did not observe it, this was due to the fact that on his photographs this satellite coincided with -2.50. The distance of the two principal maxima at this point is 0.476 Ångström units, so that -2.50 and $+2.22$ were only separated by 0.004 Å. U., which his echelon could no longer resolve.

In this case the advantage is very clearly apparent which the method of interference points has over the method of interference bands; for with observations with the Hilger plate only, as may be seen from Fig. 1, the strong satellites with intensities 1, 1, 2, 3, would have coalesced with the principal line to form a single, vertical, greatly broadened band.

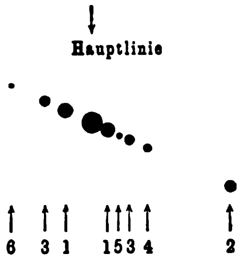


FIG. 2.—*Hg* 546.1 $\mu\mu$ with 10 mm plate by Hilger and 3 plate (D) by by Haecke.

At λ 576.9 and 579.0, the fainter satellites given by Janicki again failed to appear.

As to λ 579.0, it is to be remarked that satellite -2.02, which is only faintly indicated, could not be accurately measured. On this plate there also appears a pretty strong line which could not be a ghost. On the other hand, it could not be included among the satellites, as it did not lie in the direction of the remaining satellites. This is evidently a widely distant line. It can only be said, on the basis of the measures, that its distance is in any case more than $4 \cdot 10^{-2}\ \mu\mu$. It therefore ought to be possible to observe it with a good grating. The possibility is moreover not excluded that it is the already known line λ 580.4.[1]

b) *Bismuth.*—The bismuth amalgam lamp of quartz, previously described, was used for the production of the spectrum of bismuth. The line λ 472.2 was investigated and yielded results in perfect accordance with the former measures.

From the good agreement of the earlier measures and those communicated here, it is again to be inferred that the method of inter-

[1] Kayser and Runge, *Abhandlungen der Berliner Akademie der Wissenschaften*, 1891.

ference points, even with the use of less perfect plates, such as Plate C, yields unambiguous results, and renders it possible to distinguish ghosts from the true satellites. Every step forward in the production of plane-parallel plates will also further diminish the only disadvantage

	Gehrcke and von Baeyer, Plate C and D	von Baeyer, Hilger Plate and Plate D	I
	+0.66	0.63	1
	+1.12	1.08	3
472.2 $\mu\mu$	+2.61	2.63	2
	+3.07	3.07	1
	+3.36	3.45	1

of the method, namely, the faintness of the phenomenon; and it is to be expected that in this point also the performance of the echelon will be at least reached.

As to the splitting of the spectral lines, the above investigation shows that the number of the certainly proven satellites has again increased, and it seems probable that every increase in the resolving power will always bring further new satellites into visibility.

ON THE BRIGHTNESS OF THE INNER EDGE OF THE PENUMBRA IN SUN-SPOTS (SECOND NOTE)

By S. CHEVALIER

Some time ago I wrote a note to show that in photographs made at the Zô-sè Observatory the inner edge of the penumbra of sun-spots was very generally brightened, just as it appears to be when looked at in a good telescope. Since this note was written, improvements have fortunately been made at this observatory in photographing the sun. I consequently feel obliged to return to the subject.

The photographs of the sun, taken daily, as far as possible, at Zô-sè, vary in diameter from 63 to 65 mm. When atmospheric circumstances are favorable, they show distinctly the small granules of the photosphere, but fail to represent all the details of sun-spots. Hundreds of features observed at the eyepiece and traced on drawings are wanting on the photographs. This is certainly due, in large part, to the smallness of the picture. The nucleus especially, unless crossed by some brilliant "bridge" or filled with only half-obscure material, appears quite dark. All around the nucleus there is a very faint ring, visible only with the greatest difficulty. Outside of this ring begins the inner edge of the penumbra, which looks very generally brighter than the outer one. This is the fact which in my first note I tried to set forth as a common occurrence in sun-spots.

But I have now strong evidence that this brighter ring which skirts the umbra in these photographs is not really the actual edge of the umbra. Within the brighter ring there is a fainter one, just mentioned, probably consisting of the sharp ends of the filaments which run still farther inside the nucleus and which must be considered as belonging to the penumbra. So that what appears on our ordinary photographs, taken at Zô-sè, as the edge of the penumbra, is as a matter of fact at some distance from the nucleus. On ordinary photographs the nucleus always comes out much larger than it ought to and obscures the very minute details along the brim.

Before inquiring into the cause, the fact must be clearly demonstrated. For this purpose, among many instances, I will select two.

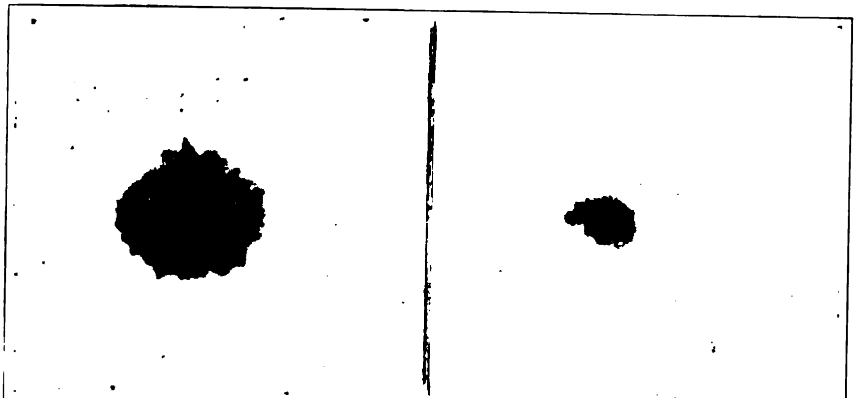
In each case I will put before the reader two images of the same sun-spot. One, which for the sake of brevity I will hereafter call *A*, is an enlargement to 16 diameters of a photograph made in the ordinary way, viz., on a plate at the focus of the telescope. The other, which I will call *B*, is a photograph obtained by the direct enlargement of the image formed at the focus of the telescope. The enlargement being 16 diameters, the image of the sun on this scale would have a diameter of one meter. The plate taken at the focus of the telescope has been enlarged to just the same size in order to facilitate a comparison between the two pictures.

The first set of photographs was made on July 17, 1906. *A* was made at $9^h\ 10^m$ and *B* with the enlarging camera half an hour later (Plate XV). *A* came out remarkably fine and clear, as appears from the plate, which shows distinctly the granules of the photosphere and many features of the spot. *B* did not come out so well as far as photographic art is concerned; but nevertheless it is far in advance of the other for representing the real features of the spot. There is no need of pointing out to the reader how much larger the nucleus is on *A*. As to the penumbra, the brightest part, which appears on *A* as forming the very brink of the inner edge, is clearly shown on *B* to shine at some distance from the nucleus; in the intervening space between the bright ring and the nucleus the sharp ends of the slender filaments are seen progressively fading away into the dark center.

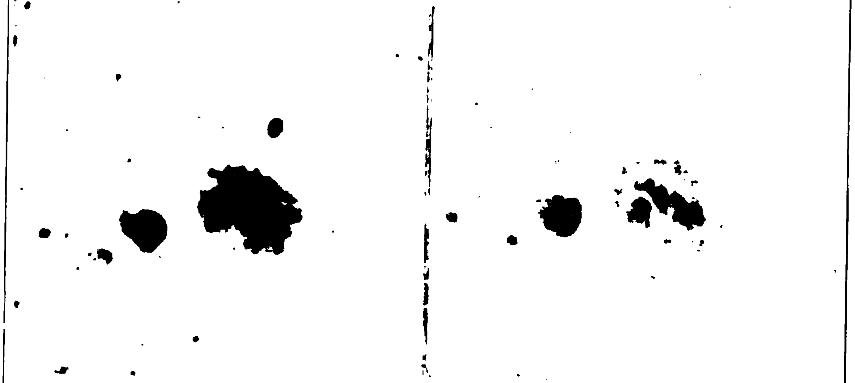
The other set of photographs represents a sun-spot which was near the center of the sun on September 14, 1906. *A* was taken at 8^h in the usual way and enlarged to the size of *B*, viz., to 16 diameters. *B* was obtained as above, nearly half an hour later. As it is the best photograph I ever made of a sun-spot, and contains many interesting particulars, I have printed it in three graduated tones. The reader will certainly notice at first sight how very much larger the nucleus is on *A*. No one can fail to notice that the brightest part of the penumbra, which on *A* is bordering the umbra, is distinctly not so on *B*.

It being now clear that the smaller photographs, whatever their clearness may be, fail to show the minute features of a sun-spot, what is the cause of this deficiency? In my opinion it is to be found in the

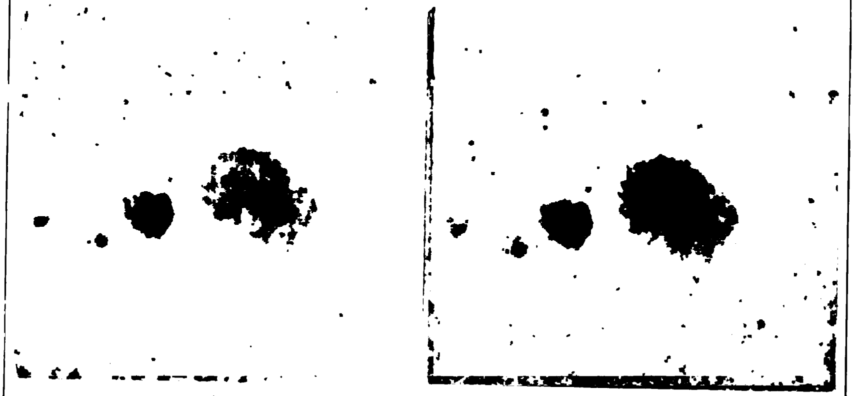
PLATE XV



A. 17 th July 1906 B



A. 14 th September 1906 B



B 14 th September 1906 B

1^{mm} nearly $2'$

small size of the picture. The breadth of the filaments, especially at their sharper end, is, on the enlarged image, only two or three tenths of a millimeter. The same object on the image at the focus of the telescope must therefore have a size of only one or two hundredths of a millimeter. It does not seem that so narrow lines can be printed without confusion on a silver-bromide film. Consequently, where the filaments are shining more brightly their confusion causes a large patch of sensitive film to be blackened. At points where the sharpening of the filaments renders their light both narrower and fainter, their confusion prevents any action of the light to be marked on the film. The granules of the photosphere are very small, their diameter varying from 3″ to 1″ or a little under 1″, and yet they come out distinctly where the filaments of the penumbra appear only in patches. That is true; but the diameter of these granules, even those of only 1″, is still on our original smaller photographs three hundredths of a millimeter; that is, is about three times greater than the breadth of the filaments. Now, it is not very often, but only in the case of an exceptionally clear and calm atmosphere, that the minutest granules are clearly printed on the film. It is therefore not surprising if the filaments, which are considerably thinner, have as yet never been distinctly seen even on our best plates, where they form confused masses, somewhat like skeins of threads placed too far for distinct vision, as shown in Fig. *A*. But they do come out separately in Fig. *B*, since they are enlarged sixteen times *before* being impressed on the film.

On these photographs, series *B*, there are many particulars well worth being attentively considered. I have printed three copies on graduated tones, just to show as far as possible all the details visible on the plate itself. As it is much to be feared that a half-tone engraving, even of the best quality, as those in the *Astrophysical Journal*, will not reproduce the minute details, I have still enlarged the original plate threefold. On that last figure (Plate XVI) one second of arc is equal to 1.6 mm.

In many places around the penumbra the granules of the photosphere are evidently stretched forth in the direction of the filaments, especially on the south, at the origin of the bright "bridge" crossing the west part of the umbra.

At the outer edge of the penumbra there is a ring considerably darker than the middle or brighter ring, and even darker than the inner edge. But it is neither continuous nor identical all around. It is far more strongly marked in the north than in the south of the nucleus; and even in the north it is here and there cut with bright filaments.

Inside this ring comes the brighter part of the penumbra, the breadth and form of which are far from being uniform around the nucleus. Then comes the inner edge formed by the sharp ends of the filaments, which as they are narrowing begin at the same time to fade away from their previous brightness.

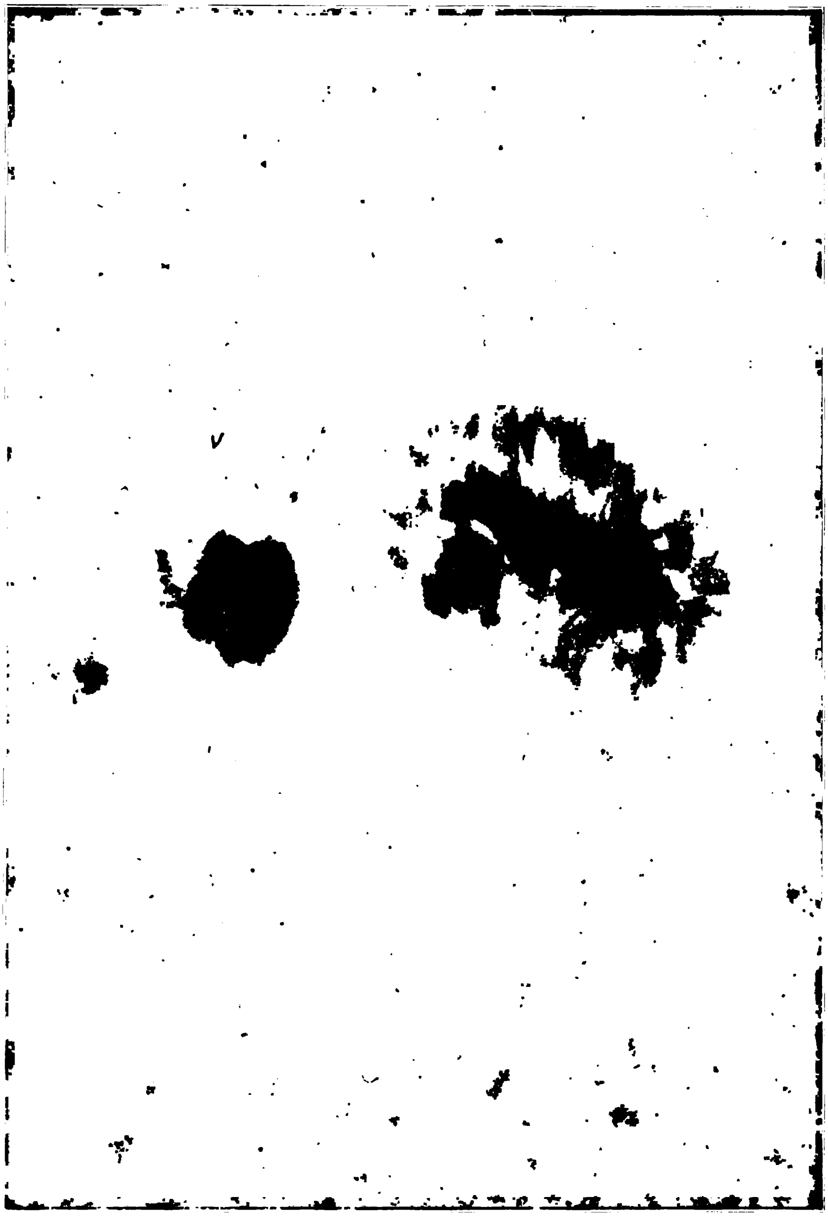
Let us suppose streamers of luminous matter, flowing from the outside toward the center, which would first bend somewhat downward as they cross the outer dark ring, then rise up again on the brighter ring, and lastly, on reaching the edge of the nucleus, begin to rush downward, apparently along a descending slope, and grow thinner and fainter as they gradually melt into it. As far as I can understand a drawing, these would show precisely the same form and bending which strike me when I consider the lines of the filaments.

This therefore seems to give strong support to the opinion that the dark nucleus is a hole in the photosphere; not, of course, a vacuum, but a hole filled up with materials which must be at the same time of lighter density and less brilliancy. The lighter density is shown by the fact that the filaments bend down into it, while on a fluid of greater density they would run upward.

I do not intend in any way to construct a theory of sun-spots, but only to point out what seems to be represented in these interesting photographs.

ZÔ-SÈ OBSERVATORY, NEAR SHANGHAI,
March 22, 1907

PLATE XVI



September 14, 1906. 1 mm = 0″.6, nearly.

Minor Contributions and Notes

ON THE VELOCITY OF METALLIC PARTICLES IN THE SPARK DISCHARGE

The *Astrophysical Journal* for January 1907 contains a paper by Professor G. F. Hull in which certain experiments made by Mr. Hemsalech and myself[1] are referred to, and their explanation as given by us is called in question. Professor Hull's results have already been quoted as invalidating our conclusions; it seems therefore worth while pointing out that there is no real connection between the two sets of experiments. Our work led to the measurement of the velocities of metallic particles under certain conditions in a spark, and Professor Hull failed in his discharges to obtain a displacement of lines corresponding to these velocities; but our paper, as far as I know, contains no sentence which could have led anyone to expect a Doppler effect under the conditions under which Professor Hull experimented, and the fact that no displacement of lines was observed by him cannot therefore be taken to affect our results. If Professor Hull were right in his two contentions, (1) that metal particles of mercury and cadmium volatilized by the spark do not travel at a higher rate than 100 meters a second, and (2) that Mr. Hemsalech and I observed the propagation, not of matter, but only of a luminosity, he ought to have driven the argument to its logical conclusion, which is that a "luminosity" may carry the spectrum of a metal with it, while the metal particles themselves are left behind, giving up their spectral ghost to a phantom which rushes ahead. That his reasoning leads to this paradox I proceed to show.

Your readers may remember that Mr. Hemsalech and I worked with sparks sufficiently intense to produce singly a strong impression of the spectrum. Our method of operation indeed required this in order to secure the object of our research, which was indicated in the first two sentences of our paper in these words: "When an electric spark passes between metal electrodes, the spectrum of the metal

[1] *Phil. Trans.*, **193**, 189, 1899.

appears not only in immediate contact with the electrodes, but stretches often across from pole to pole. It follows that, during the short time of the duration of the spark, the metal vapors must be able to diffuse through measurable distances."

The problem is thus clearly stated. There is evidently no metal vapor in the spark-gap before the discharge has set in, and the first appearance of a metal line proves without question the presence of the metal which might possibly have traveled more quickly than the experiment indicates, and be rendered luminous only at a later stage; if it traveled more slowly, the paradoxical result already mentioned would follow. Our measurements therefore give the lowest possible values of the velocities, and give them with a directness which cannot be invalidated by the absence of any Doppler effects. In the case of cadmium vapor, to which Professor Hull more especially refers, the metal lines appear at a distance of 2 mm from the electrode about the millionth part of a second after the discharge has set in. This is certain because the position of the air lines fixes accurately the time of beginning of the spark. The "luminosity" has got to the center of the spark with an average velocity of about 500 meters per second, and it has found the metal there. The question therefore reduces itself to this. Can the luminosity show the lines of cadmium before the cadmium has reached the spot?

In referring to our own discussion of our results, it will be noted that we favored the view that the motion of the metal particles from the electrode to the center of the spark was a process of diffusion which follows the explosive volatilization of the metal by the spark; we actually showed by calculation that the order of magnitude was the same as that of the vapor treated as if it were flowing from a place of compression into a vacuum. It is clear, therefore, that we did not intend to apply our results to the case of continuous successions of discharges where ultimately some equilibrium in the distribution of the density of the metal vapor must be reached. But if we turn to Professor Hull's experiments, we find that, instead of using, as we did, single sparks taken from charged condensers, he worked with the discharges of the induction coil without capacity at all. Under these circumstances there was no reason to suppose that the effects noticed by us should be observed. Professor Hull does not

give sufficient information on the nature of the discharge he employed, but from the data given it is probable that it resembled that commonly observed in a vacuum tube. At any rate, I have frequently observed, under conditions which were not dissimilar, the cathode glow giving its characteristic spectrum in a sharp fine layer on the cathode, and the positive part of the discharge showing the bands of nitrogen. The electrodes would volatilize to some extent, but they would not be driven explosively into the center of the spark. The two sentences by which he attempts to bring his own experiments into connection with ours rest on an exceptionally bold piece of extrapolation for which there is no justification.

There are no doubt difficulties connected with the interpretation of some of our experiments, notably the function played by the metal in the oscillations of the discharge. Attention was drawn to certain appearances pointing to the fact that the metal vapor acted as conductor in the discharge, and that the oscillation of the spark produced periodic changes in the velocities of the metal particles. Mr. Royds is at present repeating our experiments with improved apparatus, and the preliminary results obtained show that his investigation will clear up some doubtful points. While I do not deny that when an electric discharge is established, the propagation of a "luminosity" may simulate to some extent the appearance of propagation of luminous matter, and that some observed effects may thus be accounted for, I think that a careful examination of the reproductions of photographs published in our paper might have convinced Professor Hull that such an explanation cannot be applied to our experiments. Is it likely, for instance, that a "luminosity" would set out from the pole with a large velocity which gradually diminishes and ultimately settles down to rest, unless it were connected with the transport of material particles?

ARTHUR SCHUSTER

VICTORIA PARK, MANCHESTER
March 24, 1907

NOVA T CORONAE OF 1866

This object, which suddenly blazed up to the 2d magnitude on May 12, 1866, and then faded to the 9½ magnitude, proved to be a star which had been previously observed by Argelander, being *B.D.*

$+27^\circ 2765$. After this extraordinary outburst, it apparently returned to its original condition, at which it has ever since remained.

This is the first of the novae to be examined with the spectroscope. Observations were obtained of its spectrum when bright in 1866 by Huggins, who was just introducing the spectroscopic study of the stars.

As this is the only example of the novae which was known previous to its outburst—with the probable exception of *Nova Persei* of 1901 (See *H.C.O. Circular* No. 66), and as it is the brightest of these bodies at present visible, it is important that we should study it carefully, in the hope that some information as to the cause of these outbursts may be gained. The present observations are therefore intended to be a small contribution to the history of the star at the present time.

Some of the other novae which are still visible present a hazy or out-of-focus image in a large telescope, viz., *Nova Cygni* of 1876, *Nova Aurigae* of 1892, *Nova Sagittarii* of 1898, etc.

To see if anything peculiar existed in the telescopic image of *T Coronae*, I have examined it several times of late years with the 40-inch telescope. Its focus was measured with respect to that of the $9^m.1$ star *B.D.* $+26^\circ 2761$, which precedes it 1 minute of time, and is $1\frac{1}{2}'$ north. The star was also examined inside and outside the focus. There was nothing in these observations in any way to suggest a difference from the ordinary star. We must, however, remember that, so far as the extra focal images of the novae are concerned, the peculiarity has not usually made its appearance until the star has become much fainter than the present brightness of *Nova Coronae*. The strong visual light may therefore mask this peculiarity in the case of this star.

The following focal measures were carefully made:

		Inches
1903 May 4.	Focus on *Nova*. Scale	= 2.22 (3)
	on *B.D.* +26.2761	2.21 (3)
	Diff.	+0.01 inch

Of course, this quantity is too small to be a real difference in a focal determination with the 40-inch telescope, though it is in the right direction—being a little farther from the object-glass—for a nova.

The image was also tested on several other dates, though no measures were made. The star was also observed for color:

1904 May 4. "There is no color to the star."

1906 Aug. 28. "It is almost colorless, but the seeing is too bad to be certain." Later: "So far as I can judge, the star is white."

1906 Sept. 11. "It may be tinged with yellow, but not certain. It is not *white*."

For the brightness of the *Nova* the following observations were made:

1906 Aug. 28. Independent estimate $= 9^{\text{m}}.3$.
Aug. 28. $\frac{2}{10}^{\text{m}}$ less than $+26°.2761$
Sept. 11. $\frac{3}{10}^{\text{m}}$ less than $+26.2761$

In the *B.D.* these two stars are:

$+26°.2761$ $9^{\text{m}}.1$
$+26.2765$ 9.5 (*T*)

It would seem, therefore, that the *Nova* is now essentially of the same brightness it was before the outburst in 1866.

These two stars are Cambridge *A.G.C.*, 7412 and 7433, and their places, brought up to 1906.0, are:

	15^{h}	54^{m}	$32^{\text{s}}.39$	$+26°$	$12'$	$38''.8$
T	15	55	34.32	$+26$	11	10.2
	$\Delta\alpha + 1^{\text{m}}$		$1^{\text{s}}.93$	$\Delta\delta - 1'$		$28''.6$

On 1906 August 28, I compared these two stars for position of the *Nova:*

$\Delta\alpha = +1^{\text{m}}\ 1^{\text{s}}.85$ (8 transits)
$\Delta\delta = -1'\ 27''.1$ (2 measures)

It will be seen that my measures make the $\Delta\delta$ $1''.5$ smaller than the value given by the Cambridge observations. If, however, this error is divided between the two stars in the Cambridge observations, the discordance would not be greater than would be expected from good meridian positions. So there seems to be no indication of motion in the *Nova*.

A NEBULA NEAR THE *NOVA*

While examining the star, I found a faint nebula in the field with it, following. The nebula is of the 14th or 15th magnitude, and is from $5''$ to $10''$ in diameter, without any nucleus.

The following measures of this nebula and the *Nova* were made:

		$\Delta\alpha$	$\Delta\delta$
1903	April 27	$+4' 50''.9$ (1) $= +0^m 21^s.62$	$+1' 13''.1$ (1)
	April 28	$+4\ 50.5$ (1) $= +0\ 21.60$	$+1\ 13.4$ (1)
1906	Sept. 11	$+4\ 49.1$ (2) $= +0\ 21.48$	$+1\ 11.1$ (2)

The discordances in these results are purely accidental, from the faintness and indefiniteness of the nebula.

From these observations, and the Cambridge position of the *Nova*, the place of the nebula is:

$$1906.0 \quad \alpha = 15^h\ 55^m\ 55^s.85\ ; \qquad \delta = +26^\circ\ 12' 21''.9\ .$$

E. E. BARNARD

YERKES OBSERVATORY
April, 1907

REGULAR AND DIFFUSE REFLECTION

In the *Astrophysical Journal*, **24**, 351, 1906, the writer is quoted as having found that "various common minerals which reflect only *diffusively* in the region of the spectrum of shorter wave-lengths than 8 μ have bands of metallic reflection from 8.5 to 10 μ." Since then the original paper is being interpreted from the standpoint of "diffuse" and "metallic" reflection. The whole is based upon a misconception and misquotation of the original statement,[1] viz.: "The writer found the reflecting power [of various silicates] to be practically zero for the region of the spectrum up to 8 μ, followed by bands of metallic reflection from 8.5 to 10 μ."

Since the surfaces examined were in nearly all cases plane and highly polished, there was no "diffuse" reflection, which is an entirely different question from low, "practically zero," reflection; and it would indeed be very unfortunate to have such an idea (that diffuse and low reflection are synonymous) gain foothold. It is with the hope that such hazy notions may be cleared up that the present comments are written.

When energy is reflected from a plane surface, it is commonly called "regular" (or, less accurately, "specular") reflection. On the other hand, energy reflected from a rough surface suffers "diffuse" reflection. In "diffuse reflection" for each infinitesimal surface the

[1] *Phys. Rev.*, 23, 247, 1906.

ordinary laws of reflection are obeyed in full, unless the linear dimensions of the reflecting surface, or of the rugosities or inequalities on it, are small compared with the wave-length. However, the unpolished surface as a whole destroys all phase-relation between the particles in the reflected wave-front, which is no longer plane, but irregular.[1] This irregularity decreases as the angle of incidence increases, so that for a given roughness we get regular reflection. The long waves will be reflected first, then the shorter. "Smoked glass, which at perpendicular incidence will show no image of a lamp at all, will at nearly grazing incidence give an image of surprising distinctness, which is at first reddish, becoming white as the angle increases."[2]

The amount of energy reflected "regularly" from a plane surface will depend upon the reflecting power of the substance. Now, the reflecting power, R, of any substance is related to its index of refraction, n, and its absorption coefficient, k, by the equation

$$R=\frac{(n-1)^2+k^2}{(n+1)^2+k^2}.$$

For "transparent media," i. e., "electrical non-conductors,"[3] the absorption coefficient is so low that it is negligible, and the reflecting power is a function of only the refractive index. Here the reflecting power is low, only 4 to 6 per cent., and decreases with increase in wave-length. All transparent media thus far examined (except silver-chloride) show bands of selective reflection. In these bands the absorption coefficient, k, attains high values.

If k becomes sufficiently large,[4] of the order unity, the absorption affects the reflecting power, and the heat- and light-waves no longer enter the substance, but are almost totally reflected, as in metals; whence the name "bands of metallic reflection." For metals, "electrical conductors," he absorption coefficient is so large that nearly all the energy, for nearly all wave-lengths, is reflected.

If, then, the eye were sensitive to heat-waves, many substances would have a "surface color" similar to that of gold and fuchsine

1 See Wood's *Optics*, p. 36.

2 *Ibid.*, p. 37.

3 See Drude, *Optics;* also Schuster, *Optics;* Pockels, *Lehrbuch der Crystallographie*.

4 See Schuster, *Optics*, p. 250; Pockels, *Crystallographie*, 1906.

in the visible spectrum. In other words, the reflecting power of plane surfaces ("regular reflection") of "transparent media" will be low for all regions except where there are bands of "metallic reflection." It is evident that the diffuse reflection from rough surfaces of transparent media will also be selectively reflecting. Hence in the stream of energy reflected in any direction the density will be greatest for wave-lengths 8.5 to 10 μ, for silicates. In this case, however, the term "metallic reflection" is not to be used synonymously with the expression "specularly reflected," as it appears to be in this *Journal* (*loc. cit.*, pp. 351 and 353); for the latter refers to "regular reflection." Since the energy-density in every direction must be greatest for wave-lengths 8.5 to 10 μ, one would expect to detect this difference in any direction, and not simply at the angle of regular reflection.

As a whole, then, what the writer found is that the silicates reflect like transparent media for wave-lengths up to 8 μ, and like metals from 8.5 to 10 μ. In other words, it may be said that in quartz, SiO_2, the silicon ions retain the proper period of undamped electrical vibration which they would have in the element silicon (which has metallic as well as non-metallic properties), for wave-lengths 8.5 and 9.03 μ, while for all other wave-lengths the vibration periods are more damped and the reflecting power is decreased.

W. W. COBLENTZ

BUREAU OF STANDARDS, WASHINGTON
March 26, 1907.

VARIABLE RADIAL VELOCITY OF *U CEPHEI*

Measures of plates of this star, made with the single-prism spectrograph, show a variation of 95 kilometers in the radial velocity. This star is classed with *Algol* variables, and its light-variation is recognized as somewhat peculiar. It has a period of 2.5 days and a variation from the 7.1 to the 9.2 magnitude. The spectrum is of the early hydrogen type. There is indication of complexity in the lines, suggesting that both components of the system may be bright. More plates will be needed to give this interesting question a definite answer.

V. M. SLIPHER

THE SPECTRUM OF ϵ *CAPRICORNI*

Spectrograms of this star made last autumn, with the single-prism spectrograph, show its spectrum to contain bright lines. At Harvard the star was found to have a peculiar spectrum, but the bright lines were not recorded. My previous plates of the star, from which its variable velocity was discovered, were made with high dispersion and were, owing to this and their narrowness, unsuited to reveal the bright lines. On the late plates the hydrogen lines are paired—the sharp, dense dark line with a weak bright line above it. As is usual with this type of peculiar spectrum, the emission line is more intense in the lower members of the hydrogen series. On the recent plates, which unfortunately extend over only a short period of time, the dark hydrogen and metallic lines were displaced toward the red by an amount corresponding to a velocity of 45 kilometers on October 8, which had decreased to 35 kilometers by October 27. In addition to these dark lines the spectrum contains also helium absorption lines, which are, however, very broad and indefinite, and do not share with the other dark lines the shift toward the red. Measures on some of the less diffuse ones indicate, on the other hand, a small displacement toward the violet. It seems probable, therefore, that the dark hydrogen and metallic lines belong to one component, and the diffuse helium and bright hydrogen lines are due to the other component, and that the weak bright lines are blotted out by the intense dark ones, except when the relative velocity of the two stars is sufficient to separate well the two lines.

V. M. SLIPHER

LOWELL OBSERVATORY, FLAGSTAFF, ARIZ.
February, 1907.

REVIEWS

Generalkatalog der Potsdamer photometrischen Durchmusterung des nördlichen Himmels, Enthaltend die Gröszen und Farben aller Sterne der *B. D.* bis zur Grösze 7. 5. VON G. MÜLLER und P. KEMPF. *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, Nr. 52, Siebzehnter Band, 1907.

Following closely Part IV, containing the final zone measures, comes the general catalogue giving the revised magnitude and color results for the northern heavens, with the stars arranged in order of right ascension. This noble volume contains 14,199 stars, the various columns giving data as follows:

1. Current number.
2. *B. D.* number.
3. R. A. for 1900 to seconds of time.
4. Declination for 1900 to tenths of a minute of arc.
5. *B. D.* magnitude.
6. Measured magnitudes, usually two, one each by Müller and Kempf.
7. Photometer used.
8. Mean estimated color.
9. Mean magnitude.
10. Remarks, giving Bayer's letter or Flamsteed's number; Struve's or J. Herschel's number for doubles.

The exceeding usefulness of this general catalogue puts the astronomical world under great obligations to its authors; but the valuable introduction is equally noteworthy and deserves a more extended notice than can be given here. The catalogue is not a mere rearrangement of the results of the different zone catalogues; but these have been discussed, their mutual relations investigated, and systematic corrections applied so as to make from the four parts one homogeneous whole. All this has been carried out with the scientific thoroughness and critical ability which are characteristic of the staff of the Potsdam Observatory. The completed work therefore stands alone in the literature of stellar photometry, and does much toward putting that branch of the science on a level with the astronomy of position.

The discussion in the introduction falls under two heads: first, the corrections to be applied to the measures by the two observers and with the four different arrangements of the photometers, with respect to

magnitude and color; second, a comparison of the magnitudes, thus corrected, with the various Harvard catalogues. The original observations were planned to furnish the data required for the comparisons first mentioned. An equal number of observations were made on each star by each observer, and the mean used, so that only the difference in instruments needed consideration. Two photometers were used: the smaller, D, with an objective of 135 mm aperture; the larger, C, with objectives of (I) 67, (II) 36.5, and (III) 21.5 mm. Since the magnitudes of the 144 fundamental stars had been determined with D, the results obtained with C were reduced to that standard. The systematic differences of magnitude between the two photometers, due to star colors, were so small, 0.01 and 0.02, as to be negligible. On the contrary, the progression due to magnitude was pronounced; the corrections to C_I ranging from −0.09 at magnitude 3.5 to +0.09 at magnitude 7.4, and the corrections to D ranging from −0.11 at 4.8 to +0.11 at 9.0. These differences seemed due to the relative brightness of the standard and measured stars in each instrument; the standards used with C averaging 5.4, those used with D 6.8. The corrections applied to C_{II} and C_{III} showed no progression, and were 0 and −0.03 respectively.

The color estimates were also reduced to the system of photometer D, the corrections applied to C_I and C_{II}, in terms of the small Potsdam steps (18 between white and red), increasing from 0 for white stars to 2 for the more intensely colored ones. A comparison with Osthoff's color-scale (799 stars in common) showed the Potsdam step equivalent on the average to 0.4 of Osthoff's unit, but this quantity varied from 0.1 to 1.1 in the means for the different groups.

Before comparing with Pickering's results, it was necessary first to investigate the relations between the various Harvard catalogues. Although Vol. **45**, like the Potsdam Durchmusterung, contains all the stars to magnitude 7.5, it was found impossible to compare with it directly, as the magnitudes of 5217 of its stars were taken from older catalogues, and considerable systematic differences were found between the various volumes, thus depriving Vol. **45** of the character of a homogeneous *Durchmusterung*. After deducing these systematic differences, ranging from 0 to 0.2 or 0.3, the brighter Potsdam stars, measured with C_I and C_{II}, were compared with Harvard **44**, and the fainter stars measured with D, with Harvard **45**. The following formula expresses the difference between the two systems:

$$\text{Potsdam-Pickering} = +0^{m}.220 + 0^{m}.024\, m - 2^{m}.25 - 0^{m}.027\, C - 0^{m}.008\, m - 2^{m}.25\, C\,,$$

in which m indicates magnitude and C color (white, 0; yellowish white, 1; whitish yellow, 4; and yellow, 6). The residuals left after applying this

formula are tabulated on page xxxii. They range from 0 to 0.09 magnitude, the mean value, without regard to sign, being 0.02 for groups containing on the average 113 stars.

The general conclusions may be condensed as follows:

1. Accidental errors in measures with the Zöllner photometer can be confined to a few hundredths of a magnitude.

2. Systematic errors, in spite of the utmost precautions, may amount to $0^m.1$ per magnitude.

3. Each series of photometric measures should be corrected for systematic differences before being combined with another series.

4. Star colors have a greater effect on magnitude measures than had been supposed.

5. Visual estimates of star colors apply only to the particular eye and instrument used.

Though not mentioned by the authors, it occurs to the reviewer that the above conclusions suggest that further advances in stellar photometry, especially in the allowance which must be made for color, can best be made by photographic methods.

J. A. PARKHURST

Cours d'Astronomie. Première partie: Astronomie théorique. Par H. ANDOYER. Paris: A. Hermann, 1906. Pp. 221, with numerous diagrams. Fr. 9.

This first part of Professor Andoyer's work is divided into fifteen chapters. It is not printed from type, but reproduces a very distinctly hand-printed manuscript. The principal object of the book is stated by the author to be the study of the apparent motions of the celestial bodies. After an introductory chapter on spherical trigonometry, the author considers the various systems of co-ordinates used in astronomy (chapters 2-5). To refraction, parallax, and aberration the succeeding three chapters are devoted. Chapter 9 gives a short initiation into the principles of celestial mechanics, upon which he draws not inconsiderably in chapters 10-12, which treat respectively of the theory of precession and nutation, and the apparent motion of the sun, with special reference to the notion of time. The last three chapters (13-15), to which group in a fashion parts of chapter 12 likewise belong, deal with the motion of the movable celestial bodies, sun, moon, planets, and satellites, and the last chapter in particular with the eclipses of these bodies.

A book which comes from the pen of so eminent a scholar as Professor Andoyer has of course in a high degree those remarkable qualities for

which the French textbooks have become the exponents *par excellence:* clearness and elegance. If in what follows attention is called to some minor shortcomings, they are offered with proper appreciation of those excellent qualities. The book is evidently intended for beginners; otherwise one could not understand the entire absence of references from cover to cover. The knowledge of calculus seems to be all that the author presupposes in the way of prerequisites. There is but one paragraph where higher attainments on the part of the reader are called into service—in the ninth chapter, where the author speaks of the convergence of the Fourier series and arrives at Laplace's and Rouché's result. This might well have been left out, since it is decidedly beyond the understanding of the average reader. Instead, a more detailed use of the fundamental principles of analytical mechanics might have added much to the understanding of this chapter. The author probably deliberated a good deal where to place the contents of this chapter to best advantage. Brünnow succeeded well in giving the matter a place in the early part of his book. The present arrangement likewise has advantages, and, after all, such matters depend a good deal upon the personal predilections of the author.

The very complete introduction to spherical trigonometry, which comprises many details for purely geodetic students, creates the impression in the reader that this line of investigation will be taken up in the further issues of Andoyer's work. But the next following chapter somewhat disillusions him, when the short paragraph given to the ellipsoidal figure of the earth is considered. The custom which prevails in some textbooks, of considering "reduced" latitude and "geocentric" latitude as synonymous terms, is adopted in this treatise too. This is unfortunate, especially since Helmert's fundamental treatise distinctly defines reduced latitude in precisely the manner in which the eccentric anomaly is defined in the elliptic motion. Andoyer's "auxiliary" angle is Helmert's reduced latitude. A figure which shows distinctly the three different kinds of latitude would be quite in place. The figure given in the text does not convey clear conceptions, since a point M at an elevation MM' is made the starting-point for the discussion. It is the surface point to which the figure should refer.

In conclusion, attention is called to the contents of the last three chapters, which in previous textbooks on the same subject have received but scant recognition, and which open the way to a more vivid conception of the subject-matter that evidently will follow this very excellent first part of Professor Andoyer's book.

KURT LAVES

MARCH 19, 1907

Physical Optics. By Robert W. Wood. New York: The Macmillan Company, 1905. Pp. xiii + 546; figs. 325 and four plates. $3.50.

Every reader of Professor Wood's *Physical Optics* must be impressed with the value of the book as a compendium of the best modern views on optical phenomena. And it is a great satisfaction to find a book so full of the most valuable theoretical and experimental data which is written in a clear, forceful, and original style, always from the standpoint of the physicist rather than from that of the mathematician or the mere statistician.

Although the author makes no claim to originality in the mathematical treatment, he is to be congratulated on his excellent choice and arrangement of the mathematical material, and on the clear, concise, and illuminating discussion with which he has amplified it. The general methods of Verdet are followed in the earlier chapters, while specific paragraphs are credited to their various originators: e.g., Airy's treatment of diffraction by a circular aperture, Runge's theory of the concave grating, and Michelson's theory of the echelon. Drude's unsurpassed treatment, based on the electro-magnetic theory, has been closely followed in some of the later chapters.

One of the most interesting chapters in the book is that on "The Theory of Dispersion." The Cauchy, Sellmeier, Helmholtz, and Ketteler-Helmholtz formulae are discussed in order, and followed by Drude's treatment from the electro-magnetic standpoint. The fascinating experimental results of Rubens, Nichols, Aschkinass, Pflüger, and of Professor Wood himself, are given in illustration and verification of the formulae.

The chapters on "Optical Properties of Metals," "Rotatory Polarization," and "Magneto-Optics" are rich in valuable theoretical and experimental material, and the chapter on "Transformation of Absorbed Radiation" includes the remarkable results recently obtained by Professor Wood with sodium vapor. The chapter on "The Laws of Radiation" is also one of great interest, including a brief statement of the theoretical results of Larmor, Stefan, Boltzmann, Wien, and Planck, and the experimental results of Nichols and Hull, Lummer, Pringsheim, Rubens, Kurlbaum, Pflüger, and others.

One cannot read the book without being impressed anew with the large amount of wonderfully ingenious qualitative work which Professor Wood has done, especially in the domain of anomalous dispersion and the optical properties of metallic vapors.

The author apologizes in his preface for the large number of references to his own work, but one is glad that modesty did not prevent him from

including them, as the collecting in this way of so much of his work is one of the valuable features of the book. The several references to the "Schlieren-Methode" of Töpler, and the author's photographs of sound-waves by this method, although illustrating the points in question very well, would, however, perhaps be more in place in a general textbook than in a work on physical optics. The description of a lecture-table experiment for illustrating the phenomena of mirage and a two-page account of a mica echelon which, as he says, is "of no use as a piece of optical apparatus" and "useless as a tool for research," seem to be somewhat below the high scientific plane of the book as a whole. There are the usual number of typographical errors which creep into a first edition. They are especially conspicuous in the formulae of the chapter on "The Theory of Dispersion."

The thoroughly up-to-date character of the book is evidenced by the chapters on "The Laws of Radiation," "Transformation of Absorbed Radiation," "The Nature of White Light," and "The Relative Motion of Ether and Matter." Numerous references are found throughout the book to articles which appeared as late as 1905.

The book should be read to be properly appreciated, and it should be read by every student who wishes to be thoroughly informed on what is best and latest in physical optics. It will perhaps prove to be one of the most valuable of the many contributions for which the science of physics is already indebted to Professor Wood.

H. G. G.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* shorter articles will generally be placed and subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts type-written, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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The ASTROPHYSICAL JOURNAL is published monthly except in February and August. The annual subscription price is $4.00; postage on foreign subscription 75 cents additional. Business communications should be addressed to *The University of Chicago, University Press Division, Chicago, Ill.*

All papers for publication and correspondence relating to contributions should be addressed to *Editors of the* ASTROPHYSICAL JOURNAL, *Yerkes Observatory, Williams Bay, Wisconsin, U. S. A.*

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XXV — JUNE 1907 — NUMBER 5

THE HELIOMICROMETER[1]

BY GEORGE E. HALE

The measurement and discussion of photographs of the sun made with the spectroheliograph involve a large number of processes, some of which have been satisfactorily worked out, while others are still under consideration. Up to the present time special attention has been devoted to the following points:

1. The measurement of the heliographic positions of flocculi, with reference to the determination of the solar rotation.
2. The measurement (with the monocular micrometer of the Zeiss stereocomparator) of the relative distances from the center of the sun of corresponding points in the calcium (H_2) and hydrogen ($H\delta$) flocculi, with reference to the relative level of these objects.
3. Comparative studies, with the same instrument, of the forms of flocculi, as photographed with various lines.
4. Photometric determinations of the relative brightness, at various distances from the sun's center, of the flocculi and the adjoining photosphere, as bearing on the level of the flocculi and the absorption of the solar atmosphere.
5. Photometric measurements similar to (3), but especially adapted for the determination of the level of sun-spots.
6. Stereoscopic studies of the flocculi.
7. Studies of the flocculi with a kinetoscope, with reference to their changes in form.

[1] *Contributions from the Solar Observatory*, No. 16.

8. The measurement, by a photometric method, of the areas of flocculi in regions ten degrees square, for the study of variations in the solar activity.

Spectrographic studies have also been made, particularly of the hydrogen and calcium lines, for the purpose of interpreting the spectroheliographic results.

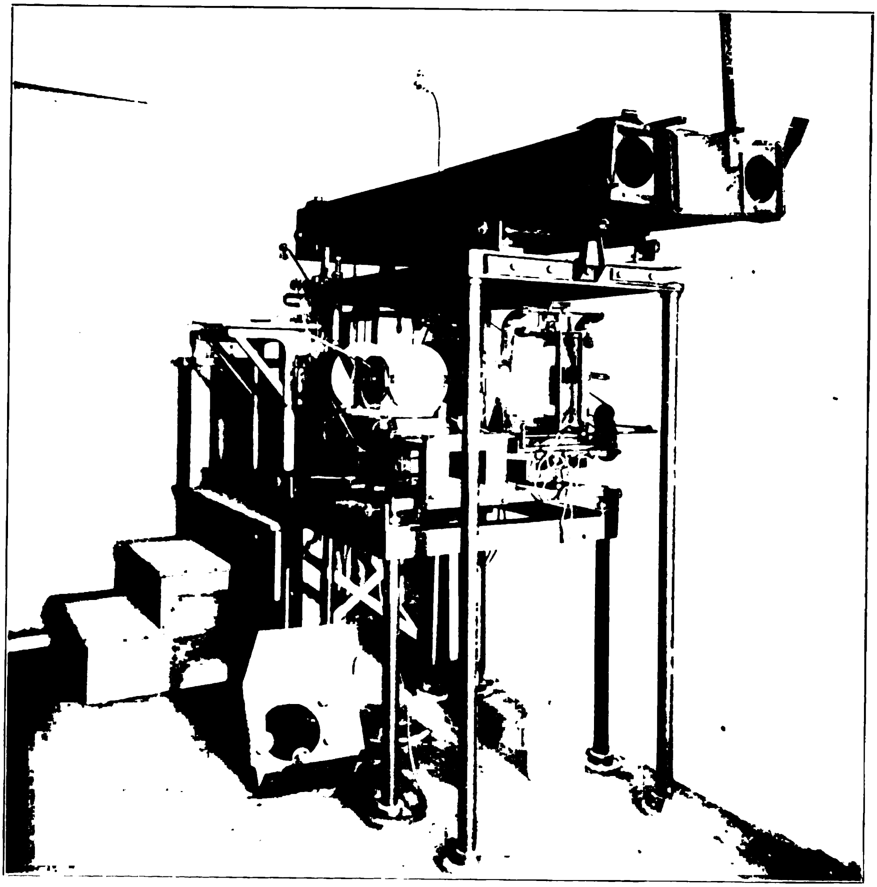
The present paper deals with the measurement of heliographic positions; the other methods of measurement and reduction will be described later.

THE HELIOMICROMETER

The determination of the heliographic positions of sun-spots, as carried on at Greenwich, involves the measurement of their polar co-ordinates, and a simple calculation, facilitated by the use of tables. As the average number of spots on each photograph is small, only a moderate amount of computing is required, and the method has apparently proved satisfactory after many years of service. For the flocculi the case is different. The average number of flocculi suitable for measurement on a single plate may reach forty or fifty, thus involving a large amount of computing in the aggregate. Furthermore, the lack of sharpness of the flocculi (as compared with spots), and their rapid changes of form, rarely permit of the precision of setting attainable in the case of spots, and render a different mode of measurement feasible. It will appear, however, that in its perfected form the instrument described in this paper is capable of giving results no less precise than the ordinary measuring machine.

The first and simplest form of the instrument, which I devised for the measurement of the Kenwood spectroheliograph plates by Mr. Fox at the Yerkes Observatory, consists of a metallic globe, with smooth white surface, upon which the solar image is optically projected. The axis of the globe being set at the inclination required by the date of the photograph, and the plate properly oriented, it is only necessary to read off the heliographic latitudes of the flocculi and their differences in longitude from the sun's center, by the aid of parallels and meridians ruled, one degree apart, on the surface of the globe. The results obtained with this simple device were so satisfactory that it is still regularly employed in the measurement

PLATE XVII



The Heliomicrometer

of the Rumford spectroheliograph plates. The essential condition to be met is that the angular diameter of the globe, as seen from the projecting lens, shall equal the angular diameter of the sun, as seen from the earth.[1]

In the second form of the instrument, as constructed in the shops of the Solar Observatory, two 4-inch (10 cm) telescopes were mounted parallel to one another. One of these pointed at a globe, 60 feet (18.29 m) distant, beside which stood the solar photograph to be measured, with its center in the optical axis of the other telescope. The images of globe and plate, as formed by the two telescopes, were brought together in a single eyepiece, by means of a half-silvered prism. After the photograph had been oriented and centered on the globe, by adjustments controlled from the observer's seat, the latitudes and longitudes of the flocculi were read off with respect to parallels and meridians, ruled one degree apart, on the surface of the globe. This instrument gave good results, but the illuminated surface of the globe (black lines on a matt silvered surface) interfered with the visibility of the minute flocculi. Experiments with cross-hairs for setting purposes, and the desire to secure a higher degree of precision than estimates to a tenth of a degree could afford, soon led me, by successive steps, to the design embodied in the "heliomicrometer," which is illustrated in Plate XVII.

The two 4-inch telescopes, of 60 inches focal length, which were used in the second form of the instrument described above, are retained and mounted immediately above the globe and the plate-carrier. These telescopes point to the centers of two optically plane mirrors, mounted on a concrete pier at a distance of about 30 feet (9.14 m) In the telescope on the right, the solar image, brilliantly illuminated by transmitted light, is seen after reflection from one of the plane mirrors. In the same way the globe, illuminated by reflected light, may be seen in the telescope on the left. The images are brought together in a single eyepiece, with the aid of a right-angle prism at the end of each telescope and a half-silvered prism, similar to that used in the monocular eyepiece of Zeiss's stereocomparator, mounted a short distance in front of the eyepiece.

[1] A full description of this instrument will soon appear in a paper by Mr. Fox and myself on the solar rotation.

As a complete description of the apparatus will be published later, only its general features are described in the present paper. The adjustments of the plate-carrier permit the photograph to be raised or lowered, moved right or left, rotated for orientation, and moved toward or from the mirror. Cross-hairs, mounted immediately in front of the plate,[1] are controlled from the eyepiece, and their intersection can be made to coincide with the flocculus to be measured. While settings are thus being made, the object-glass of the telescope which points to the globe is covered by a swinging shutter, controlled by the observer. Under these conditions the details of the solar photograph are seen with great clearness, and settings are made as easily and rapidly as in any form of measuring machine.

The entire globe has a matt silvered surface, and one hemisphere shows no lines except the equator and central meridian. These lines do not quite reach the center of the globe. At the point where they would intersect, a small circular black dot is engraved. This is brought into coincidence with the intersection of the cross-hairs, by rotating the globe in longitude and latitude. In longitude the globe has both quick and slow motions, permitting a rapid and accurate setting to be made. In latitude a single motion, consisting of a worm-gear operated by an arm with double Hooke's joint, gives sufficiently rapid motion, combined with perfect control. The longitude circle, provided with a vernier reading to tenths of a degree, or, if desired, to 2′, is read with a telescope from the eye-end. The globe is then turned back to zero longitude, and at this point the latitude circle, also reading to tenths of a degree or to 2′, is read with a second telescope.

The vertical axis that carries the fork in the opposite bearings of which the horizontal axis of the globe rests, can be set at any desired angle, read by means of a vernier and divided circle. The inclination of this axis corresponds to the tabulated latitude of the center of the sun for the date of the photograph.

It is unnecessary to give here the details of the adjustments of the apparatus. It may be added, however, that the photographs made with the Snow telescope and 5-foot spectroheliograph are oriented by the aid of control plates, on which the solar image is allowed to

[1] The same support carries the electric pen, referred to below.

drift between two exposures. A line joining two images of the same flocculus gives the east and west direction. The angle between this line and a line drawn on the plate by the image of a needle-point mounted in contact with the collimator slit provides the means of orientation. The centering of the solar image on the globe is easily effected, with the aid of a metallic screen mounted in front of the globe and bearing a black circle, adjusted so as to be exactly concentric with the globe. This circle should coincide, when the instrument is in adjustment, with a black circle etched in a glass plate permanently fixed in the adjustable plate-carrier. This circle is slightly larger in diameter than the solar photograph which can thus be centered with great precision. In order to provide for the varying size of the image, due to the varying distance of the sun from the earth, the plate-carrier can be moved along rails, so as to change its distance from the telescope with which it is observed. The necessary adjustment of focus is made by moving the object-glass of the corresponding telescope.

Measurements are confined to objects within fifty degrees of the sun's center. This is for the purpose of avoiding errors inevitable when spots or flocculi are measured in the near neighborhood of the sun's limb. Furthermore, it is not necessary, within this limited region, to change the focus of the telescope used to observe the globe.

In testing the heliomicrometer, a photograph of the sun was measured with this instrument and with an ordinary measuring machine giving polar co-ordinates. After the reductions had been completed, it was found that the latitudes and longitudes of the flocculi, measured in the two ways, agreed within the limits of error of settings (one or two tenths of a degree). The operations of measurement proved as rapid with the heliomicrometer as with the other measuring machine, and all the time required for calculations was saved. A further test was made by measuring the positions of certain sun-spots, which also appear on the Greenwich photographs. Mr. Maunder was kind enough to give me the positions of these spots as determined at Greenwich. The results correspond within a few tenths of a degree, or as closely as could be expected under the circumstances.[1]

[1] The centers of spots on spectroheliograph plates are often slightly displaced by overhanging flocculi.

It should be stated here that Dr. Frank Schlesinger, with whom I had the benefit of discussing the first globe measuring instrument used at the Yerkes Observatory, suggested, in 1902, that an ordinary theodolite, provided with a small pointer in place of the telescope, might give good results in this work. The details of the apparatus were not worked out, and I had forgotten the suggestion, when I was led by the above described steps to the adoption of the same principle, namely, the use of divided circles in place of meridians and parallels ruled on a globe. This important element in the design is therefore due to Dr. Schlesinger. It is evident that a globe is not really required (as a pointer might suffice), but it has the advantage of permitting one surface to be divided into ten-degree squares, used as described below.

My attention has been called, by Professor A. E. Douglass, to his paper published in *Popular Astronomy* (**5**, 57, 1897), entitled "The Astronomer's Globe." This globe was used for the solution of spherical triangles. The positions of markings on *Mars* were also plotted on the surface of the globe, and their latitudes and longitudes read off with the aid of two divided semi-circles, which could be moved over the surface. The same, of course, could be done with sun-spots. I do not understand that any system of optical projection was used with this globe.

It will be readily seen that the heliomicrometer can be used as a stereocomparator (with monocular or binocular vision). For this purpose it is only necessary to mount in front of the globe a second photograph, illuminated by transmitted light. Miss Ware, who is in charge of the heliomicrometer, uses this arrangement for comparing the forms of the flocculi photographed on successive days, in order to select points suitable for measurement. By means of an electric pen, controlled from the eyepiece, a small dot is made on the glass side of the plate, for purposes of identification, near each point selected.

One hemisphere of the globe is ruled with meridians and parallels ten degrees apart. When seen in projection on the globe (the axis of which is given the proper inclination for the date in question), the intersections of the squares can be marked on the glass side of the plate by means of the electric pen. Plates marked in this way are used by Miss Smith in her photometric measures of the areas of

the calcium flocculi within ten-degree regions. The data thus secured are employed in our studies on the variation of the solar activity, with reference to the condition of the sun as a whole, and its activity in different zones of latitude and longitude.

The heliomicrometer could be improved by supporting all parts of the apparatus shown in Plate XVII on a stone or concrete pier. The present instrument, which is fastened to a concrete floor and braced to an adjoining brick wall, gives very satisfactory results, but is less stable than if it had been designed anew, instead of being modified from the second form of the instrument, described above. I am indebted to Mr. Pease for working out the detailed drawings.

In view of the rapidity of measurement, and the accuracy of the results obtained with this instrument, I can recommend it for the determination of the heliographic positions of spots and flocculi shown on direct photographs or on spectroheliograph plates.

APRIL, 1907

A PHOTOGRAPHIC COMPARISON OF THE SPECTRA OF THE LIMB AND THE CENTER OF THE SUN[1]

By GEORGE E. HALE AND WALTER S. ADAMS

Prior to 1873, although the spectra of the limb and the center of the sun were compared by several observers, no differences in the relative intensities of the Fraunhofer lines were detected. In that year Hastings[2] noticed that some of the lines changed their appearance at the limb. In 1879 and 1880 he continued the investigation with improved apparatus. The results of the later observations, which were made visually with a Rutherfurd grating spectroscope, are given by Hastings as follows:

I. The most important fact of all is that the differences in the two spectra of center and limb are extremely minute, escaping all but the most perfect instruments, and all methods which do not place them in close juxtaposition.

II. Certain lines, the thickest and darkest in the spectrum, notably those of hydrogen, magnesium, and sodium, which appear with haze on either side, in the spectrum of the center of the solar disk, are deprived of this accompaniment in that of the limb.

III. Certain very fine lines (four observed) are stronger at limb.

IV. Other very fine lines (two or three observed) are stronger at center.[3]

Hastings, referring to the spectrum of sun-spots, remarks:

As is well known, such a spectrum exhibits a very strong general absorption, with a very slightly modified selective absorption. A few faint lines appear in the spot spectrum which are not otherwise seen; and a few faint lines of the ordinary spectrum are strengthened. A careful examination has persuaded me that the spectrum of a spot differs from that of the unbroken photosphere, just as the spectrum of the limb differs from that of the center of the disk, save that the variations are more pronounced.

In spite of the great interest and importance of these observations, they do not appear to have been repeated in the long interval which has elapsed since Hastings made them. A photograph by Jewell, reproduced in Young's *Manual of Astronomy*, page 221, shows that the H_1 and K_1 bands are greatly weakened at the limb, and this

[1] *Contributions from the Solar Observatory of the Carnegie Institution of Washington*, No. 17.

[2] *American Journal of Science*, **5**, 360–371, 1873.

[3] *Ibid.*, Third Series, **21**, p. 35, 1881.

effect seems to have been recognized by Deslandres in some of his papers, but we have seen no discussion of any other important changes from center to limb. In her *Problems in Astrophysics* (p. 42), the late Miss Clerke, after discussing various details of the Fraunhofer lines, writes as follows: "An embarrassing peculiarity of the Fraunhofer lines is their virtually uniform existence all over the sun's disk."

Our first photographic comparisons of the spectra of the limb and the center of the sun were made in the red region of the spectrum, mainly for the purpose of testing Halm's recent results regarding line displacements.[1] It was noticed at once that many of the lines on the plates showed slight changes of relative intensity, which promised to be of interest in connection with our sun-spot investigations. As the discussion in our last paper[2] indicates, we have been occupied with the problem of distinguishing between the changes in the appearance of lines produced by differences in temperature and by differences in the density of the vapors. In this connection, we had long since planned to take up an investigation of the spectrum of the limb as compared with the center of the sun. It was supposed, however, that the differences would be so slight that they could be detected only with difficulty, and a special means of testing suspected cases, by a photometric study of spectroheliograph plates made for this particular purpose, was devised. It will be seen from the results contained in the present paper that the differences, particularly in the more refrangible part of the spectrum, are so striking that no such device is required.

The photographs hitherto taken cover almost the entire length of the spectrum from λ 3600 to λ 7000. They were made with the Snow telescope and the Littrow spectrograph described in our previous papers on sun-spot spectra. The image of the sun on the slit has a diameter of about 6.7 inches (17.02 cm). The spectrograph is of 18 feet focal length and contains a 4-inch Rowland grating, having 14,438 lines to the inch (5700 lines to the cm). Most of the spectra were photographed in the third order, but some were taken in the

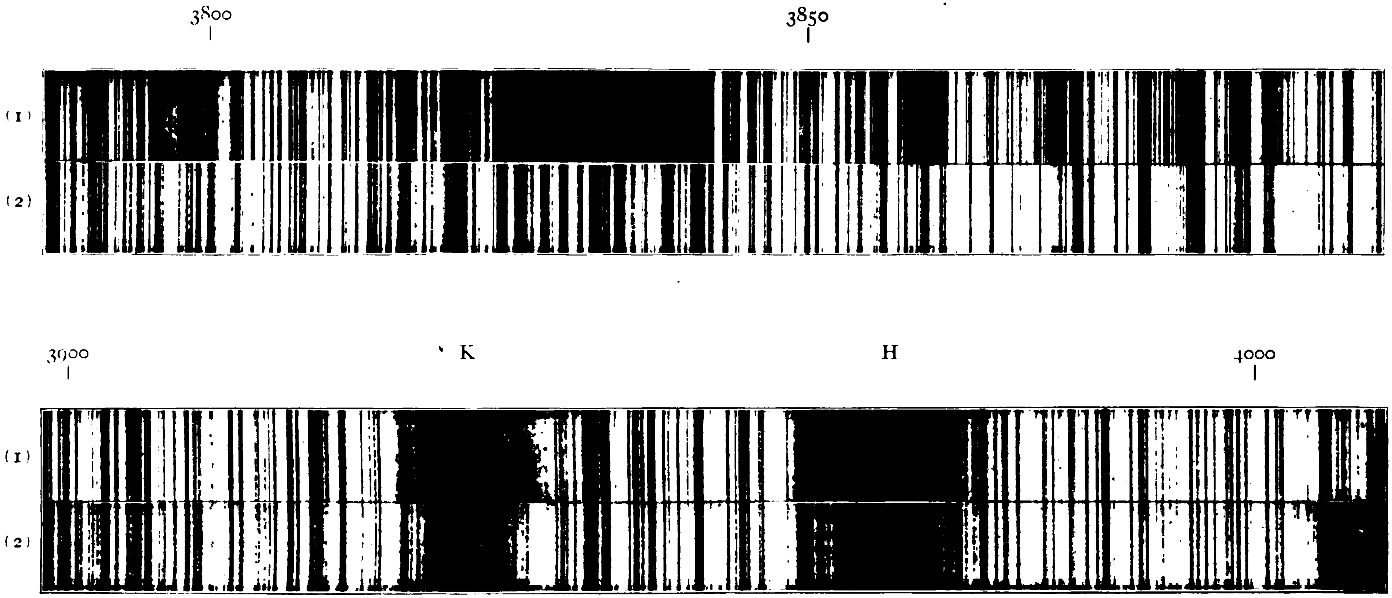
[1] *Astronomische Nachrichten*, No. 4146–47, **173**, 273, 1907.

[2] *Contributions from the Solar Observatory*, No. 15; *Astrophysical Journal*, **25** 75, 1907.

second, and a few in the fourth, the last for the study of line displacements. In making the exposures the image of the sun was adjusted so that the slit was parallel to a tangent at the limb, and usually about one millimeter within it. Precautions were always taken to prevent any light from the chromosphere from entering the slit. Hence, in all cases, the photographs of the spectrum of the limb represent the sun's disk, and not the region lying outside of it. After the exposure for the limb had been completed, a small sliding bar was moved over the slit, so as to cover the portion previously exposed, and light from the center of the sun was admitted on each side of the bar. Thus each photograph of the spectrum of the limb lies between two strips of spectrum representing the center of the sun. In the violet it is necessary to give from eight to ten times as much exposure at the limb as at the center. This ratio is reduced to four or five at the red end of the spectrum.

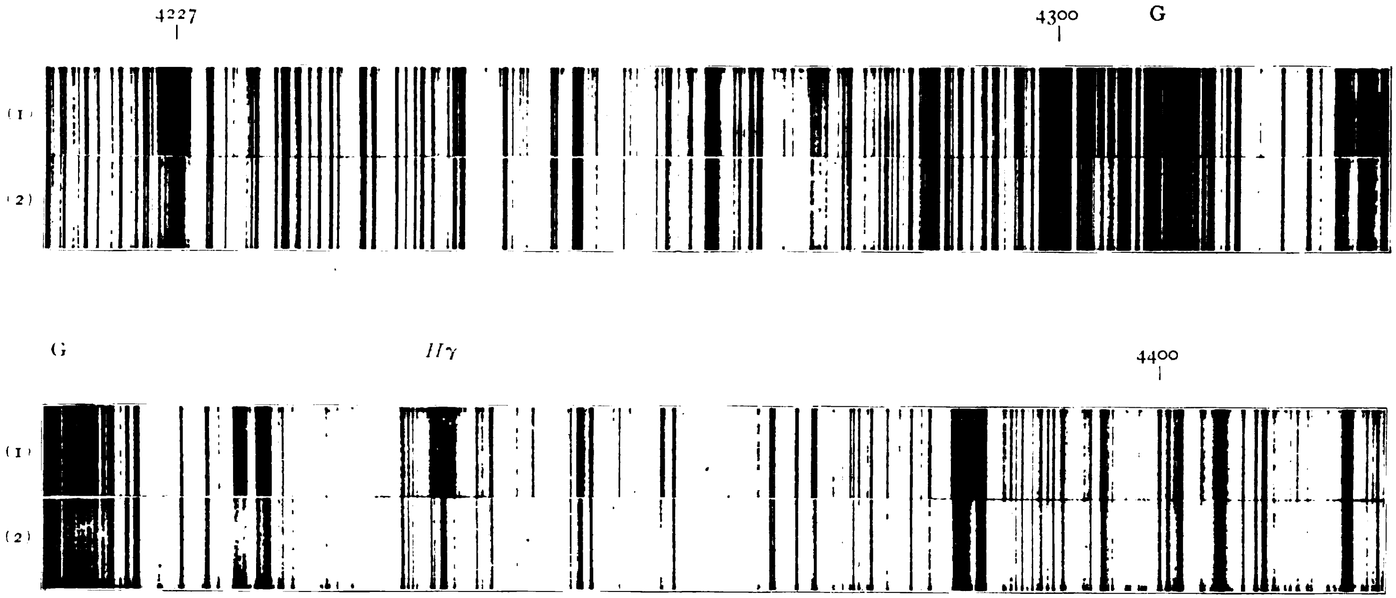
As an examination of Plates XVIII, XIX, and XX will indicate the differences in the relative intensities of the lines are not very marked in the less refrangible region, except in the case of such winged lines as D_1 and D_2, b_1, b_2, and b_4. In these cases the wings are greatly reduced in intensity, and the central portions of the lines are strengthened in the spectrum of the limb—a result which is in complete agreement with the observations of Hastings. There are some very distinct differences of line intensity to be noticed in this region, but such effects become far more marked in the blue and violet, and very conspicuous in the ultra-violet. In the region λ 3815–λ 3840 the appearance of the spectrum is greatly changed, through the almost complete disappearance of the wings which characterize the stronger lines corresponding to the center of the disk. So far as we have yet observed, in our preliminary examination of the plates, this practical elimination of the wings applies to all lines of this character. It is therefore to be regarded as the most striking feature of the spectrum of the sun's disk near the limb. In spots, on the other hand, a marked strengthening of the wings is a well-known characteristic in the less refrangible region of the spectrum. If this applies in the same degree to the much more closely crowded lines of the ultra-violet, it may be one of the causes contributing to the extreme weakness of the spot spectrum in this region.

PLATE XVIII



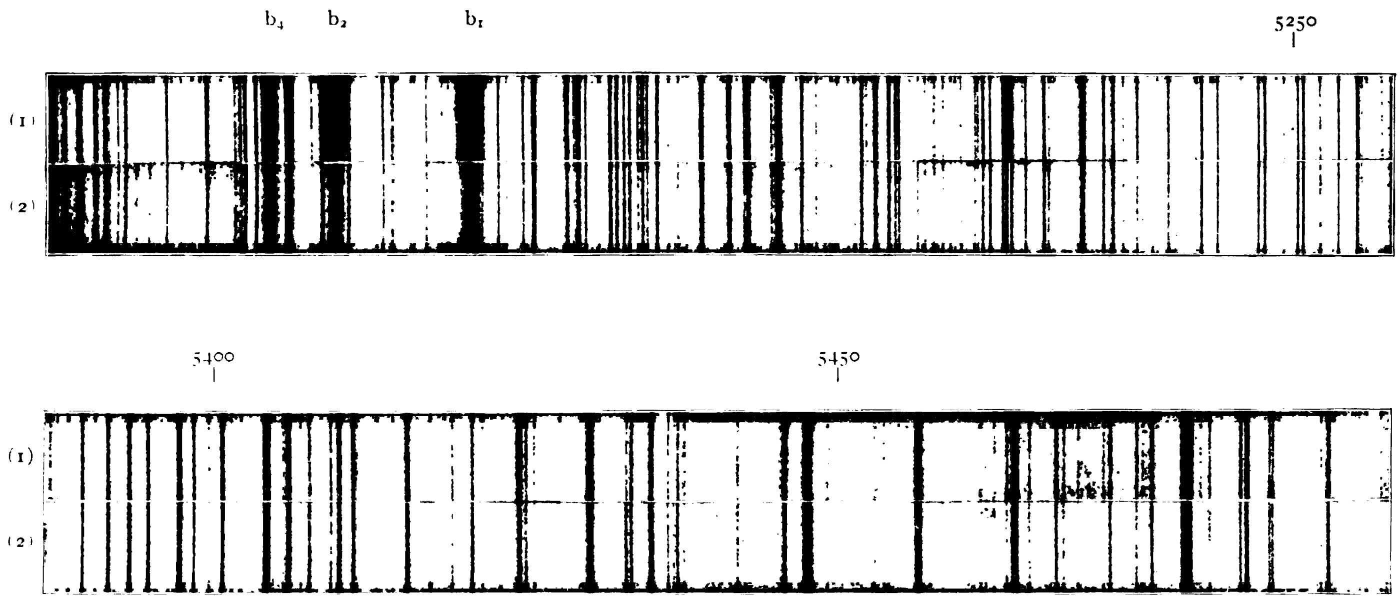
Spectra of (1) Center of Sun, (2) Limb

PLATE XIX



SPECTRA OF (1) CENTER OF SUN, (2) LIMB

PLATE XX



Spectra of (1) Center of Sun, (2) Limb

A second important distinction between the spectrum of the limb and the center lies in the changes in the relative intensities of the lines. In general, as the brief tables based upon a preliminary examination of several regions indicate, these changes of line intensity correspond closely with those observed in the case of sun-spots. In other words, lines that are strengthened in sun-spots are usually strengthened, in much smaller degree, near the sun's limb. Furthermore, lines which are weakened in sun-spots are usually weakened at the limb. Indeed, this latter effect seems to be relatively more marked than the effect of strengthening throughout the whole range of the spectrum. Some important exceptions have been noted, and others will no doubt appear in a more complete examination of the photographs. Nevertheless, the resemblance in this particular of the spectrum of the limb to that of sun-spots is very marked, especially in the less refrangible region. In the violet and ultra-violet the resemblance is partially concealed by the presence, in the case of spots, of an overlying solar spectrum. It is, however, clearly indicated. In our last paper we discussed the two principal sources to which this solar spectrum could be ascribed, namely, photospheric light and general sky illumination. Our present results on the spectrum of the center and the limb indicate that the photospheric light must probably be much the more important—a conclusion which agrees with that of Newall.[1]

In the tables which follow we give the results of our comparison of the spectrum of the center with that of the limb for three regions in different parts of the spectrum. The first of these includes a part of the G group and $H\gamma$; the second, the b lines and a portion of the green; and the third, a region in the yellow which is of considerable interest in sun-spot spectra. These results are merely preliminary and are given mainly to indicate the character of the evidence connecting changes in line intensities at the limb with those in spot spectra. In the abbreviations "Str." indicates that the line is strengthened, and "Str. +" that the effect is marked. Similarly "Wk." indicates that the line is weakened, and "Wk. +" that the amount of weakening is considerable. Lines in which the wings are reduced are indicated by "Sharp.," an abbreviation for sharpened.

[1] *Monthly Notices*, **67**, 158, January 1907.

TABLE I

λ	Element	Intensity Rowland	Spot	Limb
4299.15	*Ca*	3	Str.	Str.+
99.41	*Ti, Fe*	4		Sharp.
99.80	*Ti*	2	Str.	Wk.?
4300.21	*Ti*	3	Wk.?	N. C.
00.48	—	1 N		Wk.+
00.73	*Ti*	2	Str.	Wk.?
00.99	—	1	Wk.?	Wk.
01.16	*Ti*	2	N. C.	Wk.+
01.26	—	4		
01.33	—	1		
02.35	*Fe*	2		Wk.+
02.46	—	2		Wk.+
02.69	*Ca*	4	Str.+	Str.+
03.34	*Fe*	2	Wk.	Wk.+
03.75	—	1	N. C.	Wk.?
03.99	—	2	Wk.	Wk.+
04.10	—	4		
04.73	—	2	Wk.	Wk.+
05.27	—	1	Wk.?	Wk.+
05.48	*Fe, Sr*	1	Wk.	Wk.?
05.61	*Ti, Cr*	3		
05.87	—	2		N. C.
06.08	*Ti*	4	Str.	Wk.?
06.30	—	2	N. C.	Wk.+
06.86	—	–	Wk.	Wk.+
07.02	—	2		
07.46	—	2 N		Wk.+
07.72	—	2 Nd?		Wk.+
07.91	*Ca*	3	Str.	Str.+
08.08	*Fe, Ti*	6		Str.?
08.76	—	2 Nd?		Wk.++
09.06	—	1	Wk.	Wk.+
09.20	*Fe*	2		
09.54	*Fe*	3	Str.?	N. C.
09.79	—	1	Wk.	Wk.+
09.88	—	1		
10.27	—	2	Wk.	Wk.+
10.54	—	1	Wk.	Wk.+
10.63	—	2		
10.86	—	2	N. C.	Wk.+
11.06	—	1	Wk.	Wk.+
11.15	—	1		
11.32	—	2	N. C.	Wk.+
11.61	—	2	Wk.+	Wk.+
11.67	—	2		
12.25	—	2		Wk.+
12.46	—	2		Wk.
12.72	*Mn*	1 N		Wk.?
13.03	*Ti*	3	Wk.?	Wk.
13.19	—	1 N		Wk.
14.25	*Sc*	3	Str.	Str.+
15.14	*Ti*	3	Str.	N. C.
15.26	*Fe*	4		Str.
16.06	*Ti?*	1	Wk.?	N. C.

TABLE I—*Continued*

λ	Element	Intensity Rowland	Spot	Limb
4318.82	*Ca, Mn?*	4	Str.+	Str.+
20.91	*Sc*	3	Str.?	Str.?
21.12	*Ti?*	2	Wk.?	Str.?
21.96	*Fe*	2	Str.?	Wk.?
23.39	—	2	Wk.	Wk.+
23.67	—	1		Wk.+
23.77	—	0		Wk.+
24.01	—	3	Wk.	Wk.+
24.57	—	2	Wk.?	Wk.
25.15	*Sc*	4 }	Str.	N. C.
25.31	*Ti, Cr*	1 }		
25.52	*Ni, Zr*	1		Wk.+
25.94	*Fe*	8		Sharp.+
26.52	*Ti*	0	Str.+	Str.
26.92	*Fe*	2	Str.?	Str.?
28.08	*Fe*	2	Str.?	Wk.
30.19	*V*	0	Str.+	N. C.
30.40	—	1	Wk.	Wk.
30.87	*Ti, Ni*	2	Wk.	Wk.+
31.81	*Ni*	2	Str.	Str.+
32.99	*V*	0	Str.+	Str.?
33.92	*La*	1		Wk.+
35.10	*La*	0	Str.	N. C.
37.22	*Fe*	5	Str.	N. C.
37.72	*Cr*	3	Str.	Str.
38.08	*Ti*	4	Wk.	N. C.
38.43	*Fe*	1	Str.+	Str.
39.62	*Cr*	4	Str.+	Str.
39.88	*Cr*	3	Str.	Str.+
40.63	*H*	20	Wk.+	Wk.+Sharp.!
41.17	*V*	0	Str.+	Str.
41.53	*Ti?*	2	Wk.	N. C.
43.37	*Cr*	2 }	N. C.	N. C.
43.43	*Fe*	2 }		
44.67	*Cr*	4	Str.+	Str.+
46.99	*Cr*	1	Str.?	Wk.?
47.40	*Fe*	1	Str.+	Str.+
51.00	*Ti*	1	Wk.?	Wk.
51.22	*Cr*	3	Str.+	Str.+
51.93	*Cr, Fe*	5 }	Str.?	{ Wk.
52.08	*Mg*	5 }		{ N. C.
52.91	*Fe*	4 }	Str.+	Str.+
53.04	*V*	0 }		
55.26	*Ca?*	2	Str.	Wk.+
* * * *	* * * *	* * *	* * * *	* * * *
5137.25	*Ni*	3	Str.+	Str.+
39.43	*Fe*	4	Str.+	Str.
39.64	*Fe*	4	Str.?	Str.?
40.99	—	00	Wk.	Wk.
41.92	*Fe*	3	Str.+	Str.+
42.96	*Ni*	2	Wk.?	Wk.?
43.11	*Fe*	3	Str.	Str.
43.76	—	000 }	Str.+	Str.+
43.90	—	00 }		

TABLE I—*Continued*

λ	Element	Intensity Rowland	Spot	Limb
5144.85	*Cr, C*	00	Str.+	Str.
45.27	*Fe*	1	Str.+	Str.+
45.64	*Ti*	0	Str.	Str.
46.66	*Ni*	3	N. C.	Str. ? Wid.
47.65	*Ti*	0	Str.+	Str.+
48.22	*Fe*	2	Wk.	N. C.
48.41	*Fe*	3	Str. ?	Wk.+
51.02	*Fe*	4	Str.+	Str.+
52.09	*Fe*	3	Str.+	Str.+
52.36	*Ti*	0	Str.+	Str.+
53.41	*Fe*	1		Wk.+
53.58	—	00	Str.	
53.69	—	00		
54.24	*Co*	2	Wk.	N. C.
55.94	*Ni*	2	Str.	Wk.
56.78	*C*	000	Str.	Str. ?
56.82	*C*	00		
59.23	*Fe*	2	Wk.	N. C.
62.45	*Fe, C*	5	Wk. ?	Wk.+Sharp.
64.72	*Fe ?*	1	Wk.	Wk.
65.59	*Fe*	2	N. C.	Str ?
66.45	*Cr, Fe*	3	Str.	Str.+
67.50	*Mg*	15	Str.+	Str.+Sharp.!!
67.68	*Fe*	5		
69.07	*Fe*	3	Str.	Str.+
69.22	*Fe*	4	Wk.+	Wk.
70.04	*Fe*	0	Wk.+	Wk.+
71.78	*Fe*	6	Str.	Str. ?
72.86	*Mg*	20	Str.+	Str.+Sharp.!!
73.02	*Ti*	2	Str.	Str.+
76.30	—	000	Str.	Str.
76.73	*Ni*	1	Wk.	Wk.
77.41	*Fe*	0	Str. ?	Str. ?
77.58	*Co*	00		Str. ?
80.23	*Fe*	1	Str. ?	Str. ?
83.79	*Mg*	30	Str. ?	Str.+Sharp.!!
84.44	*Fe*	2	Wk. ?	Wk.+
84.74	*Fe, Ni, Cr*	1	Wk.	Wk.+
86.07	*Ti*	2	Wk.+	Wk. ?
88.08	*Fe*	1	Wk.	N. C. Wid.
88.86	*Ti*	2	Wk.	Wk.
89.02	*Ca*	3	Str.+	N. C.
91.63	*Fe*	4	Str. ?	Str. Sharp.
92.52	*Fe*	5	Str.	Str. Sharp.
93.14	*Ti*	2	Str.+	Str.+
95.11	*Fe*	4	Str.+	Str.+
95.65	*Fe*	2	N. C.	Str. ?
96.23	*Fe*	1	Str.	Str.
96.61	*Cr*	0	Str.	Str.
96.74	*Mn, Ni*	00		Str.
97.33	*Mn*	00	Wk.	N. C.
97.74	—	2	Wk.+	Wk.+
98.89	*Fe*	3	Str.+	Str.+
5200.50	*V*	0	Str. ?	Str.

TABLE I—*Continued*

λ	Element	Intensity Rowland	Spot	Limb
5202.44	*Fe ?*	2	N. C.	N. C.
02.52	*Fe*	4		
04.68	*Cr*	5	Str.+	Str.
04.76	*Fe*	3		
05.90	*Y*	0	Wk. ?	Wk. ?
06.22	*Cr–Ti*	5	Str.+	Str.+
08.11	*Ti*	00	Str. ?	N. C.
08.60	*Cr*	5	Str.+	Str.+
08.78	*Fe*	2		Wk. ?
10.56	*Ti*	3	Str.+	Str.+
11.70	*Fe*	00	Wk.	N. C.
16.44	*Fe*	3	Str.+	Str.+
17.55	*Fe*	3	Str. ?	Str.
18.37	*Fe*	1	Wk.	Wk.
19.88	*Ti*	0	Str.+	Str.+
20.36	*Ni*	0	Wk.	N. C.
22.56	*Cr*	00	Str.	Str.
23.35	*Fe*	0	N. C.	Str. ?
24.47	*Ti*	0	Str.	N. C.
25.10	*Cr*	0	Str.	Wk.
25.20	*Cr, Ti, Fe*	00		N. C.
25.70	*Fe*	2	Str.+	Str.+
26.71	*Ti*	2	Wk. Wid.	N. C. Wid.
27.04	*Fe–Cr*	3	Str.	Str.
27.36	*Fe*	5 d ?	Str.+	Str. Sharp.
30.03	*Fe*	4	Wk. ?	N. C.
30.38	*Co, Cr*	00	Str.	Str. ?
33.12	*Fe*	7	Str.+	Str. Sharp.!
34.79	—	2	Wk.+	Wk.+
35.56	*Fe*	1	Wk.	Wk.
35.67	*Ni*	00		
36.37	—	0	Wk.	Wk.
37.49	*Cr ?*	1	Wk.	Wk.
38.74	*Ti*	000	Str.+	Str.+
39.14	*Cr*	00	Str.	Str.
39.90	—	1	Wk. ?	N. C.
42.66	*Fe*	2	Str. ?	N. C.
43.05	*Fe*	1	N. C.	N. C.
47.23	*Fe*	1	Str.+	Str.+
47.74	*Cr*	2	Str.+	Str.+
* * * *	* * * *	* * *	* * * *	* * * *
5307.34	*Fe*	7 d ?	Str.+	Str. Sharp.
08.40	*Fe*	3	Wk. ?	N. C.
09.68	*Mn*	1 Nd ?	Str. ?	N. C.
5400.71	*Fe*	3	Wk. ?	Wk.+
00.83	*Cr*	00		
04.03	*Fe*	2	Wk. ?	N. C.
04.36	*Fe*	5	N. C.	Wk. ? Sharp.
05.99	*Fe*	6	Str.+	Str. Sharp.
07.59	*Mn*	0	Str.	Str.
07.60		0		
07.82	—	00	Wk.+	Wk.
09.34	*Fe*	2	Wk.+	Wk.
10.00	*Cr*	4	Str.+	Str.+

TABLE I—*Continued* .

λ	Element	Intensity Rowland	Spot	Limb
5411.12	*Fe*	4	Wk.?	Wk.
11.43	*Ni*	1	Wk.+	Wk.
14.28	—	00	Wk.	Wk.
15.42	*Fe–V*	5	N. C.	N. C. Sharp.
18.98	*Ti?*	1	Wk.?	N. C. Wid.
20.55 }	*Mn*	0 }	Str.+	Str.+
20.61 }		0 }		
24.29	*Fe*	6	N. C.	N. C. Sharp.
24.86	*Ni*	1	Str.?	Str.
25.46	—	1	Wk.	Wk.
26.47	—	00	Str.+	Str.
29.72	—	1	Wk.?	Wk.+
29.91	*Fe*	6 d?	Str.+	Str.+ Sharp.
32.75	*Mn*	1 Nd?	Str.+	Str.+
34.74	*Fe*	5	Str.+	Str.+ Sharp.
36.07	*Ni*	2	Str.?	Str.
36.51	*Fe*	1	Wk.?	N. C.
36.80	*Fe*	1	Str.+	Str.
41.55	*Fe?*	1	Wk.?	N. C.
45.26	*Fe*	4	Wk.?	N. C.
46.80	*Ti*	2	Str.+	Wk.?
47.13	*Fe*	6 d?	Str.+	Str. Sharp.
55.67	*Fe?*	2 }	Str.	{ Wk.
55.83	*Fe*	4 }		{ Str.
60.72	—	00	Str.+	Str.
61.76	—	0	Str.	N. C.
63.17	*Fe*	3	Wk.?	Wk.?
66.61	*Fe*	3	Str.	Str.?
67.20	*Fe*	1	Str.?	Str.?
70.80 }	*Mn*	0 }	Str.+	Str.
70.88 }		0 }		
72.92	*Fe*	1	Str.	Str.
74.11	*Fe*	3	Wk.	Wk.
76.50	*Fe*	1	Wk.?	Wk.+
77.12	*Ni*	5	Str.+	Str.? Wid.
77.90	*Ti*	00	Str.+	N. C.
78.67	*Fe*	0	Wk.	Wk.
81.45	*Fe*	1	N. C.	Wk.
81.65	*Fe, Ti*	1	Str.	Wk.?
82.08	—	00	Str.+	Str.
83.31	*Fe*	1	Wk.	Wk.
83.57	*Co*	1 d?	Str.+	Str.+
90.37	*Ti*	0	Str.+	Str.+
93.71	*Fe*	1	Str.	Str.?
97.74	*Fe*	5	Str.+	Str. Sharp.

The tendency toward a weakening of the spark lines, which is unmistakable though not very marked in the less refrangible region of the spectrum, becomes conspicuous in the violet and ultra-violet. The following list, taken from Lockyer's "Tables of the Wave-Length of Enhanced Lines," indicates the behavior in this respect of the more prominent spark lines of *Fe*, *Ti*, and *V* in the ultra-violet region. Identifications not made by Rowland are entered in brackets.

TABLE II

λ	Element	Intensity Rowland	Spot	Limb
3685.34	*Ti*	10 d?	Wk.	Wk.++
3706.36	*Ti*	3		Wk.+
3721.78	*Ti*	4 d?	Wk.	Wk.++
3741.79	*Ti*	4	Wk.	Wk.++
3748.14	*Ti*	1		Wk.++
3757.82	*Cr–Ti*	4	Wk.?	Wk.++
3759.45	*Ti*	12 d?	Wk.+	Wk.++Sharp.!
3761.46	*Ti*	7	Wk.+	Wk.++
3762.01	*Ti*	3		Wk.+
3776.20	*Ti*	2		Wk.+
3813.54	*C* (*Ti*)	2		Wk.+
3814.74	-*C* (*Ti*)	3		Wk.+
3836.23	(*Ti*)	2		Wk.
3839.76	*Fe*	2	Wk.	Wk.++
3846.55	*Fe*	2		Wk.++
3863.89	*Fe*	3	Wk.	Wk.++Sharp.
3866.96	*C*–(*V*)	2	Wk.?	Wk.+
3878.88	*C–Fe* (*V*)	2	Wk.?	Wk.++
3900.68	*Ti–Fe–Cr*	5		Wk.++
3903.40	(*V*)	2	Wk.	Wk.++
3913.61	*Ti–Fe*	5 d?	Wk.	Wk.
3914.43	*Fe?* (*V*)	3		Wk.++
3916.54	(*V*)	3	Wk.?	Wk.+
3935.96	*Fe*	2		Wk.
3939.29	(*Fe*)	0		Wk.+
3987.76	*Ti?*	2	Wk.?	N. C.
4005.86	(*V*)	3		Wk.+
4012.54	*Ti*	4		Wk.

In considering the behavior of the lines of different elements, we encounter at once an interesting peculiarity, which may prove significant. In the spot spectrum the lines of titanium and vanadium are well known to be very conspicuous, because, in most cases, they are strengthened more than the lines of other substances. In the spectrum of the limb, however, while these lines seem to be affected in the same direction as in spots, the magnitude of the effect is decidedly less than in the case of manganese, iron, calcium, and other substances, which appear to behave more nearly as they do in spots.

Hydrogen offers another case of great interest, with one apparent anomaly. $H\beta$ is not satisfactorily shown on the photographs at present available, and may for the present be left out of consideration. $H\gamma$ and $H\delta$ are much sharper and narrower at the limb than at the center of the sun, largely through the weakening of the wings which accompany these lines. The central part of the lines also appears to be slightly weakened at the limb. In spots they act in the same way, but the effect is more marked. $H\alpha$ is certainly widened at the

limb and possibly somewhat strengthened. There appears to be no great change in the sharpness of the line, the edges being slightly diffuse in both cases. In spots, on the contrary, $H\alpha$ is much narrower than in the spectrum of the sun's center and is also weakened. We have as yet found no means of explaining this apparent anomaly.

Carbon and cyanogen are particularly interesting. Many lines in the violet carbon band are of unchanged intensity, or perhaps slightly strengthened, at the limb. The cyanogen fluting, which begins at λ 3884, is, on the contrary, very decidedly weakened at the limb. This is clearly shown in Plate XVIII.

The behavior of the spark lines is a matter of special interest. In addition to the lines given in the list, the very noticeable weakening of most of the lines of the G group may be mentioned. The great majority of these are unidentified by Rowland, and are probably spark lines. The more conspicuous of these lines are weakened in spots, and doubtless others would be were they not conçealed, as suggested in our last paper,[1] by the overlying solar spectrum.

The behavior of the line λ 4233.3, which is probably mainly due to enhanced *Fe*, as shown by Lockyer, is of special interest, as this line is very strong in the flash spectrum. At the limb, on the contrary, it is much weakened. There are doubtless many other cases of this kind which will require careful investigation.

The bearing of these results on solar theory cannot properly be discussed until much more material is available, particularly as regards relative intensities and line shifts. These investigations are now in progress, and we therefore reserve further discussion for a later paper, in spite of various seemingly obvious consequences which at once suggest themselves. For example, it now seems difficult to reconcile our results as to the disappearance of wings at the limb with Halm's conclusion that the dense vapors lying close to the photosphere contribute relatively more at the limb than at the center to the formation of the absorption lines. We hope at an early date to be able to discuss this point fully, as well as many more of equal interest.

APRIL, 1907

[1] *Contributions from the Solar Observatory*, No. 15; *Astrophysical Journal*, **25**, 75, 1907.

SOME NEW APPLICATIONS OF THE SPECTROHELIOGRAPH[1]

By GEORGE E. HALE

I. SOLAR PHOTOGRAPHY WITH SUN-SPOT LINES

In several previous papers I have remarked on the importance of photographing the sun with the lines affected in and near spots. In our photographs of spot spectra many lines are strengthened or weakened, not merely in the umbra and penumbra, but in extensive regions surrounding spots. This effect, conspicuous enough to show itself directly in the spectrum, is evidently within easy reach of the spectroheliograph. Furthermore, our experience in the photography of faint hydrogen flocculi warrants the inference that spot phenomena, too delicate in their effect on line intensity to be detected with the spectrograph, will be brought to light by the application of spectroheliographs of sufficiently high dispersion.

The question of dispersion is evidently of crucial importance. If the line employed is sensibly narrower than the camera slit of the spectroheliograph, the admixture of light from the adjoining continuous spectrum will tend to blot out the comparatively feeble impression resulting from the faint light of the dark line. The fact that many of the most interesting cases are represented by extremely fine lines thus points to the use of spectroheliographs of great linear dispersion. This consideration, and the desire to photograph the sun with narrow dark lines other than those affected in spots, led to the provision of four prisms for the 5-foot spectroheliograph[2] and the inclusion, in the original plan of the Snow telescope house, of a spectroheliograph of 30 feet (9.14 m) focal length.[3] The latter instrument has not yet been completed. The delay is due in part to difficulty in securing suitable prisms, and in part to the distortion of the Snow telescope mirrors in sunlight. This distortion, while inappreciable

[1] *Contributions from the Solar Observatory of the Carnegie Institution of Washington*, No. 18.

[2] *Contributions from the Solar Observatory*, No. 7, p. 6; *Astrophysical Journal*, **23**, 59, 1906. This instrument has been used systematically since October, 1905 for photographic work with some of the wider dark lines.

[3] *Contributions*, No. 2 p. 14; *Astrophysical Journal*, **21**, 64, 1905.

during the short exposures that suffice with the 5-foot spectroheliograph, would change the focus and seriously affect the definition of the solar image during the long exposures required with much higher dispersion. As explained in a previous paper,[1] it is hoped that such distortion may be overcome in our new vertical telescope, to which the 30-foot spectroheliograph will be transferred.

Meanwhile, a temporary spectroheliograph of 30 feet focal length has been successfully used with the Snow telescope. This instrument was mounted in the telescope house, between the second mirror of the coelostat and the concave mirror of 60 feet focal length. Instead of the latter, a 5-inch (12.7 cm) objective, of 13 feet (3.96 m) focal length, was used to form the solar image on the collimator slit. The spectroheliograph was built in the Littrow form, with camera slit immediately below the collimator slit; an 8-inch (20.3 cm) objective, of 30 feet focal length, serving at once as collimator and camera lens, and a 6-inch plane grating. As the entire spectroheliograph was supported in a fixed position on piers, it was necessary to cause the solar image to move across the collimator slit, and the photographic plate across the camera slit, at a slow and uniform speed. This was accomplished by mounting the 5-inch image-forming objective and the plate holder (13 feet apart) on arms supported at opposite ends of the movable carriage of the 5-foot spectroheliograph. The driving mechanism of this instrument,[2] supplemented by a counter-shaft for slower speeds, served admirably to give the necessary motion.

On account of the small aperture (only a part of which was used) and the short focal length of the image-forming objective, no appreciable change of focus occurred during the exposures. The small diameter of the solar image (1.4 inches = 3.5 cm) prevented the minor details of structure from being recorded, but enough was shown to serve as a good test of the apparatus and its possibilities.

Several photographs of each spot were made by Mr. Adams and myself on each plate, the first exposure with the camera slit set on the dark line, the other exposures with this slit set on the continuous spectrum, a short distance from the line. In order to eliminate the

[1] *Contributions from the Solar Observatory*, No. 14; *Astrophysical Journal*, **25**, 68, 1907.

[2] *Contributions*, No. 7, p. 7; *Astrophysical Journal*, **23**, 60, 1906.

effect of increased brightness of background, due to the absence of the dark line from the slit in the continuous spectrum photographs, these were repeated, with different speeds of driving.

The first lines tried were those that are strengthened in the umbra and penumbra and on the photosphere, for a considerable distance from spots. The spectroheliograph plates made with such spot lines show the umbra and penumbra much darker than they appear on the plates taken with the light of the continuous spectrum. The apparent diameter of the spot also appears to be considerably increased, doubtless through the inclusion of the dark area surrounding it.

Lines that are weakened in spots, and on the adjoining photosphere, also give definite effects, though the results so far obtained are less satisfactory than in the case of the strengthened lines.

With a large solar image and good conditions of atmosphere, such a spectroheliograph as the one described above is capable of yielding many interesting results, if used systematically with lines affected in spots and with other dark lines. The 30-foot spectroheliograph designed for the "tower telescope" should, however, be much more efficient in many respects than this temporary instrument. The dark areas frequently observed on the sun with the aid of D_3, usually in the vicinity of sun-spots, indicate that spectroheliographic records, made with the light of this line, will also prove of great value. In fact, the promise of future work with this and numerous other lines is so great that the efforts of many investigators, provided with the most powerful instruments, will be required to derive adequate results in this extensive field.

II. STEREOSCOPIC PICTURES OF THE SUN

The application of stereoscopic methods to solar work dates back to the time of De La Rue, who secured interesting results through the combination in the stereoscope of direct photographs of the sun taken at various time intervals, up to two days. These pictures, according to his descriptions, showed the spots as depressions and the faculae as elevated regions. Since his time but little appears to have been done in this field.

In the summer of 1906 I thought it would be of interest to combine, with the aid of our large stereocomparator, various plates made with

the Snow telescope and 5-foot spectroheliograph. The results were very satisfactory, showing at once the sphericity of the sun and, to most observers, the protuberant character and cloudlike aspect of the flocculi. If the interval is too short, the effect is imperceptible; if too long, the changes in form of the flocculi, and the large displacements due to the solar rotation, trouble the eye when attempting to unite the images.

The photographs reproduced in Plate XXI were taken on August 22, 1906, at $7^h\ 26^m$ A. M. and $5^h\ 21^m$ P. M. During this interval the changes in the form of the flocculi are very noticeable, but do not prevent a fairly satisfactory combination of the images when a glass positive is viewed with a stereoscope. It is doubtful whether the half-tone print will give satisfactory results, though the sphericity of the sun should be evident.

As for the appearance of the flocculi as elevated regions, this is much better shown with the aid of the original negatives, or with positives reproduced on the same scale. The stereocomparator, provided with all facilities for centering and adjusting plates, permits the observations to be made in a most satisfactory manner. It should be stated, however, that some observers see the flocculi as apparent depressions rather than as elevations. This, however, is uncommon.

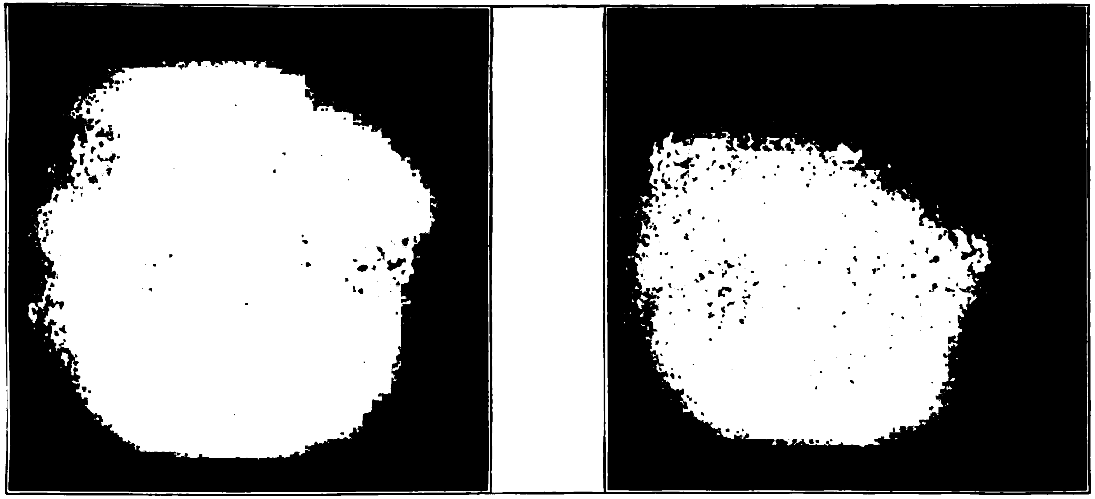
It seems probable that further studies of this character will bring useful results. It has already been possible to detect in this way linear markings on the solar surface, of great extent, which do not readily strike the eye when plates are examined singly. For purposes of measurement, and the detection of minute differences of form, the monocular eyepiece, with micrometer attachment, which is furnished by Zeiss as an accessory of the stereocomparator, is of course to be employed. It nevertheless appears probable that the stereoscopic method will prove to have certain advantages of its own, which will recommend it to those who are engaged in the study of spectroheliograph plates.

April, 1907

PLATE XXI

THE SUN, AUGUST 22, 1906

$5^h\ 21^m$ P. M. $7^h\ 26^m$ A. M.



ORBIT OF THE SPECTROSCOPIC BINARY κ *CANCRI*

By NAOZO ICHINOHE

This star ($\alpha = 9^{\mathrm{h}}\ 2^{\mathrm{m}}$, $\delta = +11^{\circ}\ 4'$; Mag. *H. P.*, 5.0; *D. C.*, 5.3; *P. D.*, 5.5) was found as a spectroscopic binary at Yerkes Observatory by Messrs. Frost and Adams, in the course of the regular programme of work on stars having spectra of the *Orion* type. Their measures of the first five plates taken (IB 256, 267, 286, 289, 305) will be found in their paper.[1] At the suggestion of Mr. Frost, I began last summer to measure the spectrograms of the star taken later than the above plates.

The spectrum of this star belongs to the later stage of development from the *Orion* type toward the Sirian type. The hydrogen lines are strong and well defined. The helium lines λλ 4009, 4026, 4144, 4388, and 4472 are scarcely visible. The carbon line λ 4267 is well seen; the silicon lines λλ 4128 and 4131 are very good for measurement, as they are quite strong and well defined. The K line is sharp and strong, and the magnesium line λ 4481 is also well defined. Besides these, many faint lines of *Fe* and *Ti*, especially *Fe* λ 4550, can be seen. But generally, on a slightly over-exposed plate, the metallic lines become very faint and difficult to measure; while the hydrogen lines have very good definition on such plates. For under-exposed plates, on the contrary, the hydrogen lines are broad and diffuse, while the *Mg* line, as well as other metallic lines, are more distinct. The only lines in this spectral region which are always distinct are *Si* λ 4128 and λ 4131.

I measured almost all the lines visible on the plates, but of course many of them are too faint to give precise values of the radial velocity, so that only the following lines were chosen for the determination of velocities:

K	3933.825	*Ti*	4344.451
Hδ	4101.890	*Mg*	4481.400
Si	4128.211	*Fe*	4549.808
Si	4131.047	*Ti*	4572.156
Fe	4233.328	*Fe*	4584.018
C	4267.301	*Ti*	4590.126
Ti	4338.084	*Ti*	4856.203
Hγ	4340.634	*Hβ*	4861.527

[1] *Astrophysical Journal*, **19**, 351, 1904.

The number of spectrograms of *κ Cancri* obtained here up to date is twenty-five, the particulars as to which follow. For all of these plates the spectrograph was used with one prism. For the collimator $\frac{a}{f}=\frac{51\text{ mm}}{958\text{ mm}}$; for the camera $\frac{a}{f}=\frac{57}{608}$. The scale-values at different wave-lengths are approximately as follows:

λ	$\frac{d\lambda}{ds}$	
3933	1 mm = 16.5 t.-m.	= 1264 km per sec.
4045	18.9	1404
4338	25.9	1788
4481	29.6	1984
4552	31.6	2083
4856	40.9	2523

The spark spectrum of titanium and iron was impressed on each plate at the beginning and end of the exposure. The slit-width was varied to suit the conditions of the atmosphere. The third column of the following table gives the G. M. T. of mid-exposure. The temperature (C.) is that within the outer case of the spectrograph. Under observer, A = Adams, B = Barrett, F = Frost. The seeing is estimated by the observer on a scale of 1–5, 5 representing the finest conditions.

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Plate	Date			G. M. T.	Exposure	Slit-Width	Temperature	Observer	Seeing
IB 256	1904	Jan.	21	22^h44^m	63^m	0.038mm	−13°8 C.	A	3
267		Jan.	23	20 44	50	.051	−12.4	A	3–2
286		Feb.	26	18 38	50	.044	− 7.8	A	2–1
289		March	8	18 36	48	.038	+ 2.2	F	3
305		April	15	15 25	50	.038	+ 0.1	A	3–2
330		April	30	15 50	56	.038	+16.2	F	3–2
471		Dec.	30	20.18	67	.041	+ 2.2	B	3–1
482	1905	Jan.	9	21 00	76	.041	−13.0	F	2–1
500		Feb.	3	18 39	50	.051	−16.6	F–B	2–3
626		Dec.	9	20 11	46	.051	− 0.5	F	3–2
633		Dec.	11	21 06	48	.051	+ 4.2	B	3–2
644		Dec.	15	22 21	50	.054	+ 0.2	F	3
653		Dec.	25	20 07	66	.051	− 2.9	B	3–1
662	1906	Jan.	26	18 54	52	.050	+ 3.2	B	3–2
713		March	23	15 59	92	.051	− 4.6	F	2–0
732		April	20	14 03	60	.059	+18.7	B	3–2
740		April	23	14 14	52	.051	+ 9.7	F	3
746		April	27	14 15	60	.051	+15.5	B	3–2
899		Oct.	31	23 34	37	.051	+ 5.2	B	3–2
940	1907	Jan.	21	21 18	120	.051	−10.2	F	1–2
969		Feb.	22	19 26	55	.051	− 8.9	B	3
1018		April	13	17 13	50	.046	− 0.6	Fox	4–2
1027		April	20	15 40	63	.046	+ 6.1	Fox	3–2
1039		April	26	14 57	74	.051	+ 3.6	F	3–2
1052		May	10	14 26	71	.051	+ 6.0	B	2

Plate No. 644 is very weak and unsuited for accurate measurement, so that it is omitted from the discussion. All of the other plates, except the five by Frost and Adams, were measured by me. The method of measurement and reduction is that regularly employed at this observatory. The number of lines on each plate used for the determination of radial velocity is not constant, varying from nine to three according to the nature of the plates, but generally five or six lines were chosen. The weight for each line is assigned during measurement, depending upon the character of line on the plate, and the weighted mean is taken for the radial velocity. The following table contains all the data for the determination of the orbit of *κ Cancri*. The first column gives the number of the plate; the second, the date expressed in Julian days; the third, the radial velocity reduced to the sun; the fourth, the number of lines used for the determination of v.

Plate No.	Julian Day	v	n	Phase	v_c	$v-v_c$
		km			km	km
256	2416482.947	+ 2.5	7	1.000	+ 9.2	−6.7
267	6503.864	+88.3	5	2.738	+81.8	+6.5
286	6537.776	+34.9	5	4.685	+31.2	+3.7
289	6548.775	+82.7	9	2.898	+83.5	−0.8
305	6586.643	+70.0	5	2.408	+75.3	−5.3
330	6601.600	+31.2	8	4.639	+34.3	−3.1
471	6845.846	−44.5	4	5.891	−47.3	+2.8
482	6855.875	+85.6	6	3.134	+84.4	+1.2
500	6880.777	+73.7	9	2.464	+76.7	−3.0
626	7189.841	+38.8	7	4.664	+32.6	+6.2
633	7191.879	−36.1	5	6.702	−34.1	−2.0
653	7205.838	+37.9	4	1.482	+38.1	−0.2
662	7237.788	+33.7	7	1.467	+37.3	−3.6
713	7293.666	−52.9	3	6.201	−50.7	−2.2
732	7321.585	+61.5	5	2.155	+67.9	−6.4
740	7324.593	− 3.6	5	5.163	− 5.1	+1.5
746	7328.504	+89.1	3	2.771	+82.3	+6.8
899	7515.982	+20.0	6	4.762	+25.7	−5.7
949	7597.888	+85.9	6	3.559	+80.8	+5.1
999	7629.810	+78.2	6	3.516	+81.8	−3.6
1018	7679.718	+71.8	5	2.280	+71.8	0.0
1027	7686.653	+76.2	4	2.822	+82.8	−6.6
1039	7692.621	+70.2	7	2.397	+75.1	−4.9
1052	7706.601	+82.2	3	3.591	+80.4	+1.8

The period was investigated last summer, and after several trials it is found to be $3^{d}.393$. Although many plates were taken after this period was obtained, all of them gave values pretty close to the

expected velocity. Accordingly, I believe that the above value of the period is accurate within one one-thousandth of a day.

Taking $3^{d}.393$ for the period, the phases were now calculated, as shown in the fifth column of the above table. For this the date of the first plate was assumed as phase $1^{d}.000$. Then the values of v were taken as the ordinates, and the phases as the abscissas of a system of rectangular co-ordinates. The points defined by these co-ordinates were plotted on millimeter paper, on a scale of 1 day = 50 mm, 1 km = 2 mm, and a smooth curve was drawn passing through these points as well as possible, and such that the curve will fulfil the conditions for the determination of the orbit by the method of Lehmann-Filhés.

The radial velocity of the center of gravity of the system is first found by shifting the axis of abscissas until the area inclosed by the curve above the axis becomes equal to that below the axis. We thus obtain

$$\text{Radial velocity of the system} = +26.3 \text{ km}.$$

Then the maximum A and minimum B of the radial velocity, z_1, z_2 and t_1, t_2 were determined as follows:

$$A = 58.2 \text{ km}, \qquad B = 77.4 \text{ km}$$
$$z_1 = 6160 \text{ sq. mm}, \qquad z_2 = -6750 \text{ sq. mm}$$
$$t_1 = 4^{d}.760, \qquad t_2 = 7^{d}.725.$$

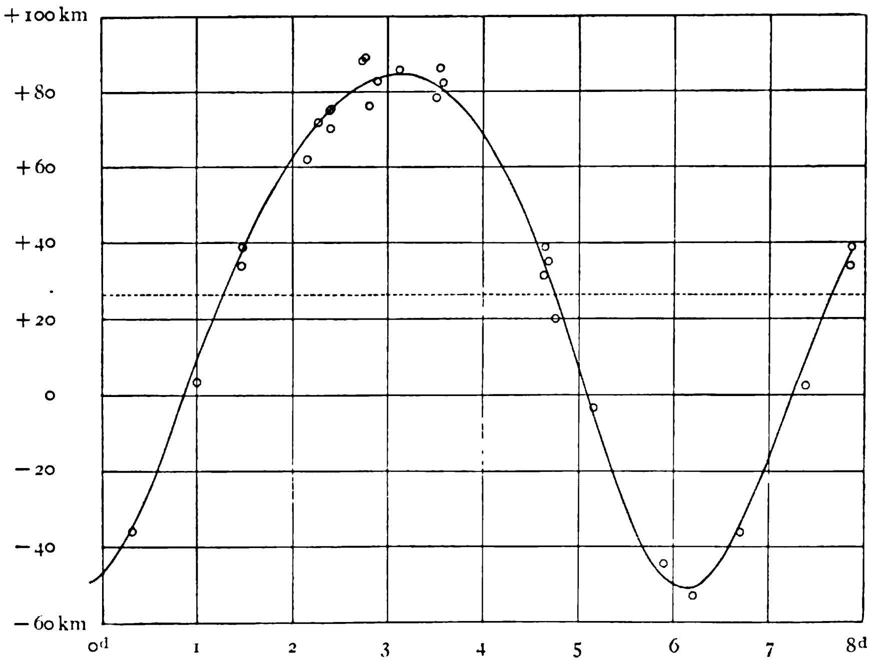
The following elements were derived:

$$
\begin{aligned}
U &= 6^{d}.393 \\
u_1 &= 81^\circ\ 51'.2 \\
\omega &= 162^\circ\ 15'.8 \\
e &= 0.149 \\
\mu &= 56^\circ.31 \\
\text{or } \log \mu &= 1.75060 \\
T &= 1904 \text{ Jan. } 6.897 \\
a \sin i &= 5.890\,000 \text{ km}
\end{aligned}
$$

In order to see if the elements satisfactorily represent the observations, an ephemeris was calculated; the computed values will be found in the sixth column of the above table. The last column of the same table gives the residuals, observation minus computation. The residuals are pretty large, ranging from +7 km to −7 km, but they may be regarded as fairly satisfactory, since all these plates

were taken with one prism. The mean error of a single plate is ±4.2 km. There is no appreciable evidence of systematic errors in the residuals.

I did not try to correct the elements further, as the observational data are not yet adequate for the calculation of the definitive orbit. The accompanying figure shows the computed curve and observed points.



Computed Velocity-Curve for *κ Cancri*, with Observed Points

The above relates exclusively to the principal component of the spectrum of *κ Cancri;* but the star shows another spectrum, as was pointed out by Frost and Adams. I was able to see the faint component of the spectrum on eighteen plates out of twenty-five. These lines are so extremely faint that it is very difficult to measure them, and the results are liable to great errors. The discussion of the orbit of the fainter component is therefore deferred until additional plates can be obtained and greater certainty assured.

Yerkes Observatory
May, 1907

ORBIT OF THE SPECTROSCOPIC BINARY β *ARIETIS*[1]

By H. LUDENDORFF

The variability of the radial velocity of β *Arietis* was announced in 1903 by Director Vogel[2] as a result of his measurements of 14 spectrograms obtained by Messrs. Eberhard and Scholz and myself with spectrograph IV (three prisms) attached to the 32.5-cm photographic refractor of the Potsdam Observatory during the period from 1902 to the beginning of February 1903. At the request of Professor Vogel, I subsequently investigated the radial velocity of this star more minutely, measuring again the fourteen plates mentioned and twenty-three additional ones obtained up to the end of 1904. The results of this investigation were published in *Astronomische Nachrichten*, No. 4090 (**171**, 149, 1906). As no determinations of the radial velocity by other observatories were available, I had to depend solely on the thirty-seven plates obtained here. I was only able to conclude that the period was $32/n$, where n is an integer $\leqq 5$. It further appeared that the radial velocity was almost constant during a considerable fraction of the period.

Thirty-nine more spectrograms of β *Arietis* were obtained by Dr. Eberhard and myself with spectrograph IV in the winter season of 1906–7, and these in connection with the thirty-seven earlier plates now permit a first determination of the orbit.

The spectrum of β *Arietis* is to be assigned to Vogel's class Ia2. The lines are all broad and diffuse; and in the region sharply defined by spectrograph IV (λ 4300 to λ 4530) only two absorption lines, *Mg* λ 4481 and $H\gamma$, could be measured. No indication of the second component of the star could be recognized in the spectrum. The magnesium line does indeed appear double on plates Nos. 1354 and 1355, as stated by Vogel in his paper referred to, but I have been unable to detect this appearance with certainty in any of the other

[1] Translated from advance proofs, sent by the author, of a paper to appear in the *Sitzungsberichte der Berliner Akademie*.

[2] *Astronomische Nachrichten*, **163**, 145, 1903.

seventy-four plates. It is therefore to be assumed that the duplicity of the *Mg* line on these two plates may be explained by special disturbances in the atmosphere of the star. Similar phenomena have often been observed here in the case of stars of the first type.

On account of the width and diffuseness of the *Mg* and $H\gamma$ lines, the measurements of the radial velocity of β *Arietis* are quite uncertain. In order to overcome as much as possible the effect of a subjective error in setting, I employed a reversion-prism in the measurements, so that the position of the spectrum could be apparently reversed through 180° after the completion of the series of measurements in the first direction; the mean of the two series was then employed.

Since the measurements of the $H\gamma$ line were decidedly more uncertain than those of the *Mg* line, they were assigned only half-weight in the reduction. It was not possible to measure $H\gamma$ at all on several of the underexposed plates (Nos. 1187, 1361, 1390, 1529, 1745, 1752, 2054, 2057, 2058, 2063, 2066). On those plates the *Mg* line was measured twice, and the mean taken from the two wholly independent series of measures. Those plates were also twice measured which gave values of the radial velocity differing from each other by more than 15 km; the plates were Nos. 1329, 1343, 1355, 1513, 1748, 1759, 2008, 2025, 2036, and 2060.

The uncertainty of the radial velocity from the measurements of a single plate may amount to 10 km, in some cases even more. The results from the sixty-five plates on which both the *Mg* line and $H\gamma$ were measured indicate in the mean that a receding velocity of the star greater by 3 km may be inferred from the *Mg* line than from $H\gamma$.

The following summary shows all the data which I employed for the determination of the orbit. It gives the number of the plates, the date, Central European Time of mid-exposure, the observers at the telescope (E = Eberhard, L = Ludendorff, S = Scholz), the radial velocity reduced to the sun, and finally, in the last column, the phase Φ computed after the period had been determined, i. e., the interval expressed in days from the next preceding maximum of the radial velocity.

No.	Date		C. E. T.		Observer	v	Φ
1187	1902 Oct.	21	9^h	48^m	E, S	-8^{km}	$+17^d$
1196		22	10	24	E, S	-2	18
1323	1903 Jan.	17	6	55	E, S	$+31$	105
1329		18	6	23	E	$+43$	106
1334		19	6	27	E, L	$+60$	0
1337		20	6	10	E, L	$+19$	1
1340		21	6	11	E, L	$+12$	2
1343		22	5	37	E, L	0	3
1349		27	7	2	E	$+2$	8
1350		28	7	41	E, L	-11	9
1354		29	5	44	L	-10	10
1355		31	9	42	E	-1	12
1356	Feb.	2	6	0	E, S	-8	14
1361		6	6	4	E, S	0	18
1363		6	8	28	E, S	-5	18
1368		15	6	24	E	-5	27
1374		16	6	13	E, L	$+6$	28
1381		19	6	30	E, L	-4	31
1383		20	6	23	E, L	-8	32
1386		24	6	30	E, L	-7	36
1389	March	2	6	57	E, S	0	42
1390		4	6	59	E, S	-5	44
1391		7	6	58	E, S	-6	47
1394		8	7	4	E	-8	48
1513	Dec.	3	8	6	E, L	$+24$	104
1521		4	8	47	E, L	$+36$	105
1526		22	8	24	E, L	-2	16
1529		25	6	8	E	-17	19
1533		28	7	34	E, L	-8	22
1537	1904 Jan.	4	6	7	E	-4	29
1549		11	6	11	L	-9	36
1584	Feb.	12	6	10	L	$+4$	68
1745	Dec.	6	6	53	L	-8	45
1748		9	8	5	E, L	$+5$	48
1752		12	6	47	E	-10	51
1755		13	6	32	L	-7	52
1759		16	6	50	L	-5	55
1994	1906 Sept.	27	10	15	E	$+10$	63
1995		28	9	31	L	-9	64
1997		30	9	45	E	0	66
1999	Oct.	1	9	33	L	-6	67
2003		8	9	32	E, L	-7	74
2005		9	10	27	E, L	-11	75
2006		10	10	24	E	$+1$	76
2008		13	9	21	E, L	$+8$	79
2009		17	8	27	E, L	$+2$	83
2014	Nov.	6	9	47	E	$+21$	103
2016		8	7	50	L	$+34$	105
2017		8	9	0	L	$+27$	105
2020		9	6	21	L	$+39$	106
2022		9	8	55	E, L	$+41$	106
2025		10	7	8	L	$+57$	0
2026		10	8	22	E	$+57$	0
2027		14	8	17	E	$+1$	4
2028		15	6	33	L	-8	5
2032		20	8	56	E, L	-5	10

No.	Date		C. E. T.	Observer	v	Φ
2036	Nov.	24	$7^h\ 13^m$	E	$-\ 2^{km}$	14
2040	Dec.	1	9 3	L	− 7	21
2043		7	6 8	E	− 8	27
2048		8	9 9	E, L	−18	28
2050		22	6 17	L	− 2	42
2054		27	7 5	E	+ 4	47
2057	1907 Feb.	11	6 41	E	− 7	93
2058		11	8 5	E	0	93
2060		12	6 17	L	+ 7	94
2061		12	7 29	E, L	+ 2	94
2062		20	6 25	L	+15	102
2063		20	7 18	L	+13	102
2064		22	8 13	L	+21	104
2066		23	8 18	E, L	+25	105
2067		25	6 29	L	+53	0
2068		25	7 13	L	+62	0
2069		25	7 52	E, L	+60	0
2070		25	8 38	E	+65	0
2073	March	3	6 45	E	− 3	6
2077		4	7 59	E	− 3	7

A graphical representation of the values v of the radial velocity shows positive maxima on the following days, of which only the second is somewhat uncertain.

1903 January 19	= J. D.	2 416 134
December 6	= "	2 416 455
1906 November 10	= "	2 417 525
1907 February 25	= "	2 417 632

The intervals of the last three dates from the first are 321^d, 1391^d, 1498^d, or $3\times107^d.0$, $13\times107^d.0$, $14\times107^d.0$. The period of revolution is therefore

$$P=107^d.0\,.$$

I would remark further that the observed values of v do not permit an aliquot part of $107^d.0$ as the possible period; $53^d.5$, or one-half of $107^d.0$, would require a maximum on 1904, December 14–15, which is contrary to the observations.

The observations show that the true value of the period can deviate from the value given by only a few hundredths of a day. This follows not only from the times of the observed maximum values of v, but also when the period is determined from certain points of the velocity-curve where its rise is steep.

If we now arrange the observations according to phase, it appears

that in consequence of the uncertainty of the measurements the observations at the same phase at times deviate pretty widely from each other. It therefore seemed advisable to me to form normal values, v_0, of the radial velocity by forming means from the observations taken at the same or closely related phases. For the phases 105^d to 0^d and 0^d to 2^d, observations at the same phase only were averaged, since the changes in v are here very large. For the others the values of v corresponding to the following values of Φ were formed into means v_0:

$\Phi = 3^d$ to 5^d	$\Phi = 27^d$ to 29^d	$\Phi = 63^d$ to 68^d
6 to 8	31 to 36	74 to 83
9 to 12	42 to 45	93 to 94
14 to 17	47 to 48	102
18 to 22	51 to 55	103 to 104

In averaging, the values of v which depend on the *Mg* line only, were given a weight $\frac{2}{3}$, the remainder a weight 1.

The spectrograms taken at $\Phi = 0^d$ all yielded values of v (+53 to +65 km) lying pretty near to the mean value (+59.1 km). None of these values differs from the mean more than the uncertainty of measurement would permit. Even the progression in the four values of v (+53, +62, +60, and +65 km), indicated on the evening of the last maximum, cannot be regarded as certainly real. I therefore made a simple assumption that the observations corresponding to $\Phi = 0^d$ were actually made at the time of maximum.

The following table contains the results of the computation of the normal values v_0 of the radial velocity. The third column contains the number n of plates on which the value of v_0 depends, and the last column contains the weight p.

Φ	v_0	n	p	Φ	v_0	n	p
0^d	$+59.1^{km}$	7	7	43^d	-3.2^{km}	4	$3\frac{1}{3}$
1	+19	1	1	48	−1.7	4	$3\frac{2}{3}$
2	−12	1	1	53	−7.0	3	$2\frac{2}{3}$
4	−2.3	3	3	66	−0.2	5	5
7	−1.3	3	3	77	−1.4	5	5
10	−0.8	4	4	94	+1.3	4	$3\frac{1}{3}$
15	−4.7	4	$3\frac{2}{3}$	102	+14.2	2	$1\frac{2}{3}$
19	−6.2	6	$5\frac{1}{3}$	104	+22.0	3	3
28	−5.8	5	5	105	+31.0	5	$4\frac{2}{3}$
34	−7.2	4	4	106	+41.0	3	3

In order to be able to employ the method of Lehmann-Filhés[1] for determining the orbit, the values of v_0 were platted as ordinates and those of Φ as abscissas in a rectangular system of co-ordinates, and a curve was drawn fitting the points thus obtained as closely as possible; this serves as the basis for the orbit. The radial velocity of the center of gravity of the system of β *Arietis* came out

$$V = -0.6 \text{ km}.$$

On the scale of the drawing ($1^d = 2$ mm; 1 km = 1 mm), the following values, expressed in square millimeters, were obtained:

$$z_1 = +128 ; \quad z_2 = -424 .$$

We also have

$$A = 59.7 \text{ km} ; \quad B = 5.5 \text{ km} .$$

In view of the uncertainty of the maximum value of v, A was taken in round numbers as 60 km.

By the formulae derived by Lehmann-Filhés I then found the following elements:

$$u_1 = 146\overset{\circ}{.}3 ,$$
$$\omega = 19\overset{\circ}{.}7 ,$$
$$e = 0.88 ,$$
$$\mu = 0.05872 = 3\overset{\circ}{.}364 ,$$
$$T = +0\overset{d}{.}1 ,$$
$$a \sin i = 22\,880\,000 \text{ km},$$
$$\frac{m_1{}^3 \sin^3 i}{(m+m_1)^2} = 0.042 \odot .$$

Since the spectrum of only one component of β *Arietis* is visible, nothing further can be determined as to the masses than the ratio given above. If we assume $m = m_1$, we should have

$$m \sin^3 i = 0.17 \odot .$$

For $i = 90°$ we should therefore have $m = m_1 = 0.17 \odot$, or the total mass of the system would amount to about one-third that of the sun.

I would add a few further remarks as to the computation of the elements. The radial velocity of β *Arietis* varies exceedingly slowly at the time when it has its largest negative value, so that the velocity-curve to that point runs almost parallel to the axis of abscissas. The consequence is that the determination of the phase to which the

[1] *Astronomische Nachrichten*, No. 3242, **136**, 17, 1894.

greatest negative ordinate B belongs is extremely uncertain. Hence z_2 was found by subtracting the quantity z_1 from the entire area below the axis of abscissas (after this had been displaced for the motion of the center of gravity.).

The eccentricity e was computed by the formulae of Lehmann-Filhés:

$$e \sin \omega = \frac{2\sqrt{AB}}{A+B} \cdot \frac{z_2+z_1}{z_2-z_1},$$
$$e \cos \omega = \frac{A-B}{A+B}.$$

The formula otherwise applicable for large eccentricities,

$$\sqrt{1-e^2} = \pi \tan u_1 \left\{ \frac{1}{2} - \frac{t_2-t_1}{U} \right\},$$

is hardly suited to the present case; since the time t_2 at which the radial velocity of the star equals, for the second time (reckoned from its maximum), the radial velocity of the center of gravity of the system, can be only very inaccurately determined on account of the very acute angle at which the velocity-curve cuts the axis of abscissas.

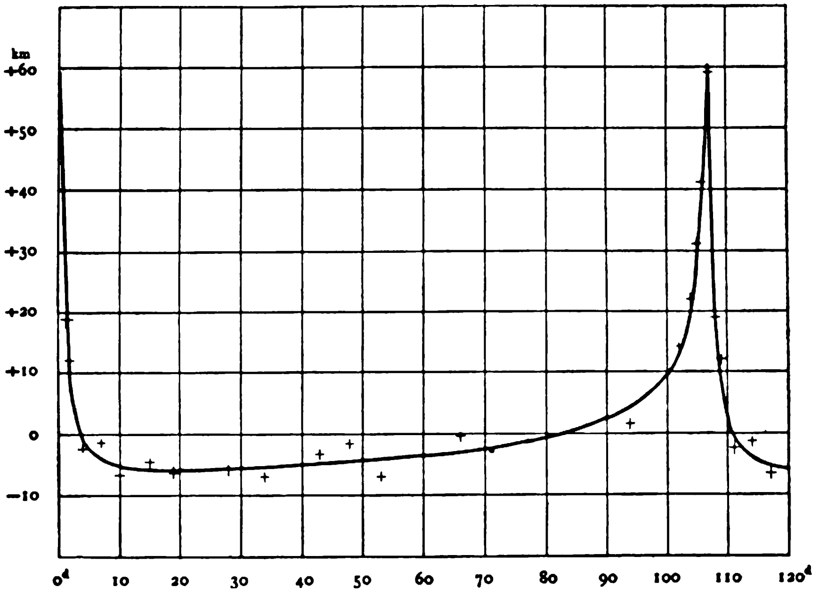
In order to test how well the elements represent the observations, I computed the following ephemeris. In so doing no attempt was made to give the fraction of a kilometer rigorously, which would be superfluous in view of the inaccuracies of the observations as well as of the elements.

Φ	v	Φ	v	Φ	v
0.0^d	+50.0^{km}	30^d	−5.8^{km}	95^d	+4.5^{km}
0.5	+43.0	35	−5.6	97.5	+6.5
1.0	+26.1	40	−5.3	100.0	+9.5
1.5	+13.8	45	−4.0	101.0	+11.0
2.0	+8.3	50	−4.5	102.0	+13.3
3.0	+2.4	55	−4.1	103.0	+16.3
4.0	−0.7	60	−3.7	104.0	+20.7
5.0	−2.4	65	−3.1	105.0	+27.6
7.5	−4.5	70	−2.4	105.5	+33.3
10	−5.3	75	−1.7	106.0	+42.3
15	−6.0	80	−0.8	106.5	+53.3
20	−6.1	85	−0.4	107.0	+59.0
25	−6.0	90	+2.0		

The normal values v_0 of the radial velocity derived from the observations are represented as follows:

Φ	O.−C.	Φ	O.−C.	Φ	O.−C.
0d	+0.1km	19d	−0.1km	77d	−0.1km
1	−7.1	28	+0.1	94	−2.7
2	+3.7	34	−1.4	102	+0.9
4	−1.6	43	+1.9	104	+1.3
7	+2.8	48	+3.0	105	+3.4
10	−1.5	53	−2.7	106	−1.3
15	+1.3	66	+2.8		

The representation is entirely satisfactory, as systematic deviations nowhere strongly appear, and the values in general are also small. It is to be noted that the two largest differences (at $\Phi = 1^d$ and $\Phi = 2^d$) correspond to values of v_0 which depend on the measurement of only



Computed Velocity-Curve of β *Arietis* with normal values of velocity.

a single plate. The mean value for weight unity, i. e., for a plate on which the *Mg* line and *Hγ* were measured, comes out as ±4.05 km. This value is not larger than would have been expected in advance from the character of the spectrum.

In view of what has been said, there is no occasion for undertaking a correction of the orbit until further observations have been obtained,

particularly such as will permit a sharper determination of the maximum velocity.

The figure gives a graphical representation of the ephemeris and of the normal values v_0 of the velocity. The unusual form of the velocity-curve is explained by the very great eccentricity of the orbit, and by the fact that the passage of the star through the ascending node nearly coincides with its passage through periastron.

The observations by no means exclude the possibility that A is somewhat larger than I have assumed it to be, in which case e would be larger than as given above. If, for instance, we take $A = 70$ km, and regard z_1 and z_2 as unchanged, which is tentatively permissible in view of the very pointed form of the curve near the maximum, we obtain:

$$u_1 = 148^\circ\!.7\,,$$
$$\omega = 18.1\,,$$
$$e = 0.90\,.$$

β Arietis has by far the greatest eccentricity of any spectroscopic binary of which the orbit is thus far known. *β Herculis* follows with an eccentricity of 0.55, and *ζ Ursae Majoris* with 0.52. Small eccentricities appear to predominate in general among spectroscopic binaries, in so far as such conclusions may properly be drawn from the slight amount of data at present available. For the twenty-six orbits of such systems at present known, including *β Arietis*, the eccentricities are distributed in the following way:

e	No.
0.00 to 0.15	15
0.16 to 0.30	3
0.31 to 0.45	2
0.46 to 0.55	5
>0.55	1

It should be mentioned further that Deslandres found $e = 0.60$ for the spectroscopic binary *θ Aquilae;* but the orbit seems to be very uncertain, and I therefore did not include that star in the above statistics.

Among visual binaries there are several having an eccentricity as great as, or even greater than, that of *β Arietis*. Aitken's "Catalogue of the Orbits of Visual Binary Stars"[1] gives for *γ Virginis*,

[1] *Lick Observatory Bulletin*, No. 84, 1905.

$e = 0.90$; for Σ 2525, even $e = 0.96$. The periods of these stars are, however, 194 and 307 years. Aside from these, eccentricities of 0.80 or over occurred in case of the binaries γ *Andromedae BC* ($e = 0.82$, $P = 55$ years), 99 *Herculis* ($e = 0.81$, $P = 65$ years), and γ *Centauri* ($e = 0.80$, $P = 88$ years). In these cases the systems are all of long period, while in the case of β *Arietis* the very large eccentricity is of particular interest especially in view of the shortness of the period.

ASTROPHYSIKALISCHES OBSERVATORIUM
Potsdam

A SPECTROGRAPHIC STUDY OF THE FOURTH-CLASS VARIABLE STARS *Y OPHIUCHI* AND *T VULPECULAE*[1]

By SEBASTIAN ALBRECHT

INTRODUCTION

On account of the extremely small displacements of spectrum lines, due to the radial velocities of the stars, it is desirable to use spectrographs of as high dispersion as possible. The amount of star light available is the principal factor in determining the upper limit of the dispersion. At present, determinations of the radial velocities of stars are made most extensively with three-prism instruments. These can be made to yield velocities reliable within a few tenths of a kilometer. The practicable limit of such an instrument, attached to the largest existing telescopes, is about the sixth photographic magnitude, which requires an exposure of approximately 2½ hours. There is urgent need for a knowledge of the radial velocities of much fainter stars. Data for the solution of important astronomical problems by non-spectroscopic methods have been obtained from a large number of stars, some of which are as faint as the twelfth visual magnitude, whereas radial velocities have really been limited to the sixth photographic magnitude. The one-prism spectrograph of the Lick Observatory was employed by Dr. R. H. Curtiss in a study of the variable star *W Sagittarii*,[2] which varies between 5.5 and 6.5 photographic magnitudes. His work showed that good velocity determinations with the one-prism instrument could be obtained, at least when the exposures were comparatively short. His average exposure was about 30 minutes. It was definitely an object of the present investigation to test the efficiency of this spectrograph for much fainter stars, requiring long exposures. The average exposures for the two variable stars selected (*T Vulpeculae* and *Y Ophiuchi*) were 75^m and 180^m, respectively. The

[1] Thesis in partial fulfilment of requirements for the degree of doctor of philosophy in the University of California.

[2] *L. O. Bulletin*, **3**, 19, 1904; and *Astrophysical Journal*, **20**, 149, 1904.

latter star, of about the eighth photographic magnitude at minimum, may be considered the practicable limit for this instrument, attached to the 36-inch refractor. In the case of a star whose light is concentrated in a few spectrum lines or bands, it is of course possible to go several magnitudes lower. For example, the spectrum of *Nova Aquilae No. 2* was successfully photographed when the star was of the eleventh visual magnitude.

The dispersion of the one-prism spectrograph is one-fifth that of the three-prism Mills spectrograph. The average radial velocity of the brighter stars is about ± 20 km per second. The equivalent displacement with the one-prism instrument, for the $H\gamma$ region, is 0.005 mm. A radial velocity of 2 km would produce a shift of 0.00002 inch (0.0005 mm). If the average radial velocity for the fainter stars is about the same[1] as for the brighter, then these small displacements are the quantities to be measured on the plates taken with the one-prism spectrograph. The results obtained are considered highly satisfactory. In the case of *Y Ophiuchi*, with an average exposure of 3 hours, the double amplitude of the velocity-curve is only 17 km. On the Mills spectrograms the same linear displacements would give a curve of $3\frac{1}{2}$ km double amplitude.

In addition to testing the possibility of extending the usefulness of the one-prism instrument for radial velocity work, it was thought that a contribution might be made toward the discovery of the causes of some of the peculiarities that are observed in short-period variable stars of the δ *Cephei* or η *Aquilae* type. Some of the more important points to be considered in this connection are: the peculiarities of, and the relation between, the light- and velocity-curves, peculiarities of the spectrum, changes in the character of the spectrum during the period of variability, and the behavior of the individual spectrum lines. The situation has been well described by Dr. Alexander W. Roberts.[2]

[1] Campbell's results seem to indicate that the velocities of the fainter stars are greater than those of the brighter stars.—*Astrophysical Journal*, **13**, 85, 1901.

[2] "Those whose researches lead them in one direction of an inquiry regarding the causes which underlie short-period variation must be impressed as well as oppressed, with the great area of uncertainty which surrounds the whole subject.

"At first sight we seem to know practically nothing of the immediate circum-

INSTRUMENT

With the exception of the addition of a temperature-case, the instrument used was the same as that employed by Curtiss in his study of *W Sagittarii* (*l. c.*). References in regard to description of the various parts of the instrument are given in that article. The iron spark, with sufficient self-induction to eliminate most of the air spectrum, was employed as the source of the comparison spectrum. The temperature-case was always put on the spectrograph between 30 minutes and an hour beforehand, in order to allow the greater part of the equalization of the temperature within the case to take place before beginning the exposure.

A PECULIARITY OF THE SPECTRA

In the variable stars of the δ *Cephei* type there is a greater richness of photographic radiation relatively to visual radiation at light-maximum than at light-minimum.[1] During the light-period the point of maximum energy on the energy-curve shifts along the spectrum, moving toward the shorter wave-lengths as the star approaches light-maximum, and back again toward the longer wave-lengths as light-minimum is approached. This fact is to a certain extent masked upon the spectrograms by instrumental and atmospheric causes, but in a long series it can readily be verified. My attention was first attracted to it by plate 13B (of *U Aquilae*), on which the region λ 4000 to λ 4200 was stronger than the $H\gamma$ region. I then

stances which produce variation of the definite type to which such stars as η *Aquilae* and δ *Cephei* belong.

"And yet the uncertainty is neither complete nor final. We are convinced, for example, that revolution and variation, or it may be rotation and variation, are connected together in some intimate relation, and that a solution of the problem of short-period variation will be obtained when we are able to declare what the nature and extent of this relation is.

"Any investigation, therefore, that purposes dealing with the measures of radial velocity obtained by spectroscopic observations must have a direct bearing on the wider problem of stellar variation. By considering the orbital movement of any binary system that also exhibits light-pulsations, we are approaching this problem from a less difficult and probably a more hopeful direction."—*Monthly Notices R. A. S.*, **66**, 320, 1906.

[1] The observations by Wilkens confirm this point. For five stars he finds the photographic range of brightness to be about one-half greater than the visual range.—*Astronomische Nachrichten*, **172**, 305, 1906.

examined all the spectrograms of *W Sagittarii* taken by Dr. R. H. Curtiss, and found the violet considerably weakened relatively to the green on the approach of light-minimum, the effect being greatest for this star between three and five days after light-maximum. The region from the H line to about λ 4200 is increased in intensity relatively to the blue and green as light-maximum is approached, and is apparently strongest at about 0.4 day after maximum.

The usual appearance of a spectrogram of a star of this type, taken on a Seed's Gilt Edge (27) plate, is a denser portion of about 300 Ångström units' length at λ 4600 or λ 4700, fading off slowly toward the violet and more rapidly toward the red. A combination of causes outside of actual changes in the star itself—such as differences in the transparency of the atmosphere, seeing, collimator setting, and emulsions—will somewhat modify this appearance. It is, therefore, not easy to determine from the spectrograms whether this shift of the point of maximum-energy along the energy-curve is entirely regular. The effect usually appears as a considerable intensification of the region λ 4000 to λ 4200, this region being then stronger than the $H\gamma$ region. On a few spectrograms it produced an almost uniform density of spectrum from the K line to λ 5000$\pm$. On plate 99B the effect is so marked that the possibility is not entirely excluded that it might have been due to a somewhat unusual outburst in the star. The plate falls close to periastron, though this may be accidental. Plates 35B of *T Vulpeculae* and 65A of *U Aquilae* show a similar appearance. 35B comes half a day after light-maximum, and 65A at maximum.

THE INDIVIDUAL SPECTRUM LINES

On account of the smallness of the dispersion, nearly all of the measurable lines on these spectrograms are blends of several components. One of the first points considered was the possibility that, for a few lines, some of the components might vary sufficiently during the light-period of the star to produce periodic shifts of measurable magnitude in their positions.[1] In this connection several factors

[1] Since then, variations in the positions of spectrum lines which are progressive from spectral type to type have been found by the writer on the spectrograms taken with the three-prism spectrograph of the Mills Observatory in Chile. (See *L. O. Bulletin*,

must be taken into account, the more important of which are the effect of varying amount of exposure, and the accuracy of measurement.

For the purposes of this study 39 lines were selected, and for each line the residuals (O.–C.)—i. e., the velocity given by the line minus the mean for all the lines measured on that plate—were formed and plotted from the measures of 34 spectrograms of *Y Ophiuchi*, arranged according to phase in the light-period. The same was done for 35 spectrograms of *T Vulpeculae*. Nine spectrograms of *Y Ophiuchi* and fifteen of *T Vulpeculae* were represented by double measures, the second measures having been made at times varying from a day to two years after the first measures. Each measure was made with violet to the left and violet to the right. The 24 spectrograms that are represented by double measures include overexposures and underexposures, and plates on which the star lines are quite fuzzy. In quality they are below the average of all the plates, and the measurements were made under varying conditions of illumination. They furnish, therefore, a severe test of the accuracy of measurement.

Following are the results briefly summarized. The probable error of measurement of a single line, as obtained from these double measures, was found to be ± 2.0 km. For only 8 per cent. of the lines did the second measure differ from the first by more than 10 km, and the greatest difference found was 23 km. The greatest error of measurement was therefore 12 km. If to the 23 km be added twice the largest error of a plate, we see that the greatest total range, on all the plates, of any individual spectrum line should be about 30 km (i. e., 15 km on each side of the zero position). The best lines should not have a range greater than half that amount. Nevertheless, the actual range averaged more than 30 km, and in several cases was as high as 50 km. A line, which from all external appearances should not differ from its zero position by more than 6 or 8 km, would occasionally differ from that position by 25 km. Almost invariably the second measure would reproduce the first within a few kilometers.

No. 126, and *Astrophysical Journal*, **24**, 333, 1906.) Also, a preliminary study of the the high dispersion spectrograms of *η Aquilae* showed very strong indications of just such variations in the positions of certain lines as were looked for on these low-dispersion plates of *Y Ophiuchi* and *T Vulpeculae*.

The following residuals (in km), obtained from the first and second measures of plate 53A, illustrate this point (only the lines that were included in both measures are here given): −3, +4; −1, −1; −7, −3; −9, −7; +4, +2; +25, +24; −5, −2; −2, −2; +10, +11; −7, −5; −21, −19; +20, +11; +9, +7; −21, −23; +6, −3; −9, −17; −4, −5; +8, +11; −3, −5; +3, −1. The line giving the residuals +25 and +24 km, on the first and second measures respectively, is to all appearances one of the best lines on the plate. On the thirty other spectrograms of the same star on which the line was measured the residuals range between +10 and −11 km. The relative displacement of this line on plate 53A is without doubt real, though the cause of the shift is uncertain. There is no evidence of any distortion of the film. This region of the plate is somewhat underexposed, and it was at first believed that this would account for the broadening of the line. Several other plates on which the exposure is about the same as on plate 53A, and some that are more underexposed, do not show the broadening. This is only one of a considerable number of similar examples.

Suffice it to state that no definite trace of a shift of any of the lines was found which is progressive with the phase of the star in its light-period. Large shifts in the positions of many lines were found, which seemed to be more or less of an accidental nature. For the present the question will be left in abeyance as to whether or not these irregular changes in the positions of some of the lines are due entirely to causes outside of the star.

THE VARIABLE STAR *Y OPHIUCHI*

The variable brightness of *Y Ophiuchi* was discovered by Sawyer in 1888. The principal data taken from the Chandler and Harvard Catalogues are as follows:

Chandler's Third Catalogue

α, 1900.0, = $17^h\ 47^m3$, δ, 1900.0, = $-6°\ 7'$. Visual magnitudes 6.2 to 7.0. Period 17.1207 days. Epoch of maximum 1882 Sept. 5; *J. D.* 2408694.25.

Harvard Catalogues

Visual magnitudes 6.1 to 6.5; Class IV; Sp. *G*.

Three hundred and nine observations by Luizet[1] in the years 1898 to 1904 were found to satisfy Hisgen's elements given in Chand-

[1] *Astronomische Nachrichten*, **168**, 351, 1905.

ler's Third Catalogue. Luizet gives the visual magnitudes as 5.9 to 6.6. The observations for the light-variations fortunately extend to within a year of the spectrographic observations that are discussed in this paper. My measures of the first eight spectrograms showed a variable radial velocity, the total range of variation being, however, small.[1] Table I gives the more important data in regard to the spectrograms of this variable.

After several trials of various ellipses with different values of the elements, the velocity-curve shown in Fig. 1 (the continuous line) was selected. The lower part of the figure shows the light-curves due to Pickering and Luizet. The elements upon which the velocity-curve is based are:

$U = 17.1207$ (light-period),
$\mu = 21\overset{\circ}{.}026$,
$T = 2.6$ days after light-maximum,
$\omega = 209\overset{\circ}{.}2$,
$K = 8.5$ km (single amplitude),
$e = 0.10$,
$V = -5.0$ km (velocity of system),
$a \sin i = 1{,}999{,}000$ km.

There is some indication of an irregularity in the velocity-curve. If a secondary curve with a period equal to half the light-period and a double amplitude of 2.5 km be superimposed upon the elliptic curve given above, the curve represented by the discontinuous line in the upper part of Fig. 1 results. This curve gives a better representation of the observed velocities except between zero and two days after light-maximum, where there is no choice either way. If we remember that the velocity for plate 10E is based on only eight lines, and should therefore be given small weight, we see that the improvement is very decided along the stretch from 3 to 7 days after light-maximum. Likewise, between 11 and 13 days the points corresponding to plates 27A and 59A should be given smaller weight than the points above the curve. This irregularity is such an extremely small quantity for the dispersion employed that we cannot place entire confidence in its reality. It would be equivalent to obtaining a secondary curve of 0.5 km double amplitude with the three-prism

[1] *Pub. A. S. P.*, **18**, 60, 1906.

Mills spectrograph. Attention is called to the fact that the light-curve shows a similar irregularity.

More prominent irregularities in velocity-curves have been observed by Campbell[1] in *ζ Geminorum* and by R. H. Curtiss in *W Sagittarii*

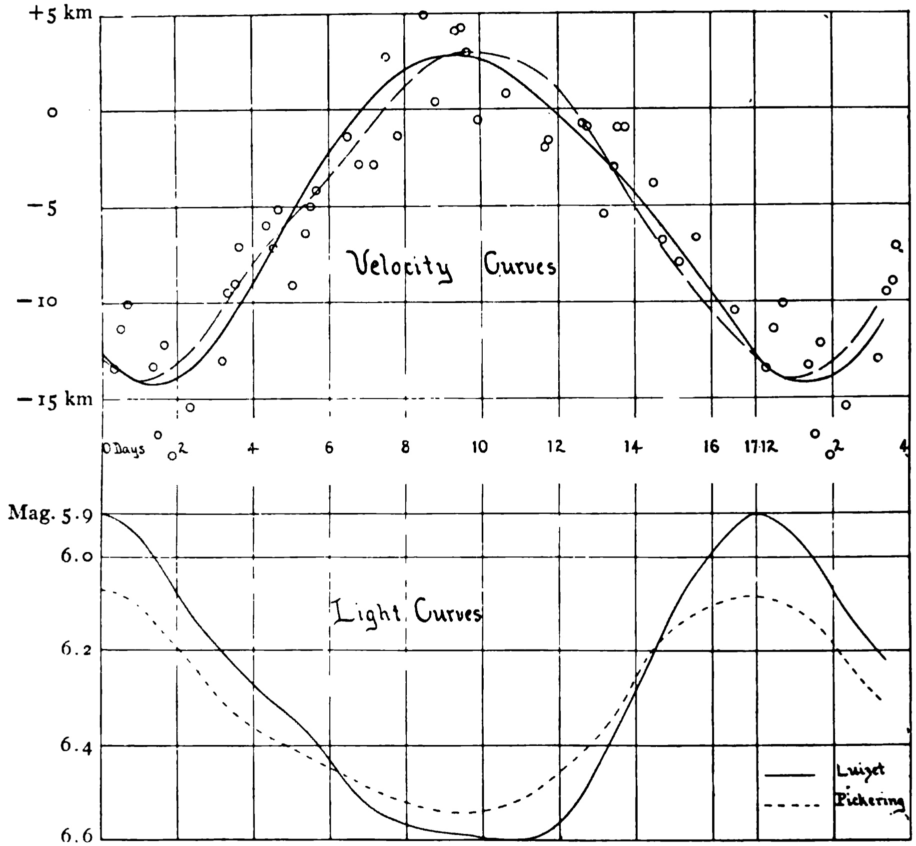


Fig. 1.—*Y Ophiuchi*

(*l. c.*). The cause of these secondary curves is still an unsettled question. Various explanations have been offered, such as the presence of a third body; the rotation of the brighter component; a resisting medium; or the effects of tidal forces, which must necessarily be large in such close binaries. Dr. Alexander W. Roberts

[1] *Astrophysical Journal*, **13**, 90, 1901.

TABLE I

Y OPHIUCHI

Plate	Date of Mid Exposure. G. M. T.	Interval since Maximum	v_S	Reduction to Sun	V	No. of Lines	Duration of Exposure	Slit-Width	Seeing	Character of Spectrogram, etc.
	1905	Days	km	km	km		min.	In units of 0.025 mm		
1D	May 10.980	13.438	20.0	+17.0	−3.0	28	120	1.4	Fair	Fair. Underexposed.
10E	June 5.812	5.022	15.3	+ 6.1	− 9.2	8	240	1.4	V. bad	Much underexposed.
12A	July 11.830	6.799	+ 8.1	−10.9	− 2.8	22	150	1.4	Poor	Underexposed. Much smoke in air.
14A	12.843	7.812	+10.0	11.4	1.4	25	240	1.4	Poor	Good.
15B	13.823	8.792	+12.1	−11.7	+ 0.4	43	180	1.4	Fair	Good.
16A	16.797	11.766	+11.3	−12.9	− 1.6	29	180	1.4	Good to fair	Fair.
17A	17.778	12.747	+12.5	−13.4	− 0.9	34	170	1.4	Fair	Good. Much smoke in air.
18A	18.782	13.751	+12.9	−13.8	− 0.9	26	150	1.4	Good to fair	Fair.
19A	19.790	14.759	+ 7.5	−14.2	− 6.7	38	150	1.4	Fair	Good.
20A	22.785	0.633	+ 5.4	−15.5	−10.1	31	130	1.4	Fair	Good.
21A	23.799	1.647	+ 3.7	−15.9	−12.2	27	150	1.4	Poor	Underexposed Thin clouds.
22A	25.762	3.610	+ 9.5	−16.6	− 7.1	41	180	1.4	Poor	Fair.
23A	26.773	4.621	+11.8	−17.0	− 5.2	30	195	1.4	Poor	Underexposed. Fair.
24A	27.789	5.637	+13.2	−17.4	− 4.2	32	200	1.4	Poor	Good. Wind 30 miles N. W.
25A	31.710	9.558	+21.9	−18.9	+ 3.0	26	180	1.4	Poor	Much smoke in air. Underexposed.
26A	Aug. 1.762	10.610	+20.0	−19.2	+ 0.8	35	180	1.4	Poor	Good.
27A	2.76	11.61	+17.5	−19.5	− 2.0	22	190	1.4	Poor	Underexposed.
28A	3.769	12.617	+19.2	−19.9	− 0.7	37	200	1.4	Fair	Somewhat underexposed. Good.
29A	6.749	15.597	+14.3	−20.9	− 6.6	38	160	1.4	Good	Good.
30A	7.740	16.588	+10.8	−21.2	−10.4	38	150	1.4	Fair	Good.
31A	8.751	0.479	+10.2	−21.5	−11.3	41	140	1.4	Fair	Good.
32A	9.747	1.475	+ 4.9	−21.8	−16.9	36	150	1.4	Good	Good. Exposure cut off for 20 min.
33A	11.810	3.538	+13.4	−22.4	− 9.0	28	210	1.4	Poor	Somewhat underexposed. Good.
34A	12.808	4.536	+15.6	−22.8	− 7.2	25	160	1.5	Poor	Smoke in air. Wind 30 miles. Unexposed.
36A	13.758	5.486	+18.0	−23.0	− 5.0	33	210	1.4	Fair	Good. Sky good.

TABLE I *Continued*

Plate	Date of Mid-Exposure. G. M. T.	Interval since Maximum	v_s	Reduction to Sun	V	No. of Lines	Duration of Exposure	Slit-Width	Seeing	Character of Spectrogram, etc.
	1905	Days	km	km	km		min.	In units of 0.025 mm		
37A	Aug 14.740	6.468	+21.0	−23.3	−1.4	30	180	1.5	Poor	Good. Sky good.
38A	15.754	7.482	+20.3	−23.6	+2.7	22	210	1.5	Good	Image of star faint. Thin clouds. Underexposed. Fair.
39A	16.764	8.492	+28.7	23.8	+4.0	17	190	1.4	Poor	Haze. Underexposed and somewhat fuzzy.
40A	17.746	9.474	+28.4	−24.1	+4.3	28	200	1.4	Fair	Underexposed. Fair.
41A	21.832	13.560	+24.2	−25.1	0.9	32	205	1.4	Fair	Good. Somewhat underexposed.
42A	22.738	14.466	+21.5	−25.3	−3.8	36	180	1.4	Fair	Good.
43A	25.736	0.343	+12.5	−25.9	−13.4	37	180	1.4	Good	Good. Clouds passed over star occasionally.
44A	26.772	1.379	+12.9	26.2	−13.3	29	160	1.4	Fair	Good.
45A	27.726	2.333	+10.9	−26.3	−15.4	38	150	1.4	Good	Good.
47A	28.724	3.331	+17.0	−26.5	−9.5	35	166	1.4	Good	Good.
49A	29.765	4.372	+20.7	−26.7	6.0	28	200	1.4	Poor	Sky fair. Wind 30 miles at end of exposure. Fair.
50A	30.774	5.381	+20.5	−26.9	−6.4	31	210	1.4	Fair to poor	Much smoke in air. Underexposed. Fair.
51A	Sept. 3.751	9.358	+31.6	−27.5	+4.1	11	210	1.4	Poor	Much smoke in air. Very much underexposed.
53A	14.706	3.102	+15.4	−28.4	−13.0	27	190	1.4	Fair	Good. Somewhat underexposed.
54A	18.698	7.184	+25.7	−28.5	−2.8	11	180	1.4	Fair	Poor. Very much underexposed.
59A	24.699	13.185	+23.1	28.5	−5.4	20	190	1.5	Poor	Much underexposed.
63A	26.682	15.168	+20.5	−28.4	−7.9	28	190	1.5	Fair	Fair. Underexposed.
	1906									
112B	Aug. 12.733	9.026	+22.1	−22.6	−0.5	12	180	1.5	Poor	Very much underexposed.
121A	21.725	1.797	+6.8	−24.8	−18.0	31	170	1.5	Poor	Somewhat underexposed. Fair.

has shown, in the interesting article referred to above, that considerable deviations of the principal bodies from the spherical form, in the case where the size of the stars is distinctly comparable to the size of their orbits, would give rise to a secondary period in the velocity-curve equal to half the primary period. This is a very interesting and suggestive explanation, though probably not a complete one. In *W Sagittarii* the secondary period is without doubt half that of the primary, whereas in the case of *ζ Geminorum* a secondary period, equal to one-third that of the primary, satisfies the observed curve better than one of half the primary period. In a complete explanation probably a number of factors must be taken into account, and in the different individual cases one or the other of these factors may become the predominant one, and thus produce differences in the period of the secondary or in other peculiarities of this class of variables. In the course of a few years, as studies of several other variables of this and related classes will become available, we may hope to be able to speak more authoritatively in regard to the characteristics that are common to all as well as the points of difference. In individual cases we may be able to pick out the predominant influences that are at work.

THE VARIABLE STAR *T VULPECULAE*

The variable brightness of *T Vulpeculae* was discovered by Sawyer in 1885. The principal data taken from the Chandler and Harvard Catalogues are as follows:

Chandler's Third Catalogue

α, 1900.0, $=20^{\mathrm{h}}\ 47.^{\mathrm{m}}2$; δ, 1900.0, $=+27^{\circ}\ 52'$.
Redness 0; visual magnitude 5.5 to 6.5
Period 4.4360 days; epoch 1885, Nov. 2, *J. D.* 2409848.95
Basis of elements, observations 1885–95.

Harvard Catalogues

Visual magnitudes 5.5 to 6.2; Class IV; Sp. *F*.

Light-curves for this variable have been obtained by Sawyer, Chandler, Yendell, Pickering, Luizet, and Terkán by visual methods, and by Wilkens by a photographic method. From observations made in 1898 and 1899 Luizet derives the following elements: period 4.43578 days; epoch of maximum *J. D.* 2409849.02 (1885, Nov. 3.02). These values of period and epoch were adopted in this

investigation. They differ but slightly from the values given in Chandler's Third Catalogue. The series of photographic observations for the determination of the light-curve made by Wilkens from July 9 to September 18, 1905, are practically simultaneous with the principal series of the spectroscopic observations. This is a very fortunate circumstance.

The binary character of the star was announced by Frost.[1] Before the announcement appeared our first series of spectrograms of the star had been obtained by Drs. H. D. Curtiss and Moore. The following data were given in regard to Frost's two observations:

Plate	Date	G. M. T.	No. of Lines	Radial Velocity
1B 379	1904, July 19	$20^h\ 5^m$	9	+15km
385	22	19 47	13	−17

These two observations fall on opposite sides of my curve, the first 11 km above and the second 3 km below it. In the figure they are represented by the crossed circles. The large residual is probably due to the small number of lines measured. Table II gives the more important data in regard to the spectrograms of this star that were obtained. When taken by others than the writer, the observer's initials are given in the column of remarks. All of the measures were made by the writer.

The solution of the orbit was made by the method of Lehmann-Filhés. After several trials of various ellipses with different values of the elements, the velocity-curve computed with the elements given below was found to reproduce the observed velocity-curve well within the error of construction of the latter. A least-square solution was therefore considered entirely unnecessary. The following are the adopted elements:

$U = 4^d43578$ (light-period),
$\mu = 81^\circ 1583$,
$T = 3^d76$ after light-maximum,
$\omega = 111^\circ$,
$K = +17.6$ km (single amplitude),
$e = 0.43$,
$V = -1.3$ km (velocity of system),
$a \sin i = 969,180$ km.

[1] *Astrophysical Journal*, **20**, 296, 1904.

TABLE II.—*T VULPECULAE*

Plate	Date G. M. T.	Interval since Maximum	v_s	Reduction to Sun	V	No. of Lines	Duration of Exposure	Slit-Width	Seeing	Character of Spectrogram, etc.
	1904	Days	km	km	km		min.			
3339E	July 12.967	3.717	−19.9	+11.7	− 8.2	31	46	1.4	Good	Good. Taken by H. D. C.
3355D	24.931	2.374	+ 7.4	+ 7.0	+15.3	17	85	1.2	Fair	Violet very much underexposed. Taken by H. D. C.
3391E	Aug. 9.947	0.647	−11.1	+ 2.3	− 8.8	23	111	1.3	Fair to poor	Overexposed. Taken by H. D. C.
3395D	15.930	2.104	+ 5.4	+ 0.1	+ 5.5	27	80	1.3	Fair	Taken by H. D. C.
3415D	20.915	2.872	+15.6	− 4.9	+10.7	25	120	1.4	Good	Good. Somewhat overexposed. Taken by H. D. C.
3471C	Oct. 2.793	1.264	+10.2	−15.7	− 5.5	37	95	1.4	Poor at times	Fair. Taken by J. H. M.
3495A	18.798	3.961	+ 1.0	−19.3	−18.3	30	60	1.3	Poor	Good. Taken by H. D. C.
	1905									
35B	Aug. 12.952	0.483	− 8.0	+ 1.3	− 6.7	37	111	1.4	Fair to poor	Good. Taken by J. H. M.
40B	27.873	2.096	+13.6	− 4.1	+ 9.5	31	80	1.4	Good	Good.
48B	28.828	3.051	+19.9	− 4.3	+15.6	33	95	1.4	Good to fair	Good.
55B	Sept. 18.810	1.854	+19.7	−11.5	+ 8.2	37	90	1.4	Good	Good.
60B	24.834	3.443	+22.0	13.4	+ 8.6	25	80	1.5	Poor to fair	Good. Somewhat underexposed.
64A	Oct. 3.720	3.457	+22.4	−15.7	+ 6.7	22	90	1.4	Fair	Good. Slightly overexposed.
66B	4.790	0.001	− 0.4	−16.2	−16.6	33	65	1.4	Fair	Good.
68E	5.828	1.129	+17.6	−16.4	+ 1.2	34	70	1.4	Good	Good. 4° difference between t_1 and t_3
70E	9.776	0.641	+ 7.6	−17.2	− 9.6	26	70	1.4	Fair	Good. *Fe* overexposed.
75E	17.800	4.230	− 1.9	−19.1	−21.0	18	70	1.4	Fair	Good.
76A	19.662	1.656	+24.4	−19.2	+ 5.2	17	80	1.4	Good	Good.
78E	20.776	2.770	+35.0	−19.6	+15.4	18	95	1.4	Poor	Fair. Somewhat underexposed.
79E	21.642	3.636	+ 9.7	−19.5	− 9.8	22	60	1.5	Good	Overexposed.
81B	23.761	1.319	+18.6	−20.1	− 1.5	20	70	1.4	Good	Fair.
82E	24.705	2.263	+26.5	−20.1	+ 6.4	29	100	1.4	Good	Fair.
83E	25.679	3.237	+36.2	−20.2	+16.0	38	100	1.4	Good	Star spt'm good. *Fe* poor on one side.
84B	27.646	0.768	+12.4	−20.5	− 8.1	40	60	1.4	Good	Good. Somewhat overexposed.
85E	27.706	0.828	+16.7	−20.6	− 3.9	24	67	1.4	Good	Some clouds? Good.
87E	Nov. 2.700	2.386	+30.7	−21.3	+ 9.4	27	90	1.4	Fair	Fair. Somewhat underexposed.
88E	5.662	0.913	+16.4	−21.5	− 5.1	20	90	1.6	Poor	Good, but grain of plate is bad.
89E	7.609	2.860	+32.9	−21.5	+11.4	24	90	1.4	Fair	Fair. Somewhat overexposed.
91E	9.684	0.499	+ 8.6	−21.8	−13.2	17	68	1.4	Good	Clouds came over. Poor; intensity drops off rapidly toward violet.
92B	10.622	1.437	+21.0	−21.7	− 0.7	26	60	1.6	Poor	Fair. Smoke in air.

TABLE II.—*Continued*

Plate	Date G. M. T.	Interval since Maximum	v_s	Reduction to Sun	V	No. of Lines	Duration of Exposure	Slit-Width	Seeing	Character of Spectrogram, etc.
	1905	Days	km	km	km		min.			
93B	Nov. 13.635	0.014	+ 2.9	−21.9	−19.0	22	62	1.5	Poor	Passing clouds. Wind 28 miles. Good exposure but spectrum somewhat fuzzy.
94B	14.610	0.989	+14.3	−21.9	− 7.6	40	62	1.4	Fair	Fair. Underexposed.
95B	21.622	3.565	+17.7	−21.9	− 4.2	35	70	1.6	Poor	Fair. t_3 went up continuously.
97B	Dec. 1.621	0.257	+ 9.1	−21.3	−12.2	25	80	1.4	Poor	Fair.
98B	4.617	3.253	+30.1	−20.9	+ 9.2	25	62	1.4	Fair	Exposure gradually shut off by clouds. Underexposed. Fair.
99B	9.606	3.809	+ 7.0	−20.3	−13.3	26	99	1.6	Poor	Wind 30–40 miles. Density uniform from K line to λ5000±.
103B	31.593	3.614	+10.0	−15.7	− 5.7	8	59	1.5	Poor	Star spectrum fuzzy. Underexposed. $t_1 = -3°$.
	1906									
104B	Jan. 4.618	3.204	+25.4	−14.6	+10.8	24	66	1.5	Fair to poor	Underexposed.
106A	Aug. 5.743	3.411	+ 7.1	+ 4.3	+11.4	31	80	1.4	Good to fair	Good.
107B	5.799	3.467	+ 6.7	+ 4.1	+10.8	40	74	1.4	Good	Good.
108A	5.910	3.578	− 4.3	+ 3.9	− 0.4	35	74	1.4	Good	Good.
109B	5.962	3.630	− 7.6	+ 3.8	− 3.8	29	74	1.4	Fair	Good.
110A	9.880	3.112	+ 9.7	+ 2.6	+12.3	30	94	1.4	Good to fair	Clouds interfered during last half hour. Somewhat underexposed.
111B	9.952	3.184	+11.0	+ 2.4	+13.4	32	96	1.4	Fair	Clouds interfered considerably. Somewhat underexposed.
113A	19.692	4.053	−22.2	− 0.6	−22.8	23	74	1.4	Fair	Good. No heating current.
114B	19.754	4.115	−20.6	− 0.8	−21.4	37	64	1.4	Fair	Good. No heating current.
115A	19.859	4.220	−18.9	− 1.0	−19.9	24	60	1.4	Fair	Good. Heating current available again.
116B	19.942	4.303	−16.7	− 1.1	−17.8	27	60	1.4	Fair	Good.
117A	20.703	0.628	− 8.9	− 1.0	− 9.9	37	76	1.4	Fair to poor	Wind 25 miles. t_3 went up without heating current. Good.
118B	20.757	0.682	− 6.3	− 1.1	− 7.4	32	76	1.4	Poor	Wind 25 miles. Good.
119A	20.833	0.758	− 4.0	− 1.3	− 5.3	31	80	1.4	Poor	Wind 25 miles. Good.
122B	21.842	1.767	+ 6.5	− 1.6	+ 4.9	30	86	1.4	Poor	Lens fogged? Somewhat underexposed

For each star the probable error of a single plate was found to be ± 1.2 km. The probable error of measurement of a single spectrum line was ± 2.0 km. From this alone we should expect a smaller

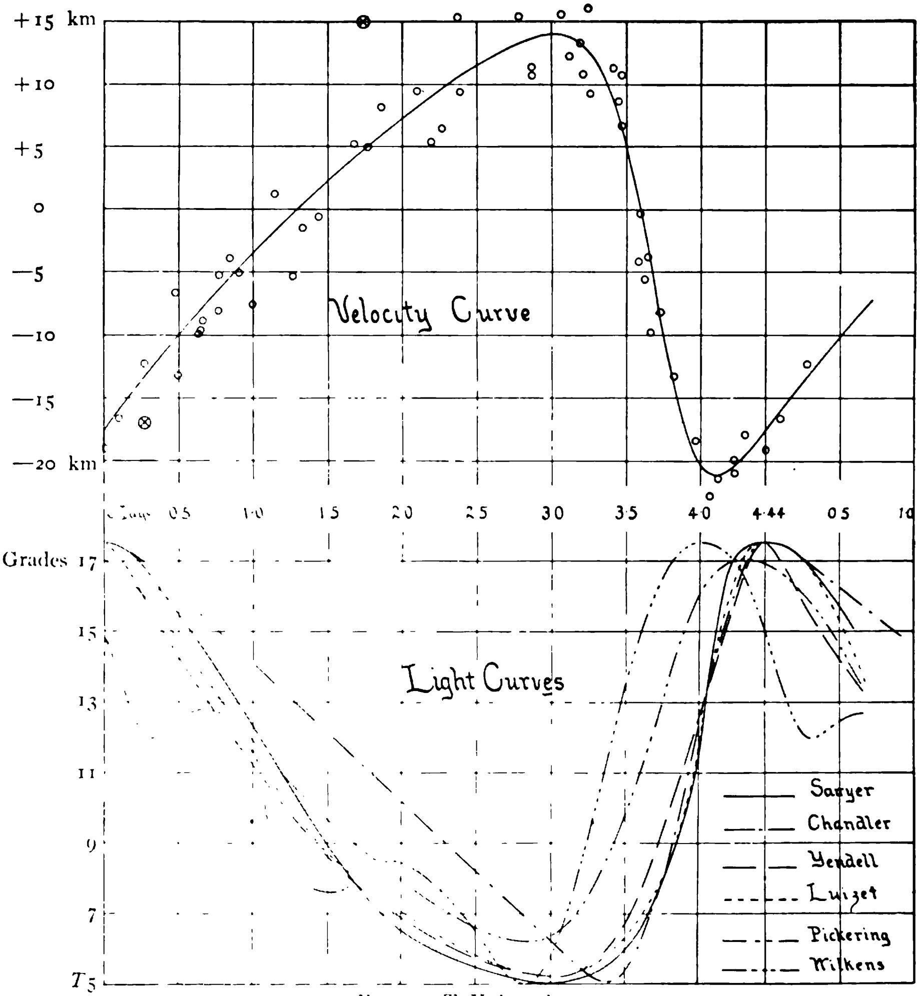


FIG. 2.—*T Vulpeculae*

value for the probable error of a plate. If we remember, however, that large accidental variations in the positions of individual lines were not uncommon (see p. 334), the value found for the probable error of a plate is readily accounted for. Also, in such long exposures as

were employed on these variables, the chances become greater for an incomplete correspondence between the star- and the comparison-spectra.

The upper part of Fig. 2 shows the velocity-curve computed with the elements above, while the lower part of the figure gives the light-curves by several observers. For Pickering's light-curve the range of variation is smaller than for the other observers. For Wilkens' photographically determined light-curve the photographic range of variation was reduced to the visual range by his ratio of photographic to visual range, which for this star was 1.5. The visually determined light-curves agree fairly well in regard to the epoch of maximum. The origin of the minor differences between these curves is not entirely clear. Luizet,[1] who compared his own curve with those of Chandler, Sawyer, and Yendell, ascribed the differences between them to a sort of personal equation. Wilkens' light-curve, determined by a photographic method, gives the epoch of maximum 0.4 day earlier than the curves which were determined by visual methods. A longer series of photographic observations is desirable to establish definitely this apparent difference between the times of maximum brightness for the visual and photographic radiations. Wilkens has drawn two secondary maxima and minima in his light-curve for this star. The curve is based on seventeen observations, which, of course, cannot be considered sufficient to establish these irregularities. My velocity-curve does not definitely show corresponding irregularities, nor does it prove their non-existence. They could be drawn in approximately the positions indicated by Wilkens, and this would reduce the residuals somewhat. One important point in connection with the velocity-curve is that it depends upon three series of observations, in three successive years, and that each series is satisfied by the same curve. There is thus no appreciable rotation of the line of apsides nor rapid change of any of the other elements.

In neither of these two stars could the variability be due to an eclipse, for in that case maximum and minimum brightness would occur at the points where the velocity equals the velocity of the system.

Perhaps the most important result of this investigation is the conclusive evidence of a much closer relationship between the light- and

[1] *Astronomische Nachrichten*, **153**, 80, 1900.

TABLE III.

Star	Period	Time Interval between Maximum Brightness and Greatest Negative Velocity	Observer	Reference
	Days	Days		
ζ Geminorum	10.15	−0.2	W. W. Campbell at L. O.	*Astrophysical Journal*, **13**, 90, 1901
η Aquilae	7.18	+0.2	W. H. Wright at L. O.	*Ibid.*, **9**, 59, 1899
δ Cephei	5.37	−0.2±	A. Belopolsky at Pulkowa	*Ibid.*, **1**, 160, 1895
W Sagittarii	7.60	+0.1	R. H. Curtiss at L. O.	*Ibid.*, **22**, 274, 1905
T Vulpeculae	4.44	−0.3	S. Albrecht at L. O.	This article
Y Ophiuchi	17.12	+1.3±	S. Albrecht at L. O.	This article
U Aquilae	7.02	+0.5±	S. Albrecht at L. O.	Not published
X Sagittarii	7.01	+0.3±	J. H. Moore at L. O.	Not published
S Sagittae	8.38	0. ±	R. H. Curtiss at L. O.	*L. O. Bulletin* 62
SU Cygni	3.85	+0.5±	J. D. Maddrill at L. O.	*Publ. A. S. P.*, **18**, 252, 1906

velocity-curves than has heretofore been believed to exist. If the light is sent out equally in all directions from the variable star, the positions of light- and velocity-maxima and minima should bear no special relation to each other, for the brightness would be independent of the direction from which the star is observed, while the radial velocity at any instant is dependent upon the direction of the observer. For different stars we should, therefore, expect the two curves to be shifted by different amounts relatively to each other around the period. For some stars greatest positive velocity would come at light-maximum, in others at light-minimum, and in most cases at other points along the light-curve. Table III of the ten variables of this class for which both light- and velocity-curves are available, shows that light-maximum and most rapid approach always occur together. Likewise, there is a time-correspondence between minimum brightness and greatest velocity of recession. We should therefore also expect that when irregularities[1] exist in both light- and velocity-curves, they will correspond to each other in position and perhaps also in shape. Of the ten stars in the above list only two have thus far shown marked irregularities in both light- and velocity-

[1] All the irregularities observed in the brightness- and velocity-curves of the stars contained in Table III fall between light-maximum and light-minimum, except in the case of *ζ Geminorum*.

curves. They are *W Sagittarii*[1] and *Y Ophiuchi;* and for these the irregularities in the two curves correspond very closely.

This establishes the fact that in the variable stars of the δ *Cephei* type the light- and velocity-variations are very intimately connected; that both are due to the same causes; and that, if the velocity-variation is dependent upon the direction of the observer, so also must the observed light-variations be dependent upon the same factor.

At present the best theory for this class of variables seems to be that they are binaries, in which one of the component stars is considerably brighter than the other. The observed velocity-variation follows mainly as a direct consequence of the orbital motion of the brighter component. The light-variation seems to be caused in some way (other than eclipse) by the influence of the darker companion. The very close correspondence between the light- and velocity-curves in regard to period and shape, and the agreement of the times of occurrence of maximum brightness with greatest velocity of approach and minimum brightness with greatest velocity of recession, would indicate that the light-variation is not so much dependent upon the position of the brighter component of the system in its orbit as upon the direction from which the star is observed. This would ascribe less direct influence to the darker companion in the matter of liberating an unusual amount of energy in a certain part of the orbit, most likely a small fraction of the period after periastron passage. Dr. Campbell has called my attention to the fact that the *Algol* variables, which are binaries of even shorter average period than the δ *Cephei* variables, show no evidence of light-variation other than that caused by eclipse, and that the apparent failure of two *Algol* components to disturb each other should make us careful in ascribing the total observed effects in δ *Cephei* variables to the mutual disturbing powers of the components. Most of the eclipse variables have earlier-type spectra (B, A, and F) than the variables of Class IV. It is not impossible that in close binary pairs having the simpler types of spectra (*Algol* variables) the mutual disturbances are less effective in producing brightness-variations than in close pairs having older types of spectra (δ *Cephei* variables).

[1] *Astrophysical Journal*, **22**, 274, 1905.

SUMMARY OF RESULTS

1. With the one-prism spectrograph and 36-inch refractor, we can obtain satisfactory radial velocities of stars of the eighth photographic magnitude, requiring exposures of three hours, provided their spectra contain well-defined lines.

2. For several variables of the δ *Cephei* or η *Aquilae* type it was found that the point of maximum energy in the spectrum is shifted toward the violet as light-maximum is approached, and back again toward the red as the brightness diminishes. From the spectrograms it was impossible to decide whether or not this shift of the point of maximum energy occurs with absolute regularity.

3. On these low-dispersion spectrograms no periodic shift in the position of any of the lines (blends) was found.

4. Velocity-curves and orbits for the fourth-class variable stars *Y Ophiuchi* and *T Vulpeculae* were obtained.

5. A comparison of the light and velocity-curves of the ten variables of this class for which both curves are available showed a much closer relation to exist between the light- and velocity-variations than has heretofore been supposed to be the case. In every observed case, light-maximum and greatest velocity of approach occur within one-fifteenth of the period of each other. Likewise minimum brightness and greatest velocity of recession occur at the same time. That is, the two curves have a very close correspondence in phase, in addition to correspondence of shape and period. If the velocity-curves were plotted with negative values above and positive below, the brightness- and velocity-curves for any star, if constructed on the proper scale, would be almost identical. The time-interval from maximum to minimum brightness is on the average about double the interval from from minimum to maximum brightness.

It is with great pleasure that I acknowledge my indebtedness to Dr. Campbell for the continued interest he has taken in this investigation. In assigning stars of the δ *Cephei* type as subjects for theses to several successive candidates for the degree of doctor of philosophy, he had definitely in mind the determination of the characteristics of these variables as a class, through the observations of a large number of these stars, as a basis for safe reasoning upon the cause or causes of their variability.

Lick Observatory
University of California
May, 1907

ON THE DISTORTIONS OF PHOTOGRAPHIC FILMS ON GLASS[1]

By SEBASTIAN ALBRECHT

INTRODUCTION

In various lines of astronomical research depending upon photographic plates, discrepancies of a considerable magnitude occasionally appeared, which seemed attributable to no definite cause. On the star-photographs taken with the Crossley Reflector, these occasional discrepancies, which seemed to be more or less accidental, usually amounted to a few tenths of a second of arc, and very rarely to as much as a second of arc, which is equivalent to a linear distance of about 0.001 inch (0.025 mm). Even though discrepancies are the exception rather than the rule, and discrepancies of the magnitude referred to above are extremely rare, nevertheless they cause considerable annoyance when extreme accuracy is desired, for the error of measurement need not much exceed 0.001 mm. It seemed highly desirable definitely to locate, if possible, the cause of the difficulty. In the case of the Crossley star-photographs it seemed for a time as though the cause must be sought for in the large mirror of the telescope. Another alternative was the study of the photographic film itself. Accordingly, in the winter of 1904, at the suggestion of Director Campbell and Dr. Perrine, the writer undertook an investigation of the distortions of the gelatine film.

The principal previous investigations on the distortions of the gelatine film were made by Scheiner, Loewy, Bergstrand, and Ludendorff. Scheiner[2] at first believed he had found a contraction in one co-ordinate and a dilation in the other, and assumed that this distortion was progressive, i. e., proportional to the distance measured. In 1897,[3] however, he says:

> Weitere Erfahrungen haben indessen gezeigt, dass man wahrscheinlich die Verziehungen nicht der gemessenen Strecke proportional setzen darf, sondern dass sie ziemlich localer Natur sind und sich häufig schon auf sehr kurze Strecken hin wieder aufheben.

[1] Thesis in partial fulfilment of the requirements for the degree of doctor of philosophy in the University of California.

[2] *Zeitschrift für Instrumentenkunde*, **11**, 394, 1891.

[3] *Photographie der Gestirne*, p. 119.

Loewy,[1] from a study of an *Eros* plate with multiple exposures, using three images of each of 82 stars, came to the conclusion that distortions of the film introduced an accidental error into the differences $x_n - x_m$ and $y_n - y_m$ which was represented by a probable error of 0."06 (=0.001 mm or 0.00004 inch).

Bergstand[2] investigated two star-plates, and from his results drew the following conclusions: Except near the edges of the plate, the distortions are, in general, not common to large parts of the plates; the affected regions are in the nature of bands or strips, which are not greater than 5 mm, and probably much smaller than this; these strips seem to have a tendency to be oftener parallel to one co-ordinate than to the other. The two plates studied by him seem to show a striking agreement both as to the amounts and trend of the distortions, and he concludes therefore that the distortions cannot always be eliminated by the use of several plates.

Ludendorff[3] investigated two of his star-plates and found that they showed large distortions which extend systematically over a considerable portion of the plates. On both plates he found several neighboring reseau-lines to be curved, the convex side being directly toward the center on one of the plates, and away from the center on the other. In some cases he found the inclination of the reseau-lines to be somewhat changed. The largest relative displacement between two neighboring lines was 10 μ. He states, however, that his endeavor was principally to find plates with strong distortions. It is well to quote some of the additional circumstances to which he calls attention.

Ganze Gruppen von Platten, welche an demselben Abend oder an naheliegenden Abenden aufgenommen wurden, weisen an demselben Randstriche Krümmungen in demselben Sinne, aber von verschiedenen Betrage auf. Auch kommen mitten unter diesen Platten ganz normale vor. . . . . Auffällig bleibt indessen, dass diese Verzerrungen an Platten, welche aus derselben Zeit stammen, mit solcher Regelmässigkeit auftreten, bei Platten aus anderen Zeiten gar nicht, und dass sie, wenigstens soweit meine Beobachtungen reichen, hauptsächlich einen bestimmten Strich (A 26) und dessen Nachbarstriche betreffen.

[1] "Sur la précision des coordinées des astres . . . . ," Third Memoir, p. 83, in *Bulletin du Comité international permanent pour l'exécution photographique de la carte du ciel*, **3**, 1902.

[2] *Öfversigt af K. Sv. Vet.-Akad. Förhandl.*, 1900, No. 2, p. 38.

[3] *Astronomische Nachrichten*, **162**, 343, 1903.

He explains these facts in the following words:

Bei näherer Überlegung sieht man indessen, dass es möglich ist, für beide Erscheinungen plausible Ursachen anzuführen. Da die Gitterkopien alle gleichartig in den Kasseten gelegen haben, so wird es sich ganz von selbst ergeben, dass sie sich auch bei allen photographischen Processen in derselben Lage befunden haben. Namentlich beim Trocknen, während dessen die Platten nahezu senkrecht standen, meist eine bestimmte Seite des Gitters die untere, horizontale gewesen sein. Eine nähere Betrachtung der nötigen Manipulationen macht es nicht unwarscheinlich, dass dies der Strich 26 gewesen ist.

He also says that he found numerous cases of local distortions. As an example of these he mentions plate No. 40 on which portions of two neighboring reseau-lines are bent in opposite directions.

From the above outline it will be seen that our knowledge of the nature and amounts of the distortions was quite indefinite. The prevailing opinion in regard to the subject was well stated by Professor H. H. Turner,[1] of Oxford, who is here quoted:

Even now it can scarcely be said that we know definitely the stage of refinement at which we must begin to expect irregular displacements of the images from distortion of the photographic film; but we have learned that they do not occur in a gross degree, and that other apparatus must be improved before we need turn our attention seriously to errors arising from such a cause.

The more important features of the plan upon which my work was begun were investigations of the effects of (*a*) the position of the plate during the processes of washing and drying, (*b*) the rate of drying, (*c*) abrupt changes in the rate of drying during the process, (*d*) change in the position of the plate while drying, (*e*) hardener. Emulsions on plate-glass were also tried. Jewell's developer was used, and the plates were $3\frac{1}{4} \times 4\frac{1}{4}$ inches (83×108 mm) in size, the same as are used with the Crossley Reflector.

FIRST SERIES—EFFECT OF POSITION OF THE PLATE

The first point considered was the effect of the position of the plate during the photographic processes. For this purpose a transit-of-*Venus* reticle was copied by contact, on a series of six plates. A Rochester (ordinary oil) lamp at a considerable distance was used as the source of light, and the exposures were made as nearly alike as possible. This gave images of the reseau-lines clear on a darkened background. The plates were Seed's Gilt Edge No. 27, ordinary

[1] *Observatory*, **27**, 396, 1904.

emulsion. Plates 1, 2, and 3 were kept horizontal (film side up) during development, fixing, washing and drying, and Plates 4, 5, and 6 vertical. Plate 5 was overexposed, and was therefore not used. The remaining five plates were measured on the Stackpole measuring-engine in orientations 0° and 180°. Seventeen intersections of the reseau-lines distributed over an area $3\frac{1}{2} \times 2$ inches were measured on each plate, four settings being made on each intersection. The readings were made to the nearest 0.0001 of an inch, the fourth decimal place being estimated, and in the reductions the means were carried one decimal place farther. In the reduction of Plates 2, 3, 4, and 6 to Plate 1, one of the intersections was used as a center, and the corrections were applied for scale-value, orientation, and center. Table I, in which the letters represent intersections, gives the residuals in units of 0.00001 inch.

TABLE I

SERIES OF PLATES FOR VERTICAL AND HORIZONTAL POSITIONS OF PLATE. RESIDUALS IN UNITS OF 0.00001 INCH (0.00025mm)

Number of Plate	a_x	a_y	b_x	b_y	c_x	c_y	d_x	d_y	e_x	e_y
2	−7	+6	−6	+28	+2	+17	+4	−13	−5	+1
3	−2	±0	−1	+2	−4	+5	+7	+8	−1	−11
4	−6	±0	+7	±0	+8	−16	−5	+7	−25	+2
6	+13	−4	+26	−1	−6	−5	−7	−3	+8	+6

Number of Plate	f_x	f_y	g_x	g_y	h_x	h_y	i_x	i_y	j_x	j_y
2	+15	−2	−10	−23	+4	−8	+8	+8	−4	+23
3	−7	−1	+5	−5	+8	−3	±0	+2	+1	−6
4	−10	+4	−37	±0	−5	+3	−6	−8	+2	−23
6	+13	+22	+4	+4	−8	+9	−4	−4	−37	+8

Number of Plate	k_x	k_y	l_x	l_y	m_x	m_y	n_x	n_y	o_x	o_y
2	−6	−6	+5	−14	−3	−1	+48	+5	−8	+8
3	+12	+6	+12	−1	−7	+8	−6	+6	+4	−10
4	−4	−4	−11	+46	−7	−4	+12	+2	−1	+16
6	−3	+5	−5	+16	+16	−3	−7	−11	+3	−15

Number of Plate	p_x	p_y	x Center	y
2	−15	−8	+3	+2
3	+6	+20	−4	+6
4	−18	+3	−2	−1
6	−8	−14	−2	−7

In order to obtain values for the mean and greatest errors of measurement, Plate I was measured by two observers, the plate having been removed from the engine in the interval. The mean difference between the two observers for thirty-four co-ordinates is 6.5 and the greatest difference 19, in units of the fifth decimal place. Fifteen of the differences were negative, fifteen positive, and four zero. $[-v]=106$, $[+v]=114$. These differences include, in addition to errors of measurement (of bisection, reading of the scales, and division errors of the scales), differences in personality between the two observers, if such differences exist. It was therefore thought allowable to consider 19 as the greatest error due to measurement alone. Accordingly all residuals over 20 in magnitude (in bold type in Table I) were regarded as being larger than the error of measurement. Omitting these large residuals, the resulting mean residual is ± 6.6. The agreement of this value with the mean error of measurement also tends to justify this course.

The conclusions to be drawn from this series of plates are as follows: It was possible to superimpose the seventeen points measured on Plates 2, 3, 4, and 6 upon the corresponding points of Plate 1, within the errors of measurement, except for occasional large accidental deviations of individual points. Plates 1, 2, and 3 were kept horizontal, and 4 and 6 vertical, during the photographic processes. Therefore, for the size of plates used ($3\frac{1}{4}\times 4\frac{1}{4}$ inches) and the region measured ($2\times 3\frac{1}{2}$ inches) it is entirely indifferent whether the plate be vertical or horizontal during the photographic treatment. A re-examination of the intersections for which large residuals were obtained showed in a few cases that the shift was due to the arrangement of the silver grains rather than to an actual movement of a portion of the film. In other cases the intersection appeared to be perfectly normal.

SERIES SECOND

The general plan for the second series of plates was as follows:[1]

A. Ordinary commercial plates (Seed 27, Gilt Edge).

I. Uniform rate of drying.

[1] As it was the principal object of this investigation to acquire a more definite knowledge of the nature and magnitude of the distortions that are to be expected under ordinary conditions in the manipulation of photographic plates, extreme conditions (according to the above outline) were avoided.

1. Rapid drying.
   (*a*) By heat.
   (*b*) By draft.
2. Slow drying (in a cool place without draft).
3. When partly dry turn plate on opposite edge and allow drying to continue at the same rate.

II. Change in the rate of drying.
1. Without turning plate over.
2. Turning plate over when changing the rate of drying.

III. Hardener.

B. Same emulsions on plate-glass.

A form, from which artificial-star plates could be printed by contact, was made in the following way. An 8×10-inch clear glass plate was spattered with small drops of drawing-ink. From these, sixteen groups of spots well distributed over the plate, with a total of forty-six images of suitable sizes, were selected, and all the remaining spots rubbed off. This pattern was then photographed on a 5×7-inch transparency plate, the resulting image of the pattern plate being approximately $3\frac{1}{4}\times4\frac{1}{4}$ inches in size. This positive was used as a form with which all the exposures of artifical-star images were made by contact printing. Twenty plates of the same emulsion (Seed 27) were exposed consecutively and as nearly alike as possible, five exposures of seven seconds' duration being made on each plate. The temperature of the room during the exposures was 54° to 57° F. Between exposures the plate was shifted (parallel to the X and Y directions) so that the corresponding images of four of the exposures were at the corners of a square approximately half an inch on each side, and the image of the fifth exposure was at the center of the square. Each plate therefore contained five exposures of the same sixteen groups (forty-six images) of artificial star-disks. The following designations were used: The groups are lettered from a to p inclusive. The images are represented by the letter of the group with a subscript. Usually the subscript numbers were assigned in the order of increasing size of disk. The exposures are lettered α to ϵ, arranged as follows: starting around the square in the X-direction from α, the remaining corners are represented by β, ϵ, and δ; and γ is at the intersection of the two diagonals.

These twenty plates were treated differently in the photographic

TABLE II

Plate No.	Development		Fixing			Washing—(In flowing water)			Drying—(All Vertical)				Remarks
	Time	Temp.	Kind	Time	Temp.	Position	Time	Temp.	Cutedge	Time	Temp.		
	min.			min.			hrs.			hrs.			
1	8	21–17°C.	Plain hypo. with sodium sulphite to prevent oxidation	30	11°C.	Horizontal. Did not rub.	3	9°±C.	down over	4	63° F.		Almost dry when turned over.
2	8	21–17		30	11	Horizontal. Did not rub.	3	9	down over	4	63		
3	8	21–17		30	11	Horizontal. Did not rub.	3	9	up over	4	63		
4	8	21–17		30	11	Horizontal. Did not rub.	3	9	up over	4	63		
5	8	21–17		30	12	Horizontal. Rubbed lightly.	2½	9	down over	6 1±	52 75+	In dark room In assembly room	2/5 dry when turned over.
6	8	21–17		30	12	Horizontal. Rubbed lightly.	2½	9	down over	6 1±	52 75+	In dark room In assembly room	2/5 dry when turned over.
7	8	21–17		30	12	Horizontal. Rubbed lightly.	2½	9	down over	6 1±	52 75	In dark room In assembly room	½ dry when turned over
8	8	21–17		30	12	Horizontal. Rubbed lightly.	2½	9	down over	6 1±	52 75+	In dark room In assembly room	¼ dry when turned over.
9	8	20–16		30	13	Horizontal. Did not rub.	3	9	down	1½	75+	On desk near stove	
10	8	20–16		30	13	Horizontal. Did not rub.	3	9	up	4±	63		Alum bath after washing.
11	8	20–16		30	13	Horizontal. Did not rub.	3	9	down	4±	63		Alum bath after washing.
12	8	20–16		30	13	Horizontal. Did not rub.	3	9	down	4±	63		Alum bath after washing.
13	8	20–16		20	13½	Vertical. Did not rub.	4	9	down	¾	67	Fanned till dry	One half fixed much slower. Cut in film.
14	8	20–16		20	13½	Vertical. Did not rub.	4	9	down	1½	75+	Near stove	
15	8	20–16		20	13½	Vertical. Did not rub.	4	9	up	20±	50–52	Without draft	
16	8	20–16		20	13½	Vertical. Did not rub	4	9	down	20±	50–52	Without draft	
17	8	20–16	Cramer's chrome-alum solution	75	13½	Vertical. Rubbed.	4	9	up	20±	50–52	Without draft	
18	8	20–16		75	13½	Vertical. Rubbed.	4	9	down	20±	50–52	Without draft	
19	8	20–16		75	13½	Vertical. Rubbed.	4	9	down	¾	67	Fanned till dry	
20	8	20–16		75	13½	Vertical. Rubbed.	4	9	down	1½±	75	Near stove	

processes, and constitute the material for the study of Part *A* of the the general plan as outlined above. Table II contains the principal data in regard to their treatment.

Of these plates Nos. 20, 13, and 6 were selected for measurement. They represent a variety of different conditions in the treatment. Plate 20 was fixed in Cramer's chrome-alum fixing solution; Plates 6 and 13 were fixed without the use of any hardener; one-half of Plate 13 fixed clear very much slower than the other half; Plates 13 and 20 were washed in the vertical position, Plate 6 in the horizontal position; Plates 6 and 20 were rubbed lightly with the fingers at the beginning and end of washing; Plate 20 was dried fairly rapidly in a warm room; Plate 13 was fanned briskly until dry; when Plate 6 was partly dry it was turned over and the temperature of drying changed from 52° to 75° F. The remaining plates were kept in reserve, to be measured later if it should be deemed advisable. Exposure γ on Plate 20 was taken as the standard to which the exposures on the same and other plates were reduced. Table III contains the measures of Plate 20γ. The reduction consisted in computing by least squares for each exposure the orientation and center corrections, and then applying these corrections to the individual images. The first half of Table IV gives the residuals (reduced readings on the images of the exposure under consideration minus the readings on Plates 20γ) in units of 0.00001 inch. Column 8 contains the means of columns 2 to 7 inclusive, and may therefore be looked upon as being the errors—both of measurement and of possible distortions—contained in exposure 20γ. Columns 9 to 14 inclusive give the residuals of columns 2 to 7 free from the errors in the standard exposure. Columns 8 to 14 give, therefore, the errors, both of measurement and distortion, for all the exposures for which the measures were reduced, including the standard exposure. Column 8 is in all respects similar to columns 9 to 14.

These residuals do not exhibit any evidence of general distortions. They show that it is possible to superimpose the forty-six images of any one of the exposures upon the corresponding forty-six images of the standard exposure within the errors of measurement, except for occasional large deviations of individual images. In the reductions the only corrections applied to the measures were for orientation

TABLE III

MEASURES OF EXPOSURE 20γ. STANDARD

Image	X	Y	Image	X	Y
	Inches	Inches			
a_1	+1.37220	+0.63252	i_1	+0.00312	+0.02350
a_2	+1.40292	+0.65730	i_2	+0.02148	+0.00042
a_3	+1.37908	+0.68480	j_1	+1.22858	−0.39250
b_1	+0.79238	+1.13332	j_2	+1.17052	−0.35050
b_2	+0.75240	+1.13010	k_1	−0.76720	−0.61312
b_3	+0.69948	+1.11008	k_2	−0.78495	−0.63310
c_1	+0.04840	+0.89590	k_3	−0.81718	−0.56308
c_2	+0.02368	+0.84462	l_1	−1.34562	−0.51180
c_3	−0.03320	+0.93892	l_2	−1.33685	−0.58785
d_1	−1.07100	+0.94532	m_1	+1.31190	−1.10922
d_2	−1.03768	+0.99265	m_2	+1.29708	−1.10702
d_3	−1.00698	+0.97450	m_3	+1.32748	−1.15560
d_4	−0.94438	+1.09078	m_4	+1.29845	−1.17382
e_1	−1.70625	+1.08228	n_1	+0.07812	−0.83482
e_2	−1.72230	+1.04370	n_2	+0.04072	−0.82970
e_3	−1.78522	+0.95872	n_3	+0.03955	−0.88841
f_1	+0.68900	+0.37985	o_1	−1.45818	−1.05158
f_2	+0.74265	+0.38090	o_2	−1.49360	−1.03122
f_3	+0.73288	+0.36008	o_3	−1.49638	−1.06692
g_1	−0.63635	+0.33485	o_4	−1.48262	−1.05298
g_2	−0.70440	+0.23180	p_1	−1.69795	−0.51146
h_1	−1.52612	+0.34778			
h_2	−1.44290	+0.33450			
h_3	−1.43722	+0.38050			
h_4	−1.39555	+0.30578			

and center. No scale-correction was needed; i. e., the actual linear distances between the images remained the same within the area covered by the images (3.7×2.8 inches). Several of the images were within ⅛ inch (3 mm) of the edge of the plate.

A word of explanation is desirable in regard to an apparent systematic trend of the residuals over limited regions of some of the exposures. For example: Near the end of column 13γ eleven residuals of both X and Y are negative; the first few in column 6γ are positive; the first few of column 20β are negative. On Plate 6 the effect cannot be due to the sudden change in the rate of drying, because part of the affected region was dry before the change was made, while another part was the last to become dry. Although these residuals are in magnitude within the limits of the errors of measurement, they might be mistaken for minute general distortions. Without much doubt, however, their true origin may be found in the varying conditions of illumination and temperature during measurement. The measures

were made during the winter months when the weather conditions were unsettled, and alternate passing of clouds and clear sky produced marked changes in the illumination of the plate. In some cases the effect could be directly traced to this cause. Besides, the large number of images (230) on each plate rendered it impossible to complete the measures in both orientations on the same day. Where a small number of images is to be measured on a plate, constant conditions can usually be secured during the measures.

For Part B of the second series an 8×10-inch plate-glass plate, coated with a film of Seed's 27 ordinary emulsion, was used. In each corner of the plate five exposures were made in the same manner as on Plates 1 to 20. The plate was developed, fixed, washed, and dried in the ordinary way, and then cut into four parts. These four parts were numbered from 21 to 24, inclusive. The γ exposure of Plate 21 was measured and reduced to 20γ, by applying corrections only for orientation and center. It was here also found possible to superimpose the forty-six images of 21γ upon the corresponding images of 20γ within the errors of measurement, except for a few individual larger residuals. In so far as we are justified in drawing conclusions from this one plate, we may say that in the matter of distortions of the film plate-glass offers no advantages over the ordinary commercial plates.

Through this one plate-glass plate a partial connection is also made between $3\frac{1}{4}\times 4\frac{1}{4}$-inch and 8×10-inch plates. The absence of any evidence of general distortions on Plate 21 would tend to show that the greater extent of the film on the larger plate does not introduce a sagging of the film due to its greater total weight (i. e., a sort of cumulative effect).

A large number of spectrograms of high and low dispersion were examined for evidences of distortion, and the results obtained are entirely in accord with the conclusions drawn from the investigation of the artificial-star plates. No cases of general distortions were found, i. e., none of the kind with which this investigation is concerned.[1] A considerable number of local distortions were found. These were confined in each case to one or a few adjacent spectrum

[1] Occasionally a plate will show blisters which are visible to the unaided eye These blisters are caused by the omission of some necessary precaution in the treatment of the plate, usually by too high a temperature of one of the solutions or by too great a difference in the temperatures of the various solutions. As the gelatine (before it is treated with a hardener) dissolves at 90° F., the temperature of the solutions should

RESIDUALS: IMAGE ON

Image	Plate 20 a X	Plate 20 a Y
a_1 ...	± 0	−10
a_2 ...	+34	−13
a_3 ...	+18	− 6
b_1 ...	+23	+ 8
b_2 ...	+11	−35
b_3 ...	+27	− 1
c_1	+28	−19
c_2	+ 2	+ 1
c_3	+ 1	− 5
d_1 ...	+ 6	−23
d_2 ...	+15	−13
d_3 ...	+18	− 2
d_4 ...	+14	−12
e_1	−25	+39
e_2	−32	+15
e_3	−25	−46
f_1	+21	+ 1
f_2	+19	+23
f_3	+10	+56
g_1 ...	+11	−41
g_2 ...	+11	−13
h_1 ...	− 4	−11
h_2 ...	+ 4	− 7
h_3 ...	± 0	− 2
h_4 ...	+ 6	+ 1
i_1	−14	+ 3
i_2 ...	+ 4	− 2
j_1	+21	+10
j_2	+11	± 0
k_1 ...	−13	−12
k_2 ...	− 1	− 3
k_3 ...	+13	+ 2
l_1	− 5	+ 1
l_2	− 1	−17
m_1 ...	+10	−21
m_2 ...	+13	− 1
m_3 ...	+25	± 0
m_4 ...	+21	+ 5
n_1 ...	+16	− 2
n_2 ...	+12	+ 7
n_3 ...	+15	+ 8
o_1 ...	− 4	−21
o_2 ...	−16	− 7
o_3 ...	−30	+10
o_4 ...	− 2	+ 8
p_1 ...	− 9	−16

lines, and very rarely covered an area as much as $\frac{1}{4}$ mm square. The greatest displacement observed amounted to 0.02 mm, which on the Crossley star-photographs is equivalent to about one second of arc, and on the spectrograms taken with the one-prism spectrograph of the Lick Observatory to about 80 km radial velocity per second (for the $H\gamma$ region). Such large displacements as this are, however, extremely rare. The great majority of the displacements observed are less than one-fourth of this amount.

Some of the characteristic shapes of the distorted lines are: a sine-curve, a question-mark, an irregular crinkle, an abrupt bend, a gradual bend which is sometimes shared by two and occasionally by three adjacent spectrum lines, while the lines immediately on either side of the distorted ones are perfectly straight.

The distortions seem to be principally of two different kinds. In one kind, aside from the bending or twisting, the appearance of the line is normal as regards width, sharpness of the sides, and structure of the grain. In this case the most plausible explanation for the distortion is an actual movement of a minute portion of the film. In the other kind the distorted shape of the line seemed to be due in some cases to the peculiar arrangement of the silver grains, and in others to local differences in the sensitiveness of the film. The occasional non-uniformity in the sensitiveness of the film is usually very local, but at times it extends over a considerable portion of the plate. Probably an extreme case of the latter was shown on one of my lantern slides, on which a non-sensitive streak, varying in width from 0.1 to 0.05 inch (2.5 to 1.25 mm), extends more than half-way across the plate. Very rarely also one of the distortions would have the appearance of having been caused by slight movements of a minute area of the film immediately around a grain of impurity.

SUMMARY OF RESULTS

1. For the size of the plates used ($3\frac{1}{4}\times4\frac{1}{4}$ inches) it was found to be entirely indifferent whether the plate be vertical or horizontal during development, fixing, washing, and drying.

2. Within the range of the observations, hardener, the rate of drying, and changes in the rate of drying and in the position of the

not be allowed to become higher than 75° F. The distortions which were considered in this investigation were those which occur when ordinary (good) care is taken in the treatment of the plate.

plate during the process of drying introduced no general distortions of the gelatine film.

3. Local distortions were found on artificial-star plates and on spectrograms. These distortions were confined in each case to an area equal to a small fraction of a square millimeter. The largest lateral displacement found at any point in the distorted area was 0.02 mm, while the great majority were less than one-fourth of this amount. Some of these displacements are several times as large as the errors of measurement, and their possible effects must be taken into account where great accuracy is desired.

4. These distortions seem to be principally of two different kinds: one was due to an actual movement of a minute portion of the film, the other was an apparent shift of the image due to the peculiar arrangement of the silver grains or to local differences in the sensitiveness of the film.

5. The results obtained from one plate-glass plate showed no advantages of the plate-glass over the ordinary commercial plates in the matter of distortions of the film.

6. If the results obtained in this investigation for small plates be found to apply with equal force to larger plates, it will follow that the assumption which is the basis for the use of the reseau is not well founded. The assumptions involved, briefly stated, are as follows: First, general distortions exist; second, they differ in different parts of the plate; third, they may be assumed to be linear within the squares of the reseau, i. e., over a stretch of 5 mm or more. The supposed advantages of the reseau over the method of referring all the measures to a common center rest entirely upon the validity of these three assumptions. If the reseau can be dispensed with, there will be a saving of the labor involved in making the large number of settings on the reseau-lines and in the reductions of the measurements.

Throughout these investigations on photographic distortions I had the valued advice and assistance of Dr. Perrine, who had long been interested in the problem, and it is a pleasure to acknowledge my indebtedness to him. It should also be said that fully half of the measures of the first series were made by him.

LICK OBSERVATORY
UNIVERSITY OF CALIFORNIA
May, 1907

REVIEWS

Atlas Stellarum Variabilium. Series IV, containing those variable stars which are included in the limits of the *B. D.* charts in declination and magnitude. By J. G. HAGEN, S. J., Director of the Vatican Observatory. Berlin: Felix L. Dames, 1907. Two volumes, charts and catalogue. 4to. 100 Marks.

Hagen's great atlas of variable stars is now well known and quite indispensable to the observer in this field. The present series completes the original plan, though extensions are projected by the author and in part ready for the press. This series is the largest of all, containing 100 fields, the variables being adapted for observation with apertures of 8 to 16 cm This series is therefore the most useful one to possessors of small telescopes.

The charts resemble those in Series I, II, and III, in that they are all on the same scale and that each contains one variable at the center; but differ from them in several particulars. The inner red square is one degree on a side, therefore on half the scale of the previous series. This square shows all the *B. D.* stars and such fainter ones, down to magnitude 11, as are needed for identification or for comparison stars when the variable is near minimum. The outer parts of the chart contain certain *B. D.* stars down to magnitude 8–9. In regard to the accuracy of the positions, the introduction states that the brighter stars are taken from the best catalogues, chiefly those of the *Astronomische Gesellschaft.* The fainter stars were located, partly by original observations, partly from Harvard photographs, and some from the *B. D.* catalogue. As a check, the reviewer compared the twenty stars in the inner square of the chart for *U Pegasi,* with a photo-

HAGEN NO.	ERRORS	
	R. A.	Dec.
2	.....	$-0\overset{\prime}{.}38$
6	$-1\overset{s}{.}2$	......
10	-6.5	......
11	-1.6	......
14	-3.4	$+0.61$
15	$+2.2$	......
16	.....	$+0.30$
19	-4.3	......

graph. After allowing for the distortion in right ascension caused by the projection used, the co-ordinates of six stars were in error by more than one second of time. Errors greater than 0.'25 in declination were found for only three stars. For the twenty stars the mean errors were only $\pm 11^s$ and $\pm 0.'1$, respectively.

The catalogue sheets give in the usual form the data needed for the reduction of the observations. Estimates of the brightness of the comparison stars were made in grades by Hagen and Hisgen, but the method of reduction to magnitudes is peculiar to this series, being obtained by a graphic process, based on photometric magnitudes furnished by Professor Pickering for a part of the stars in each field. These Harvard magnitudes are given in the fourth column of the catalogue sheets. They are mostly from unpublished observations, and therefore cannot, at present, be reduced to the system of either of the Harvard catalogues.[1] The column of notes gives, for the brighter stars, the Potsdam colors and magnitudes rounded to the nearest 0.1.

This division of the *Atlas* makes a fitting introduction to a new series of publications from the Vatican Observatory, and places the astronomical world under renewed obligations to the author.

J. A. PARKHURST

YERKES OBSERVATORY
May, 1907

FABRY AND BUISSON'S *Wave-Lengths of Standard Lines.*

In accordance with resolutions adopted at the Oxford meeting of the Solar Union in 1905, Fabry and Buisson[2] have recently measured the wave-lengths of eighty-four standard lines in the region of the spectrum comprised between λλ 6500 and 3600. The green line of mercury, produced by a Cooper-Hewitt lamp, was chosen as the primary standard. The value of this line was determined by careful comparisons with the red and green lines of cadmium measured by Michelson and Benoit. From this line were then deduced the values of the other standard lines produced by the passage of a current of from 6 to 3 amperes at a voltage of 120 between iron electrodes 8 mm in diameter, the method of comparison being that hitherto used by Perot and Fabry, but photographic instead of visual. The interferences (fringes at infinity) were caused by two plane-parallel silver-on-glass mirrors. The spectrum was produced by a plane-grating used with auto-collimation.

[1] See Müller and Kempf, *Potsdam Photometrische Durchmusterung, General Katalog*, p. xxii.

[2] *Comptes Rendus*, 143, 165, 1906.

The values given below are for wave-lengths in air at 15° C. and at a pressure of 760 mm; and each is the mean of several measures on different plates. The discordance between independent measures rarely amounted to one part in a million. One of the lines belongs to manganese, which is present as an impurity in commercial iron. In the region about λ 5800 it was necessary to supplement the iron lines with four nickel lines, in order that the interval between any two standard lines should not exceed 50 Å. U. The wave-lengths of Rowland and of Kayser and Runge may be reduced to the same scale by dividing by 1.00003.

3606.687	4375.939	5012.075	5658.837
3640.396	4427.318	5049.829	5709.398
3677.634	4466.558	5083.346	*Ni* 5760.845
3724.385	4494.576	5110.418	5763.015
3753.620	4531.159	5127.367	*Ni* 5805.213
3805.351	4547.857	5167.495	*Ni* 5857.761
3843.266	4592.661	5192.364	*Ni* 5892.883
3865.531	4602.947	5232.960	5934.685
3906.486	4647.439	5266.570	5952.740
3935.823	4678.858	5302.319	6003.040
3977.750	4707.290	5324.198	6027.060
4021.877	4736.788	5371.500	6065.494
4076.645	4753.147	5405.782	6137.701
4118.556	4789.660	5434.532	6191.570
4134.689	*Mn* 4823.524	5455.618	6230.733
4147.682	4859.759	5497.523	6265.148
4191.446	4878.229	5506.785	6318.030
4233.619	4903.327	5535.420	6335.344
4282.411	4919.009	5569.634	6393.613
4315.093	4966.107	5586.772	6430.860
4352.745	5001.883	5615.660	6494.994

S. B. B.

A GENERAL INDEX TO THE ASTROPHYSICAL JOURNAL

The preparation of an index to the first twenty-five volumes of this Journal, covering the twelve and one-half years from January, 1895, to June, 1907, is now under consideration. Such an index would doubtless prove of great convenience to the workers in astrophysics and to libraries. The possibility of its publication will depend upon the number of advance orders received. If 200 subscriptions are obtained, the index can probably be issued at a cost of about $1.50; if 300 advance orders should be given, the work will certainly be undertaken, with the expectation of its publication in the autumn of 1907, and the price will probably be somewhat less than $1.50.

All subscribers and librarians who would purchase such an index, if issued, are therefore requested to notify the editors at once by postcard of the number of copies for which they will subscribe.

Address, Editors of the Astrophysical Journal, Williams Bay, Wisconsin, U. S. A.

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SUBJECTS

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AUTHORS